A Modified Particle Swarm Optimization for Optimal Power Flow

Jong-Yul Kim†, Hwa-Seok Lee* and June-Ho Park**

Abstract – The optimal power flow (OPF) problem was introduced by Carpentier in 1962 as a network constrained economic dispatch problem. Since then, it has been intensively studied and widely used in power system operation and planning. In the past few decades, many stochastic optimization methods such as Genetic Algorithm (GA), Evolutionary Programming (EP), and Particle Swarm Optimization (PSO) have been applied to solve the OPF problem. In particular, PSO is a newly proposed population based stochastic optimization algorithm. The main idea behind it is based on the food-searching behavior of birds and fish. Compared with other stochastic optimization methods, PSO has comparable or even superior search performance for some hard optimization problems in real power systems. Nowadays, some modifications such as breeding and selection operators are considered to make the PSO superior and robust. In this paper, we propose the Modified PSO (MPSO), in which the mutation operator of GA is incorporated into the conventional PSO to improve the search performance. To verify the optimal solution searching ability, the proposed approach has been evaluated on an IEEE 30-bus test system. The results showed that performance of the proposed approach is better than that of the standard PSO.

Keywords : Mutation Operator, Optimal Power Flow, PSO Algorithm

1. Introduction

Optimal Power Flow (OPF) is a useful tool in planning and operation of a power system [1]. The OPF problem can be described as the optimal allocation of power system controls to satisfy the specific objective function such as fuel cost, power loss, and bus voltage deviation. The control variables include the generator real powers, the generator bus voltages, the tap ratios of transformer and the reactive power generations of VAR sources.

Therefore, the OPF problem is a large-scale highly constrained nonlinear non-convex optimization problem [1]. To solve it, a number of conventional optimization techniques such as nonlinear programming (NLP) [2], quadratic programming (QP) [3], linear programming (LP) [4], and interior point methods [5] have been applied. All of these mathematical methods are fundamentally based on the convexity of objective function to find the global minimum. However, the OPF problem has the characteristics of high nonlinearity and nonconvexivity. Therefore, conventional methods based on mathematical technique cannot give a guarantee to find the global optimum. In addition, the performance of these traditional approaches also depends on the starting points and is likely to converge to local minimum or even diverge. Recently, many attempts to overcome the limitations of the mathematical programming approaches have been investigated such as Genetic Algorithm (GA), Evolutionary Programming (EP), and Evolution Strategies (ES). Their applications to global optimization problems become attractive because they have better global search abilities over conventional optimization algorithms. The OPF problem has been solved with Evolutionary Programming (EP) [6]. The proposed EP based OPF were evaluated on an IEEE 30-bus system and the obtained results were compared with those obtained using a conventional gradient-based method. An enhanced GA with adaptive crossover and mutation based on the fitness statistics of population was applied to minimize the active power loss in the transmission network [7]. Recently, Bakirtzis et al. applied an enhanced GA to solve the OPF problem [8].

Particle Swarm Optimizer (PSO) is a newly proposed population based stochastic optimization algorithm. The main idea is based on the food-searching behavior of birds and fish [9]. Compared with other stochastic optimization methods, PSO has comparable or even superior search performance for some hard optimization problems in real power systems. In [10, 11] however, some modifications such as breeding and selection operators are considered to make the PSO superior and robust. In this paper, the mutation operator of the GA is incorporated into the

† Corresponding Author:Power Conversion & System for RES Group, Korea Electrotechnology Research Institute, Changwon, 641-120, Korea (jykim@keri.re.kr)
* Koje College, Koje, Korea (hslee@koje.ac.kr)
** Pusan National University, Busan, Korea (parkjh@pusan.ac.kr)
Received 9 October 2006 ; Accepted 30 June 2007

conventional PSO, so it can produce superior performance. To verify the optimal solution search ability, the proposed approach is applied to the fuel cost minimization problem of an IEEE 30-bus power system.

2. Particle Swarm Optimization

2.1 Standard PSO Algorithm [12]

The PSO is a population based optimization method first proposed by Kennedy and Eberhart [9]. PSO technique finds the optimal solution using a population of particles. Each particle represents a candidate solution to the problem. PSO is basically developed through the simulation of bird flocking in a two-dimensional space. Some of the attractive features of the PSO include ease of implementation and the fact that no gradient information is required.

Suppose we have to find out the global minimum of multi-modal function $f(x) = f(x_1, x_2 \cdots x_n)$ in n-dimensional space. In PSO, each particle i (i=1,⋯,N) in the population p is characterized by three vectors (x_i, v_i, p_i) which represent their temporal position x_i $= (x_{i1}, x_{i2}, \cdots x_{in})$, velocity $v_i = (v_{i1}, v_{i2}, \cdots v_{in})$, and the best position $p_i = (p_{i1}, p_{i2}, \cdots p_{in})$. The fitness of each particle is given by the function value $f(x_i)$.

Since we look at the minimization problem in this report, the lower the function value the better the fitness. Each particle stores its best position P_i called personal best, p-best, which gives the best fitness in memory. They can also consult their neighbor's best position. Most simply, the neighbor is the whole population (fully connected topology), and therefore, the neighbor's best is the best position among personal bests of the whole population. Hence, the position P_g is called global best. Now each particle i moves around the search space, and renews its velocity component j using its past experience (personal best) and the population's experience (global best) as follows,

$$v_{i,j} = v_{i,j} + c_1 r_1 (p_{i,j} - x_{i,j}) + c_2 r_2 (p_{g,j} - x_{i,j}) \quad (1)$$

The parameter c_1 and c_2 are the acceleration constant, and r_1 and r_2 are the uniform random numbers within the range [0, 1].

If $v_{i,j}$ is larger than a predefined velocity $v_{\max}$ called maximum velocity, it is set to $v_{\max}$. Similarly, if it is smaller than $-v_{\max}$, it is fixed to $-v_{\max}$.

Then the particle changes its position by the "equation of motion":

$$x_{i,j} = x_{i,j} + v_{i,j} \quad (2)$$

To improve the performance, the inertia-weight was introduced by Eberhart and Shi [15] who added an inertia weight in updating the equation of standard PSO.

$$v_{i,j} = \omega v_{i,j} + c_1 r_1 (p_{i,j} - x_{i,j}) + c_2 r_2 (p_{g,j} - x_{i,j}) \quad (3)$$

The parameter ω is called inertia weight, which controls the exploration (global search)-exploitation (local search) tradeoff. When realizing the above equation, ω may be a constant factor or it may decrease linearly in a range or other appropriate form. Suitable selection of inertia weight can provide a balance between global exploration and local exploitation.

Shi and Eberhart [16] recommended a time varying inertia weight that linearly decreases with ω_{ini}= 0.9 at the initial step, iteration=0 and ω_{fin} = 0.4 at the final step, iteration= $MAX_{iteration}$:

$$\omega = (\omega_{ini} - \omega_{fin}) \times (MAX_{iteration} - Iteration) / MAX_{iteration} + \omega_{fin} \quad (4)$$

In [15], Clerc has also introduced a constriction factor K that has enhanced the particle's ability to control velocities. Therefore, Vmax or -Vmax is not necessary if the following equation is satisfied. Clerc has suggested 0.792 as a good settlement for K and in this case φ is equal to 4.1.

$$v_{i,j} = K(v_{i,j} + c_1 r_1 (p_{i,j} - x_{i,j}) + c_2 r_2 (p_{g,j} - x_{i,j})) \quad (5)$$

Now the constriction factor K is defined by using

$$K = \frac{2}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|} \quad (6)$$

$$\varphi = c_1 + c_2, \varphi > 4 \quad (7)$$

The comparison between inertia weight and constriction factor demonstrates that the latter generates better solutions in general [16]. So, in this paper, constriction factor is combined with standard PSO.

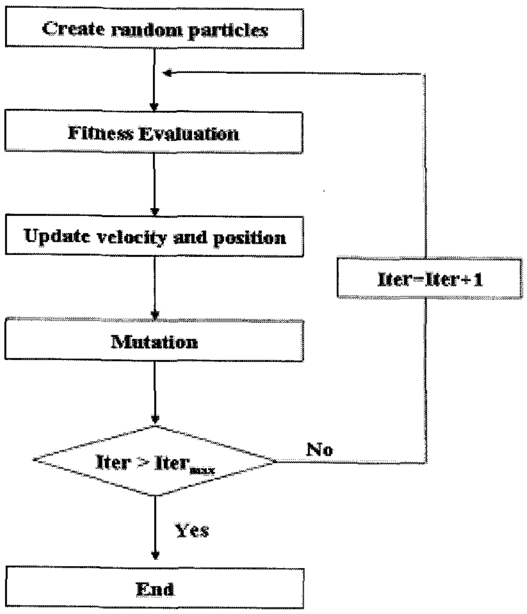


Fig. 1. Flowchart of PSO with mutation process

2.2 Modified PSO Algorithm

In recent research, some modifications to the standard PSO are proposed mainly to improve the convergence and to increase diversity. Angeline [10] showed that PSO can be improved by adding the selection process, which is similar to that of the GA. Through this process, the local search capability of PSO is enhanced and diversity is reduced, which is contradictory to the objective of natural selection. In [11], PSO is modified by adding a reproduction step to standard PSO, so we've called this "Breeding". In the PSO searching process, there is a possibility to converge on a local minimum in early stage. In breeding PSO, the parent's selection process doesn't depend on the fitness of particles, thereby preventing the best particles from dominating the breeding process, and preventing premature convergence. In this paper, the mutation process of the GA is applied to escape from local optimal point and search in different areas of the search space. In this paper, the mutation process is executed after the velocity and position update are calculated as shown in Fig. 1.

A more detailed procedure of the mutation process is as follows:

Step 1) To each particle, assign a randomly generated mutation probability P_r

Step 2) Compare the P_r with threshold probability P_m

Step 3) If $P_r < P_m$, generate the new particle by Equation (8) and replace the particle

$$Mutation(X_{i,j}) = random() \times (X_{\max,j} - X_{\min,j}) + X_{\min,j} \quad (8)$$

where random() is uniform random numbers within the range [0, 1].

Step 4) If $P_r > P_m$, go to Step 2)

For evaluating the performance of modified PSO, both the standard PSO and the modified PSO were tested on four benchmark minimization problems. These four functions have been commonly used in the performance evaluation of other evolutionary computation Algorithms such as GA, ES, EP.

(Dejong-4)

$$f_1(x_1,\bullet\bullet\bullet,x_{30}) = \sum_{i=1}^{30} ix_i^4, -512 \le x_i \le 512 \quad (9)$$

(Rastrigin)

$$f_2(x_1,\bullet\bullet\bullet,x_{20}) = 200 + \sum_{i=1}^{20} (x_i^2 - 10\cos(2\pi x_i)) \quad (10)$$

$$-5.12 \le x_i \le 5.12$$

(Colville)

$$\begin{aligned} f_3(x_1,\bullet\bullet\bullet,x_4) &= 100(x_2 - x_1^2) + (1-x_1)^2 + 90(x_4 - x_3^2) + (1-x_3)^2 \\ &+ 10.1((x_2-1)^2 + (x_4-1)^2) + 19.8(x_2-1)(x_4-1) \end{aligned} \quad (11)$$

$$-10 \le x_i \le 10$$

(Ackley)

$$f_4(x_1,\bullet\bullet\bullet,x_{30}) = -20\exp(-0.2\sqrt{\frac{1}{30}\sum_{i=1}^{30} x_i^2}) - \exp(\sqrt{\frac{1}{30}\sum_{i=1}^{30}\cos(2\pi x_i)}) \quad (12)$$

$$+20 + e, \quad -30 \le x_i \le 30$$

For each test function, 20 independent runs were carried out respectively. The PSO and modified PSO parameters were set to C_1=2.05, C_2=2.05, and K=0.792, which is the typical value of standard PSO. The mutation threshold probability was set to 0.05 in modified PSO. The 100 of population size and 2,000 of maximum iteration were used for all problems.

Tables 1 and 2 present the results of 20 runs. In the table, iteration means the average iteration of 20 runs which is satisfied with the convergence criteria of 10^{-3} and 10^{-5}. Time presents the required time to meet convergence criteria. The results shown in Tables 1 and 2 present that modified PSO provides better results in comparison with the standard PSO for all test functions. Especially, standard PSO cannot converge for the Rastrigin function, otherwise modified PSO convergence is successful.

Table 1. Results of 20 executions with convergence criteria of 10^{-3}

Function	Standard PSO		Modified PSO	
	Iteration	Time (sec)	Iteration	Time (sec)
DeJong-4	403	0.453	390	0.453
Rastrigin	-	-	1359	0.75
Colville	615	0.171	575	0.187
Ackley	404	0.468	364	0.4278

Table 2. Results of 20 executions with convergence criteria of 10^{-5}

Function	Standard PSO		Modified PSO	
	Iteration	Time (sec)	Iteration	Time (sec)
DeJong-4	471	0.515	457	0.515
Rastrigin	-	-	1520	0.828
Colville	1166	0.25	1066	0.312
Ackley	613	0.625	579	0.562

3. Optimal Power Flow Problem Formulation

The OPF problem can be formulated as a constrained optimization problem as follows:

$$Minimize \quad f(x,u) \tag{13}$$

$$s.t. \; g(x,u) = 0 \tag{14}$$

$$h(x,u) \le 0 \tag{15}$$

Where x is the state variable such as slack bus power, load bus voltage, generator reactive power, and so on. u is a set of controllable variables like generator real power outputs with the exception of the slack bus power output, generator voltages, transformer tap ratios, and reactive power generations of VAR sources.

In this paper, the objective function of OPF is minimization of fuel cost for all generators which can be formulated as Equation (16).

$$Min \quad F(P_g) = \sum_{i=1}^{N_g} (a_i + b_i P_{gi} + cP_{gi}^2) \tag{16}$$

Where $F(P_g)$ is the total fuel cost (\$/hr) of all generators; P_{gi} is the active power output generated by the i_{th} generator; a_i, b_i, c_i are fuel cost coefficients; and N_g is the total number of generators.

The equality constraints $g(x,u)$ are the nonlinear power flow equations which are formulated as follows:

$$P_{gi} - P_{di} - V_i \sum_{j=1}^{N_b} V_j Y_{ij} \cos(\theta_i - \theta_j - \varphi_{ij}) = 0 \quad i = 1,\cdots,N_g \tag{17}$$

$$Q_{gi} - Q_{di} - V_i \sum_{j=1}^{N_b} V_j Y_{ij} \sin(\theta_i - \theta_j - \varphi_{ij}) = 0 \quad i = 1,\cdots,N_g \tag{18}$$

Where P_{gi} and Q_{gi} are the active and reactive power generations at bus i; P_{di} and Q_{di} are the active and reactive power demands at bus i; V_i and V_j are the voltage magnitudes at buses i and j respectively; θ_i and θ_j are the voltage angles at buses i and j respectively; φ_{ij} is the admittance angle; Y_{ij} is the admittance magnitude; and N_b is the total number of buses.

The OPF inequality constraints, $h(x,u)$ represent limits of control variables and state variables. In this paper, the constraints for stable system operation and limits of control variables are considered.

The system operation constraints consist of the transmission line loadings, load bus voltages, reactive power generations of the generator, and active power generation of the slack generator. These variables should be within the set lower and upper limits.

$$S_i \le S_i^{Max} \quad i = 1,\cdots,N_l \tag{19}$$

$$V_{di\min} \le V_{di} \le V_{di\max} \quad i = 1,\cdots,N_b \tag{20}$$

$$Q_{gi\min} \le Q_{gi} \le Q_{gi\max} \quad i = 1,\cdots,N_g \tag{21}$$

$$P_{gs\min} \le P_{gs} \le P_{gs\max} \tag{22}$$

Concerning control variables, active power output and voltage of generators, transformers tap ratio, and shunt capacitors are restricted by lower and upper limits as follows:

$$P_{gi\min} \le P_{gi} \le P_{gi\max} \quad i = 1,\cdots,N_g - 1 \tag{23}$$

$$V_{gi\min} \le V_{gi} \le V_{gi\max} \quad i = 1,\cdots,N_g \tag{24}$$

$$t_{i\min} \leq t_i \leq t_{i\max} \qquad i = 1, \cdots, N_t \tag{25}$$

4. Application Results

The proposed modified PSO was tested on the IEEE 30-bus system as shown in Fig. 2 with quadratic generation cost curves for minimizing the total fuel cost. The IEEE 30 bus system consists of 30 buses and 41 branches. It also has a total of 15 control variables as follows: five unit active power outputs, six generator-bus voltage magnitudes, and four transformer-tap settings.

Table 3. gives details of the generator data and coefficients of quadratic generation cost curve.

Transformers are in-phase transformers with assumed tapping ranges of 10%. The lower voltage magnitude limits at all buses are 0.95pu, and the upper limits are 1.1pu.

In order to demonstrate the effectiveness of the proposed approach, standard GA and PSO were also evaluated and compared with modified PSO. The simulation parameters of each optimization algorithms are listed in Table 4.

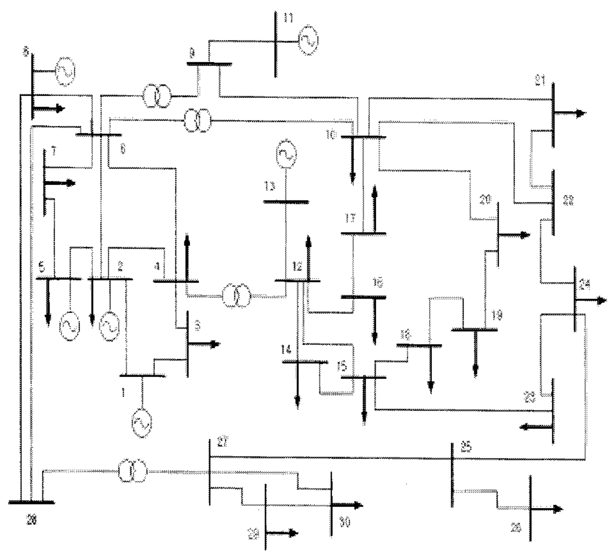


Fig. 2. IEEE 30 bus system

Table 3. Generator data and cost coefficients

Bus No.	$P_G^{\min}$ (MW)	$P_G^{\max}$ (MW)	$Q_G^{\min}$ (MVar)	$S_G^{\max}$ (MVA)	Cost coefficients a	b	c
1	50	200	-20	250	0.0	2.00	0.00375
2	20	80	-20	100	0.0	1.75	0.01750
5	15	50	-15	80	0.0	1.00	0.06250
8	10	35	-15	60	0.0	3.25	0.00834
11	10	30	-10	50	0.0	3.00	0.02500
13	12	40	-15	60	0.0	3.00	0.02500

Table 4. The simulation parameters of each algorithm

	GA	PSO	MPSO
Max Iteration	200	200	200
Population	50	50	50
Crossover rate	0.9	-	-
Mutation rate	0.1	-	0.005
C_1	-	2.05	2.05
C_2	-	2.05	2.05
K	-	2.974	2.974

Table 5. Tap ratio of transformers

No. of Transformer	1(4-12)	2(6-9)	3(6-10)	4(28-27)
Tap ratio	0.910	0.900	1.018	1.005

Table 6. Power output and voltage of generators

Bus Number	Voltage (p.u.)	P_G (MW)	Q_G (MVar)
1	1.099	178.12	2.41
2	1.078	48.61	5.48
5	1.046	21.16	22.13
8	1.053	21.15	19.77
11	1.054	12.27	24.34
13	1.053	10.90	32.33
Total	-	292.23	106.46

For all tested algorithms, the population size is taken equal to 50, and the maximum number of iterations is set to 200. For each algorithm, 20 independent runs were carried out. The tested algorithms were implemented in Visual C++ and executed on a Pentium 1.8 GHz machine.

The simulation results of OPF using the MPSO algorithm are represented in Table 5 and Table 6. Table V shows the optimal tap ratio of transformers and Table VI shows the power output and voltage of generators.

The best results of 20 test runs are tabulated in Table 7 in comparison to those obtained from other algorithms. It can be observed from Table 7 that the total cost obtained by the proposed MPSO is 799.58\$/h. With the same condition, the problem was solved by using GA and PSO with optimal costs of 802.12\$/h and 801.26\$/h, respectively. It is clearly indicated that the proposed MPSO outperforms the GA and standard PSO.

Table 7. Optimization results and comparison

Unit No	Bus No	GA	PSO	MPSO
1	1	176.13	179.35	178.12
2	2	48.21	49.01	48.61
3	5	20.68	19.95	21.16
4	8	22.84	20.04	21.15
5	11	11.94	12.88	12.27
6	13	13.13	11.55	10.90
Total PG(MW)		292.95	292.79	292.23
Total Cost($/hr)		802.12	801.26	799.58

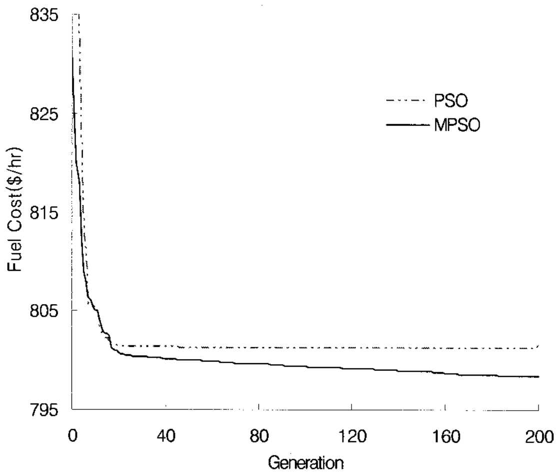


Fig. 3. Convergence of PSO and MPSO

Fig. 3 presents a graph that shows the improvement tendency of MPSO with an increase in the number of iterations. In this Figure, the standard PSO seems to be trapped in a local optimum point in early stage and cannot escape from this point until reaching the maximum iterations. However, the MPSO can escape the local optimum point by mutation activity, which can make the MPSO improve the solution continuously.

5. Conclusion

Particle Swarm Optimizer (PSO) is a newly proposed population based stochastic optimization algorithm. Compared with other stochastic optimization methods, PSO has comparable or even superior search performance for some hard optimization problems in real power systems. In recent research, some modifications to the standard PSO are proposed mainly to improve the convergence and to increase diversity. Many research findings indicate that PSO can be improved by the addition of a new operator such as selection and breeding processes. Such added operations can make the PSO more effective and robust in terms of searching optimal solution. In this paper, the MPSO with mutation process was proposed and applied to the OPF problem. By introducing the mutation process, MPSO prevents early convergence and provides better performance than the standard PSO.

Acknowledgements

This work has been supported in part by EERSI (R-2005-7-064), which is funded by MOCIE (Ministry of Commerce, Industry and Energy).

References

[1] J. A. Momoh, J. Z. Zhu, "Improved interior point method for OPF problems," IEEE Trans on Power System, vol. 14, pp.1114-1120, Aug. 1999.

[2] M. H. Bottero, E D. Caliana, and A. R. Fahmideh-Vojdani, "Economic dispatch using the reduced hessian," IEEE Trans on Power Appra. Syst., vol. 101, pp. 3679-3688, Oct. 1982.

[3] G. E Reid and L. Hasdorf, "Economic dispatch using quadratic programming," IEEE Trans on Power Appara. Syst., vol. 92, pp. 2015-2023, 1973.

[4] B. Stott and E. Hobson, "Power system security control calculation using linear programming," IEEE Trans on Power Appara. Syst., vol. 97, pp. 1713-1731, 1978.

[5] G. L. Torres and V. H. Quintana, "An interior pint nonlinear method for nonlinear optimal power flow using voltage rectangular coordinates," IEEE Trans on Power Syst., vol. 13, pp. 1211-1218, 1998.

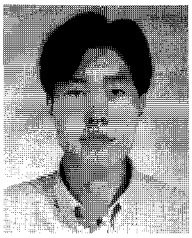
[6] Q. H. Wu and J. T. Ma, "Power system optimal reactive power dispatch using evolutionary programming," IEEE Trans on Power Syst., vol. 10, pp. 1243-1249, 1995.

[7] Q. H. Wu, Y. 1. Cao, and J. Y. Wen, "Optimal reactive power dispatch using an adaptive genetic algorithm," Electrical Power and Energy System, vol. 20, pp. 563-569, 1998.

[8] A. G. Bakirtzis, P. N. Biskas, C. E. Zoumas, and V. Pehdis, "Optimal power flow by enhanced genetic algorithm," IEEE Trans on Power Syst., vol. 17, pp. 229-236, MAY 2002.

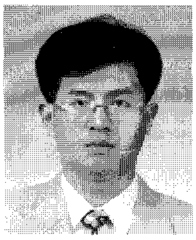
[9] J. Kennedy and R. C. Eberhart, "Particle swarm optimization," IEEE international Conference on Neural Networks, vol. 4, pp. 1942-1948, 1995.

[10] Angeline P. J, "Evolutionary Optimization Versus Particle Swarm Optimization: Philosophy and Performance Difference," The 7th Annual Conference on Evolutionary Programming, pp. 601-610, San Diego, 1998.
[11] Lovbjerg M, Rasmussen T. K., Krink T, "Hybrid Particle Swarm Optimizer with Breeding and Subpopulations," Proceedings of the Genetic and Evolutionary Computation Conference, pp. 469-476, Morgan Kaufmann, 2001.
[12] Masao Iwamatsu, "Locating All the Global Minimum Using Multi-Species Particle Swarm Optimizer: The Inertia Weight and The Constriction Factor Variants,' Proceedings of the Evolutionary Computation Conference, pp. 816-822, Vancouver, 2006.
[13] Y. Shi and R. C. Eberhart, "A modified particle swarm optimizer," Proceedings of the IEEE International Conference on Evolutionary Computation, pp. 69-73, Anchorage, Alaska, 1998.
[14] Y. Shi and R. C. Eberhart, "Parameter selection in particle swarm optimization," in Evolutionary Programming V, Lecture Notes in Computer Science, vol. 1498, Edited by V.S. Porto et al., pp. 591–600, Berlin, 1998.
[15] Clerc M, "The Swarm and the Queen: Towards a Deterministic and Adaptive Particle Swarm Optimization," Proceedings of the Congress of Evolutionary Computation, pp. 1951-1957, Washington, July 1999.
[16] R. C. Eberhart and Y. Shi, "Comparing Inertia Weights and Constriction Factors in Particle Swarm Optimization," Proceedings of the IEEE Conference on Evolutionary Computation, pp. 84-88, San Diego, July 2000.
[17] O. Alsac and B. Scott, "Optimal load flow with steady state security," IEEE Trans on Power Appara. Syst., vol. 93, pp. 745-751, May-June 1974.



Jong–Yul Kim

He received his B.S and M.S degrees in Electrical Engineering from Pusan National University and is currently employed by the Korea Electrotechnology Research Institute (KERI). His research interests are power system analysis, RES system, and AI to power systems.



Hwa-Seok Lee

He received his B.S., M.S. and Ph.D. degrees in Electrical Engineering from Pusan National University in 1991, 1993, and 1997, respectively. He is currently an Associate Professor at Koje College. His research interests include power quality, intelligent systems, and distribution automation.

June-Ho Park

He received his B.S., M.S. and Ph.D. degrees from Seoul National University of Korea in 1978, 1980, 1987, respectively, all in Electrical Engineering. He is currently a Professor at Pusan National University. His areas of interest are intelligent systems applications to power systems. Dr. Park has been a member of the IEEE Power Engineering Society, Control System Society, and Systems, Man and Cybernetics Society.