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Fast Envelope Estimation Technique for Monitoring Voltage Fluctuations

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Abstract – Voltage quality problems such as voltage sag, swell, flicker, undervoltage, and overvoltage have been of great concern for both utilities and customers over the last decade. In this paper, a new approach based on the H_∞ algorithm to monitor voltage disturbances is presented. The key idea of this approach is to estimate the amplitude of the fundamental component of distorted and noisy voltage waveform instantaneously, and then the information can be extracted from the estimated envelope to identify and classify different voltage related power quality problems. The H_∞ algorithm is characterized by a fast tracking, unlike that of existing techniques. The H_∞ algorithm outperforms the Kalman Filter (KF) by its fast convergence and robust tracking performance against non-Gaussian noise. The paper investigates the effects of various types of noise on the performance of the H_∞ algorithm. Digital simulation results confirm the validity and accuracy of the proposed method. The proposed H_∞ algorithm is examined by tracking the flicker produced by a resistance welder simulated in the PSCAD/EMTDC package.

Keywords : Envelope tracking, Estimation, H_∞, Kalman Filter, Power quality, Voltage fluctuations

1. Introduction

One of the general power quality issues in distribution systems is the voltage fluctuation. Voltage fluctuations encompass a broad band of power quality problems such as voltage sags, voltage swells, voltage flicker, undervoltage, and overvoltage. For power quality monitoring and control, it is necessary to measure the load quantities such as magnitude of voltage and current, and phase angle [1], [2]. However, these waveforms are corrupted with measurement noise due to non-ideal transducer characteristics as well as other characteristics that cannot be accurately modeled by a Fourier series. Therefore, it is imperative to build a good estimation technique that is able to estimate the fluctuations in signal amplitude and phase in noisy environments and hence to classify power quality events.

There are several tracking techniques reported in the literature among which discrete Fourier transform (DFT), Wavelet Transform (WT), Artificial Neural Networks (ANN) and Kalman filter (KF) are the most popular. The DFT-based fast Fourier transform (FFT) for spectral analysis is a well-known technique and is widely used due to its low computational requirement [3]. In this approach, the coefficients of individual harmonics are computed by implementing FFT on digitized samples of a measured waveform in a time window. However, there are several performance limitations inherent in the FFT application: 1) the waveform must be stationary and periodic, 2) the window length of data must be an exact integer multiple of the lowest considered frequency. Failing to satisfy these conditions will result in leakage and picket fence effects and hence inaccurate waveform frequency analysis. Moreover, the DFT-based algorithm can cause computational error and may produce inaccurate results if the signal is contaminated by noise [4]. The WT was employed for envelope tracking in [5]. The main disadvantage of WT is the delay introduced by its batch processing. The ANN technique, which is based on back-propagation [6], requires too much data to get trained and may lead to inaccurate results due to the random-like behavior and large variations in the voltage fluctuations. The ADALINE, a version of ANN, has been used for fast and accurate envelope tracking [7], [8]. However, it has been pointed out that the noise rejection of the ADALINE is achieved on the account of convergence speed [7]. In the KF approach, a state variable mathematical model of the signal, including all possible harmonic components, is used. The KF technique converges in a relatively short time using a small number of samples [9]. However, it requires a priori statistical noise properties. Moreover, its performance is degraded under the non-Gaussian white noises.

This paper presents a new approach based on H_∞ algorithm for on-line tracking and classification of voltage fluctuations. This technique does not require any a priori knowledge about the noise properties and hence it shows better performance than the KF estimator. The robustness of the H_∞ filter is originated from its design which allows

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for a user-defined tolerance. This unique design produces accurate estimation in the presence of non-Gaussian noise [10-12]. In addition, the H∞ algorithm offers a fast and precise tracking of various voltage fluctuations regardless of the initial values of the estimate vector.

The focus of this work is to investigate and compare the proposed H∞ algorithm with the KF for instantaneous envelope tracking. The performance of the proposed algorithm for envelope tracking is tested by two schemes. In the first scheme artificial signals generated in MATLAB are utilized, and in the second one a practical system is modeled in the PSCAD/EMTDC simulation package. The results are discussed to demonstrate the potential of the proposed technique. The conclusion is drawn based on their performance from different points of view.

2. The State-Space Model of Fluctuated Voltage

The disturbed voltage waveform in distribution systems can be modeled by the following time varying sinusoidal signal

$$v(t) = Z(t)\sin(\omega t + \phi(t)) \tag{1}$$

where $Z(t)$ is the magnitude of the voltage under disturbance, ω is the supply angular frequency, and $\phi(t)$ is the phase angle of the fundamental component. Equation (1) can be written as

$$v(t) = A(t)\sin\omega t + B(t)\cos\omega t \tag{2}$$

where $Z(t) = \sqrt{A^2(t) + B^2(t)}$ and $\phi = \tan^{-1}\left(\frac{B(t)}{A(t)}\right)$. We are interested in estimating the variables $x_1 = A(t)$ and $x_2 = B(t)$, which represent the in-phase and quadrature-phase components of the signal given in (2). These variables, represented by vector X, are often denoted by the term state variables and are governed by the state equations

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}_k \tag{3}$$

where w_1 and w_2 allow the state variables for random walk (time variation) and the subscripts on the vectors represent the time step. The measurement equation, which includes the signal and noise, can be represented at discrete instants of time as

$$z_k = \begin{bmatrix} \sin\omega t_k & \cos\omega t_k \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix} + V_k = \mathrm{H}_k \mathrm{X}_k + V_k \tag{4}$$

where V_k signifies random measurement noise and t_k is the k^{th} sampling time. Vector H_k in the measurement equation relates the state vector X_k to the measurement z_k at time t_k. The variance of the measurement noise V_k is equal to R_k and the covariance matrix for the process noise W_k vector is given by

$$E\left[\mathrm{W}_k \quad \mathrm{W}_k^T\right] = \begin{cases} \mathrm{Q}_k, & i = k \\ 0, & i \neq k \end{cases} \tag{5}$$

where $E\left[\mathrm{W}_k \quad \mathrm{W}_k^T\right]$ is the expected value of $\left(\mathrm{W}_k \quad \mathrm{W}_k^T\right)$ and superscript T denotes for the transpose operation. Fig. 1 is a block diagram of a general envelope tracker based on the model presented above. It is noteworthy that the model presented in (3) and (4) is a general representation of the voltage envelope. This model can be used to detect and classify most power quality problems related to voltage, including sag, swell, flicker, and interruption. The following section discusses the origin of the proposed H∞ algorithm and compares it with KF as an adaptation algorithm.

3. The H∞ Filtering Algorithm

The concept of the KF estimation is based on the minimization of the mean square error estimate of the state vector X_k. The KF algorithm assumes that the noise is a white Gaussian distribution and its statistical properties are known in advance. However, its performance is degraded under the non- Gaussian white noises. The mathematical representation of the KF is given in Appendix I.

In this correspondence, a new estimator known as H∞ filtering has emerged as an innovative estimation technique.

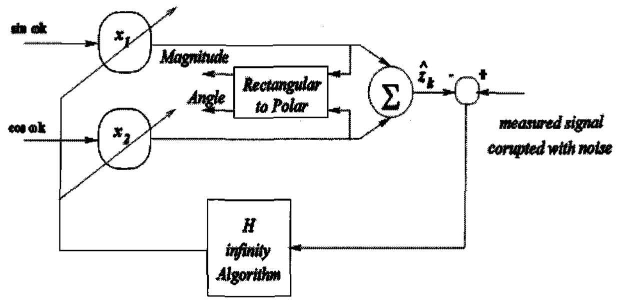

Fig. 1. Block diagram of voltage fluctuations tracking model

The H_∞ filtering estimator has the ability to estimate the state variable X_k from measurements corrupted by white Gaussian or non-Gaussian noise. It is a minimaximization problem where the maximum energy of the estimator error over all disturbances (modeling errors and additive noises) is minimized. There are two main differences between the H_∞ filtering and the KF algorithms. Firstly, the H_∞ filter requires no *a priori* knowledge of the noise statistics. It assumes that the noise signals have a finite energy. Secondly, the design criterion of the H_∞ filter is based on minimization of the worst-case estimation error resulting from modeling error and additive noise provided that these disturbances have a finite energy level [12].

Consider the state and measurement equations (3) and (4) where neither V_k, measurement additive noise, nor W_k, modeling error of the process, are known. In the KF estimation, we are interested in the optimal estimation of X_k. However, in the H_∞ filtering estimation, we are interested in the estimation of an arbitrary linear combination of X_k using the measurements $\{z_i, i \le k\}$, i.e., $s_k = LX_k$, where L is a vector. The design criterion of the H_∞ estimator is to limit the estimation error, $e_k = s_k - \hat{s}_k$, to an acceptable value for any disturbances W_k, V_k and the uncertainty of the initial state vector value X_0. The performance measure is specified by:

$$\underset{W_k, V_k, X_0}{\mathrm{SUP}} \frac{\|e_k\|_C^2}{\|X_0 - \hat{X}_0\|_{P_0^{-1}}^2 + \|W_k\|_Q^2 + \|V_k\|_R^2} \le \gamma^2 \qquad (6)$$

where SUP refers to supremum, $\hat{X}_0$ is the estimate of X_0, the notation $\|W_k\|_Q^2$ is mathematically described as $\|W_k\|_G^2 = W_k{}^T Q W_k$, $C \ge 0$, $p_0^{-1} > 0$, $Q > 0$ and $R > 0$ are the weighting matrices, and $\gamma^2 > 0$ is the desired level of disturbance attenuation. p_0^{-1} represents a positive square matrix that reflects *a priori* knowledge on how close the initial guess $\hat{X}_0$ is to X_0. The strategy of the H_∞ filter is to find an optimal estimate for s_k among all possible $\hat{s}_k$ (i.e., the worst case performance measure). The weighting elements, C, Q, P_0 and R, are selected by the designer to achieve adequate performance.

The objective of the H_∞ filter, as indicated in (6), is to process the measured signal samples to produce an estimate of the state vector $\hat{X}_0$ and at the same time to guarantee that the effect of disturbance (noise) is bounded and governed by a prescribed value γ. Following the mathematical analysis in [12], the H_∞ filtering/estimation algorithm can be mathematically described by the following set of equations

$$\hat{X}_{k+1} = \hat{X}_k + K_k(z_k - H_k\hat{X}_k) \qquad (7)$$

$$K_{k+1} = (P_k^{-1} - \gamma^{-2}\overline{C} + H_k^T R^{-1} H_k)^{-1} H^T R^{-1} \qquad (8)$$

$$P_{k+1} = (P_k^{-1} - \gamma^{-2}\overline{C} + H_k^T R^{-1} H_k)^{-1} + Q \qquad (9)$$

where K_k is the gain of the H_∞ filter and $\overline{C} = L^T C L$.

4. Simulation Results

Several test cases of the H_∞ algorithm for the envelope tracking are carried out to evaluate its performance. Primarily, the first case examines the dynamic performance of the proposed tracker with the aid of MATLAB for a given mathematical waveform. The second case deals with the implementation of a real system that has voltage flicker produced by a resistance welder. EMTDC/PSCAD software is used in the latter test to validate the accuracy of the proposed technique. The sampling rate for all the test scenarios is set at 64 samples per cycle. This low sampling rate indicates that the implementation of the proposed algorithm does not require any expensive hardware.

4.1 Performance Evaluation

To demonstrate the effectiveness of the proposed H_∞ algorithm for tracking various voltage fluctuations (represented in Fig. 2(a)), the voltage waveform with step changes. This signal mimics various voltage fluctuations

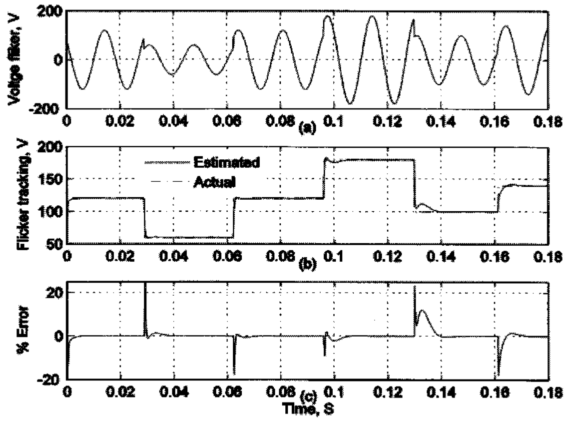

Fig. 2. Performance of the H_∞ algorithm for tracking of voltage power quality problem

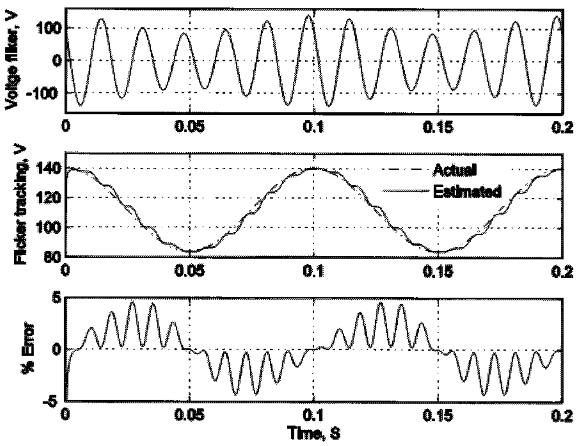

Fig. 3. Flicker tracking with the H_∞ filter

such as voltage sag in the second period, voltage swell in the fourth period, under-voltage in the fifth period, and over-voltage in the last period. With the zero initial estimate vector $X(k)$, the actual and estimated envelopes are illustrated in Fig. 2(b). Fig. 2(c) depicts the percentage error in envelope tracking. It is worth mentioning that steady state error is zero. The sharp tails in the percentage error occur at the moment of abrupt changes in the measured voltage signal, and stay for a duration less than 1 ms. Fast convergence and precise tracking are evident from this result compared to the aforementioned other techniques found in the literature.

A different test case, shown in Fig. 3, is run to assess the on-line dynamic performance of the H_∞ algorithm to estimate the envelope of a sinusoidal modulated flickered voltage waveform which is given by

$$v(t) = 28*(4+\cos 62.8t)*\cos(376.8t+\pi/4) \quad (10)$$

The initial value of the estimate vector is set to zero. The estimated phase angle of the fundamental voltage is portrayed in Fig. 4(a) and its estimation error is indicated in Fig. 4(b). Tight tracking with error less than 5% and a fast convergence is revealed for both the envelope and the fundamental phase angle. It is noteworthy that the proposed H_∞ algorithm can successfully replace the Phase Locked Loop (PLL) circuit that is used to measure the phase in many applications. This demonstrates the ability of the proposed algorithm for the on-line tracking of a broad band of envelope shapes.

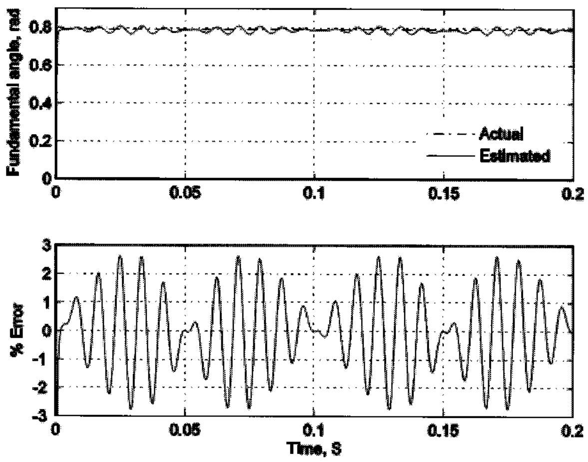

Fig. 4. Phase angle estimation using the proposed H_∞ algorithm

4.2 Noise Immunity

In the previous subsection, the H_∞ algorithm is tested using a clean (noise-free) signal. The objective of this section is to investigate the effect of measurement noise on the performance of the H_∞ algorithm for envelope tracking. Different types of noise corruption are considered to compare between the immunities of the H_∞ algorithm and the KF, respectively.

Fig. 5 illustrates a comparison between the envelope estimation errors of the signal represented in (10) with additive 2% white Gaussian noise. It is obvious that the proposed H_∞ algorithm outperforms the KF in terms of

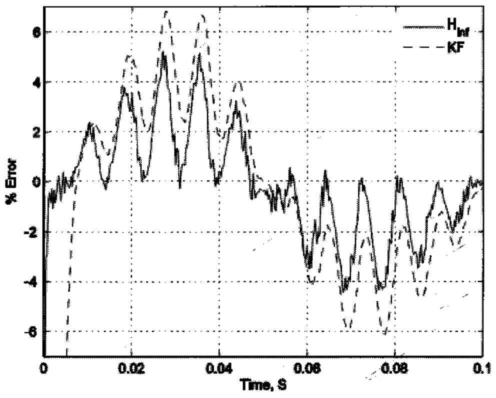

Fig. 5. Performance evaluation of the H_∞ filter against the KF for white noise measurements

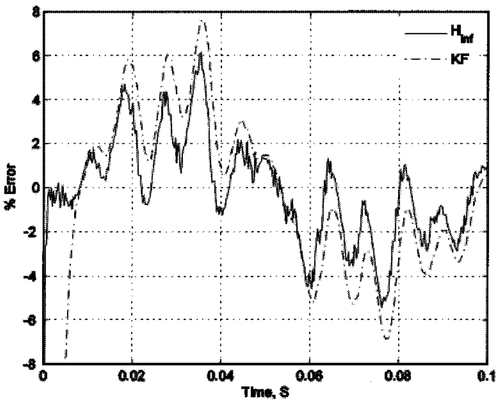

Fig. 6. Performance evaluation of the H_∞ filter against the KF for biased noise measurements

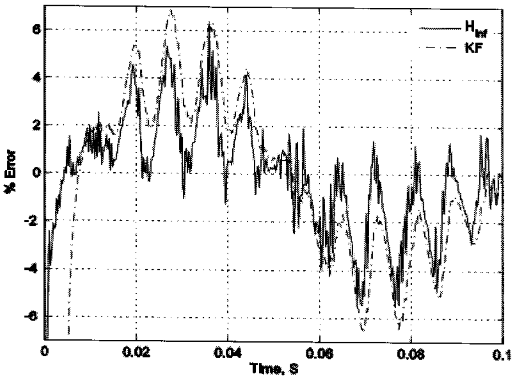

Fig. 7. Performance evaluation of the H_∞ filter against the KF for colored noise measurements

reduced estimation error and fast tracking. Comparing Fig. 5 with Fig. 3(c), one can observe that the ranges of the estimation error, with and without the white noise, are almost the same for the H_∞ algorithm. This is due to the nature of the white noise which has a zero mean. If the signal is corrupted with a biased random noise (dc component is not zero), the estimation error will increase slightly as indicated in Fig. 6.

Another type of noise is considered to validate the effectiveness of the H_∞ algorithm for envelope tracking application. The signal provided by (10) is superimposed by a colored noise given by following an 8th order auto regressive formula

$$\begin{aligned} v_c(k) = &-0.085v_c(k-1) + 0.191v_c(k-2) \\ &+ 0.046v_c(k-3) + 0.023v_c(k-4) \\ &+ 0.12v_c(k-5) + 0.155v_c(k-6) \\ &- 0.132v_c(k-7) - 0.76v_c(k-8) \\ &+ v_w(k) \end{aligned} \tag{11}$$

where $v_c(k)$ and $v_w(k)$ are the colored and white noise signals, respectively. Fig. 7 indicates the estimation error for both the H_∞ algorithm and the KF. This result is consistent with the previous cases where the H_∞ algorithm offers a lower estimation error and faster tracking compared to the KF.

The proposed H_∞ algorithm is superior to the KF and is immune against different measurement noise. The H_∞ algorithm provides the optimal estimate such that the effect of the worst disturbance (noises) on the estimation error is minimized. The estimation error does not exceed 6%. The performance of the H_∞ algorithm makes it attractive for voltage fluctuation metering devices and control applications.

4.3 EMTDC Simulation

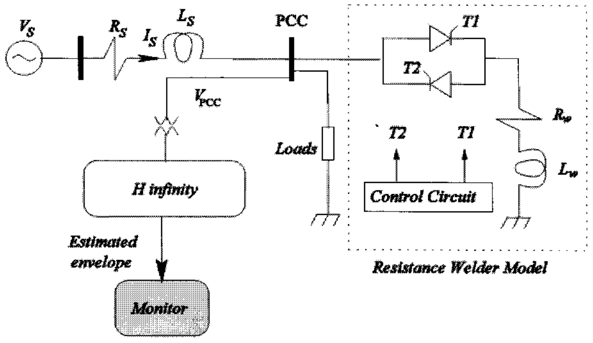

Fig. 8 System configuration

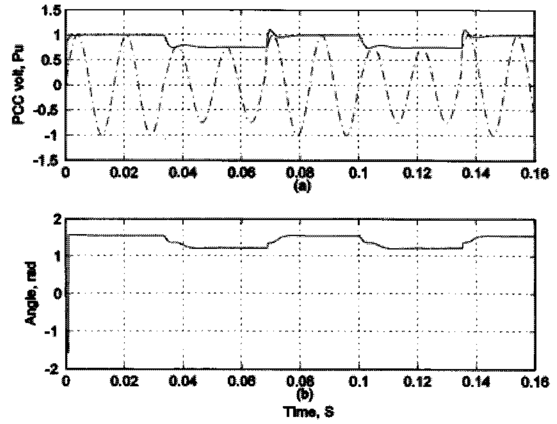

Fig. 9. Tracking of the resistance welder flicker

A practical system including a resistance welder, a common flicker source in industrial distribution systems, is simulated in the EMTDC package to execute more thorough analysis on the proposed tracking algorithm. The parameters of this system are provided in Appendix II. A resistance welder is realized as an ac voltage regulator using a back-to-back thyristor switched inductor as shown in Fig. 8. The control block of the welder based Integral Cycle Control (ICC), and the H_∞ algorithm are implemented using Fortran under the EMTDC environment. The welder is controlled to switch on and off for any number of half-cycles with any required delay in the firing angle of the thyristors. Consequently, the harmonics effect may also be considered. Usually the power rating of the welder is a significant portion of the total power demand and the voltage at the Point of Common Coupling (PCC) encounters substantial and repetitive voltage dips (sag). At the PCC, the utility serves other customers who undergo voltage fluctuations. The duration of the voltage dips at PCC is ruled by duty cycle of the welder.

Fig. 9(a) indicates the tracking of the pu voltage at the PCC when the weld-cycle is four cycles with a 50% duty. The phase angle estimation of the H_∞ algorithm is reflected in Fig. 9(b). The proposed H_∞ algorithm accurately tracks the voltage envelope as well as the phase angle with sharp variations. As observed from the first study case, the H_∞

algorithm is fast converge. This property is responsible for accurate smoothing of the estimated envelope.

5. Conclusion

This paper investigates the application of the H_∞ algorithm for envelope tracking of various types of voltage fluctuations. Unlike the KF which is designed to handle white Guassian noise, the H^∞ algorithm is a robust tool in estimating the fluctuated voltage envelope in the presence of non-Gaussian noise. The H_∞ algorithm gives the optimal estimate such that the effect of the worst disturbance (noises) on the estimation error is minimized. A fast response, accurate tracking, and robustness of the proposed H_∞ algorithm are revealed from the simulation results. In addition, this paper examines the immunity of the proposed H_∞ algorithm for the envelope estimation against various types of noise. A practical system containing a resistance welder is simulated to validate the accuracy of the proposed algorithm for the envelope tracking. The powerful characteristics of the proposed H_∞ algorithm such as fast convergence, if it is compared to the available techniques in the literature, low sampling rate and less dependency on the initial values, make the H_∞ algorithm a better candidate for control applications and render it suitable for implementing a low cost flicker meter.

Appendix I: Kalman Filter Algorithm

This appendix provides the mathematical representation of the KF. The updated estimate with measurement is given by

$$\hat{X}_{k+1} = \hat{X}_k + K_k(z_k - H_k\hat{X}_k) \qquad (12)$$

The Kalman gain is computed from

$$K_{k+1} = P_k H_k^T (H_k P_k H_k^T + R)^{-1} \qquad (13)$$

and finally the error covariance matrix is updated as follows

$$P_{k+1} = (I - K_k H_k)P_k + Q \qquad (14)$$

where $Q = E[W_k * W_k^T]$ is the covariance matrix of the process noise and $R = E[V_k * V_k^T]$ is the covariance matrix of the observations. It should be noted that the covariance parameters, **Q** and R, of the KF algorithm are different from the weighting parameter of the H_∞ algorithm.

Appendix II: System Data

V_s(rms/line)= 110 V, f=60 Hz, L_s=7 mH, and R_s=1 Ω. The arc welder parameters are R_W=10 Ω and L_W= 100 mH. The simulation time step is 0.05 ms.

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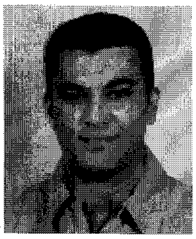

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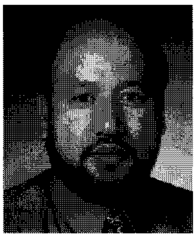

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