A Linearized Transmission Model Based Market Equilibrium In Locational Pricing Environments

Manho Joung* and Jin-Ho Kim†

Abstract – In this paper, we have investigated how transmission network constraints can be modeled in an electricity market equilibrium model. Under Cournot competition assumption, a game model is set up considering transmission line capacity constraints. Based on locational marginal pricing principle, market clearing is formulated as a total consumers' benefit maximization problem, and then converted to a conventional optimal power flow (OPF) formulation with a linearized transmission model. Using market clearing formulation, best response analysis is formulated and, finally, Nash equilibrium is formulated. In order for illustration, a numerical study for a four node system with two generating firms and two loads are presented.

Keywords : Cournot competition, Deregulated electricity markets, Electricity market equilibrium model, Game theory, Nash equilibrium, Power system network constraints

1. Nomenclatures

- P_i: Demand curve at node-i
- $q_i^{j,k}$: Production of the generator-j of firm-k at node-i
- q^k: Production profile of the generating firm-k
- $C_i^{j,k}$: Cost of the generator-j of firm-k at node-i
- C^k: Total cost of generating firm-k
- p_i: Locational market clearing price at node-i
- q_i: Demand quantity at node-i
- π^k : Payoff function of firm-k
- B_i: Benefit function of consumer at node-i
- $q_i^{j,k,Br}$: Best response of the generator-j of firm-k at node-i
- $q_i^{j,k,Ne}$: Nash equilibrium of the generator-j of firm-k at node-i
- $q^{k,Br}$: Best response function of the generating firm-k
- $q^{k,Ne}$: Nash equilibrium of the generating firm-k
- M: Number of generating firms
- N: Number of nodes(buses)
- L: Number of branches
- N^G: Number of nodes at which generation exists
- N^D: Number of nodes at which demand exists
- M_i^k : Number of generators owned by firm-k at node-i
- M^k : Number of generators owned by firm-k

† Corresponding Author: Dept. of Electrical Engineering, Kyungwon University, Korea (kimjh@kyungwon.ac.kr)
* Dept. of Electrical and Computer Engineering, University of Texas at Austin, USA (joung@mail.utexas.edu)
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2. Introduction

Many electricity markets are adopting the locational marginal pricing mechanism for improved promotion of market efficiency. Locational marginal prices (LMPs) are, therefore, used in many electricity markets to settle energy transactions. Generally, when a single slack bus is used in the power-flow formulation for calculating the prices, LMP can be decomposed into three components: 1) the reference price at the single slack bus (which is also the angle reference bus, 2) the marginal price of the transmission losses, and 3) the marginal price of the network constraints that are enforced in the power-flow model. Namely, LMP implies the price at which the electricity is consumed at each node. Due to the physical characteristics of the electricity transmission network, electricity is lost when it is transmitted from supplying nodes (*i.e.*, supplying buses) to consuming nodes (consuming buses), and additional generation must be supplied to provide energy in excess of that consumed by customers. Moreover, the capacity limitation of the transmission network of electricity systems prevents full uses of system wide cheap electricity. Therefore, an electricity price at each node, *i.e.* the price at which the electricity is consumed at each node, is differently decided depending on the network topology and energy configuration.

Until now, most of the researches have been focused on the market equilibrium and bidding strategy of generators without consideration of transmission constraints [1-5]. Since the electricity market can be modeled by an oligopoly, several papers have presented the influences of individual players' market power on the spot market equilibrium [6-8].

Although it is well known that network constraints play important roles in the efficient operation of electricity markets, the electricity market equilibriums under transmission network constraints are not extensively explored by researchers. In [9], based on the interaction between ISO and generators, impacts of network constraints on the spot market are presented. In [10], in a simple two-node system, it is suggested that the pure strategy equilibrium can be eliminated by the inclusion of transmission network constraints. To study the influence of congestion on the bidding strategies, a separate curtailment algorithm for congestion management is developed in [11].

In this paper, unlike the conventional studies, locational market clearing problem has been formulated as a modified version of the classical optimal power flow (OPF) problem. Therefore, we can observe that the conventional OPF formulation can be directly applied to the proposed method with very little modification. Using the optimization result, the nodal price for each node and the corresponding payoff of each generating firm can be derived. Based on the payoff of each generating firm, the Nash equilibrium strategy of each generating firm is obtained by the fixed point of the best response functions.

To obtain an analytic result, we have considered an electricity market with M generating firms, while the transmission system is modeled by N nodes (buses) and L branches. This paper is organized as follows. Section 2 briefly describes the market and game model. Section 3 offers the locational market clearing formulation and best response analysis for obtaining Nash equilibrium. In Section 4, the proposed algorithm is applied to a sample power system consisting of 4 buses, 2 demands, 2 generating firms, and 4 transmission lines and the simulation results are provided. Finally, the conclusion drawn from the study is provided in Section 5.

3. Model Descriptions

3.1 Market Model

Consider M generating firms in real-time electricity energy markets that use the locational marginal pricing scheme. The markets are cleared by an independent system operator through electricity energy auctions. Generating firms are competing with each other in order to serve the demand by determining their generating quantities (Cournot competition). The underlying power system consists of N nodes and L branches. Out of N nodes, there are N^D nodes at which demand exists and N^G nodes where generation exists. Here, we allow firms to own generation capacity at multiple nodes. Moreover, multiple generators can exist at a single node. No transmission losses are considered. Each branch l, where l = 1, 2, …, L, has capacity limit $\bar{l}$.

The demand at node-i, where i = 1, 2, …, N^D, is assumed to be characterized by an inverse demand function denoted by $P_i : \Re_+ \to \Re_+$, and is represented by an affine curve with a negative slope:

$$P_i(q) = -\alpha_i q + \beta_i \text{, where } \alpha_i, \beta_i \in \Re_+ \tag{1}$$

The production cost function of a generator is assumed to be of a quadratic form and the cost function $C_i^{j,k} : \Re_+ \to \Re_+$ of the generator-j of firm-k at node-i is represented as:

$$C_i^{j,k}\left(q_i^{j,k}\right) = \frac{1}{2} a_i^{j,k}\left(q_i^{j,k}\right)^2 + b_i^{j,k} q_i^{j,k} + c_i^{j,k} \tag{2}$$

where i = 1, 2, …, N^G, j = 1, 2, …, M_i^k, M_i^k is the number of generators owned by generating firm-k at node-i, k = 1, 2, …, M, $q_i^{j,k}$ is the production of generator-j of generating firm-k at node-i, and $a_i^{j,k}, b_i^{j,k}$, and $c_i^{j,k}$ are parameters. The total cost function $C^k : \Re_+^{M^k} \to \Re_+$, where $M^k = \sum_{i=1}^{N^G} M_i^k$, of generating firm-k for the production profile $q^k = \left[q_1^k, q_2^k \cdots, q_{N^G}^k\right]$, where $q_i^k = \left[q_i^{1,k}, q_i^{2,k} \cdots, q_i^{M_i^k,k}\right]$ is obtained as:

$$C^k\left(q^k\right) = \sum_{i=1}^{N^G}\left[\sum_{j=1}^{M_i^k} C_i^{j,k}\left(q_i^{j,k}\right)\right] \tag{3}$$

Let $p_i : \Re_+^{\sum_{k=1}^{M} M^k} \to \Re$ denote the nodal price function at node-i representing the market clearing price at node-i for given productions of all generators. Note that this function is different from the inverse demand function P_i at node-i. Then, the payoff function of generating firm-k is represented as:

$$\pi^k\left(q^1, \cdots, q^M\right) = \sum_{i=1}^{N^G}\left[P_i^n\left(q^1, \cdots, q^M\right)\sum_{j=1}^{M_i^k} q_i^{j,k}\right] - C^k\left(q^k\right) \tag{4}$$

3.2 Game Model

The considering market is modeled as an oligopolistic market game. In the game model, generating firms are the players and their quantity decisions are players' strategies. Generating firms' profits from the energy market are modeled as players' payoff functions. The solution is defined by Nash equilibrium $q^{Br} = \left[q^{1,Br}, \cdots, q^{M,Br}\right]$, where

$$q^{k,Br} = \arg\max_{q^k} \pi^k\left(q^{1,Br}, \cdots, q^{k-1,Br}, q^k, q^{k+1,Br}, \cdots, q^{M,Br}\right), \quad \forall k = 1, 2, \cdots, M \tag{5}$$

4. Equilibrium Analysis

4.1 Market Clearing Formulation

In the locational marginal pricing scheme, market clearing price profile $p = [p_1, p_2, \cdots, p_N]$ achieves the best short-term market efficiency by maximizing social welfare. The social welfare is defined as the total sum of producers' and consumers' surplus and this equals total consumers' benefit minus total generation costs. Consider that quantity profile $q^G = \left[q^1, q^2 \cdots, q^M\right]$ is given. In this case, social welfare will be maximized by maximizing consumers' surplus since total production costs are already determined. Following demand theory, we can interpret the demand function as measurement of marginal benefit. Therefore, consumers' benefit $B_i(q_i)$ at each node-i = 1, 2, …, N^D, where q_i is the demand quantity at node-i, which will be obtained by integrating the inverse demand function at node-i:

$$B_i(q_i) = \int_0^{q_i} P_i(q)\, dq \tag{6}$$

Total consumers' benefit $B\left(q^D\right)$, where $q^D = \left[q_i^D\right]_{i=1}^{N^D}$ is the demand quantity profile, which will be the sum of consumers' benefit at all nodes:

$$B\left(q^D\right) = \sum_{i=1}^{N^D} B_i(q_i) \tag{7}$$

The market is now cleared by finding the optimal demand quantity profile $q^{D*} = \left[q_i^*\right]_{i=1}^{N^D}$ to maximize total consumers' benefit considering energy balancing and transmission line capacity constraints. Given generation and demand quantities at each node, Kirchhoff's laws determine the flow quantity on every transmission line and this makes transmission line capacity constraints expressed as nonlinear inequality constraints. Let $f_l\left(q^G, q^D\right)$ denote the function representing the line flow on the branch-l = 1, 2, …, L. Then, the optimization problem is represented:

$$\begin{aligned} &\max_{q^D} B\left(q^D\right), \\ &s.t. \sum_{i=1}^{N^G}\left\{\sum_{k=1}^{M}\left[\sum_{j=1}^{M^k} q_i^{j,k}\right]\right\} = \sum_{i=1}^{N^D} q_i^D, \\ &f_l\left(q^G, q^D\right) \le l^{\max}, \forall l = 1, 2, \cdots, L \end{aligned} \tag{8}$$

Here, we can observe that the conventional optimal power flow (OPF) formulation can be directly applied with very little modification. In the conventional OPF problem, the total production costs are minimized with respect to generation quantities with given demand and the energy balancing and transmission line constraints. In our formulation, by considering demand quantity as generation quantity and vice versa and by defining total costs by negative value of total benefit, we can obtain the OPF formulation for market clearing. Using the optimization result, the nodal price function value $p_i\left(q^1, q^2, \cdots, q^M\right)$ for each node i = 1, 2, …, N, and, in turn, the payoff function value $\pi^k\left(q^1, q^2, \cdots, q^M\right)$ of each generating firm k = 1, 2, …, M, can be evaluated.

4.2 Best Response Analysis

One of the methods for obtaining Nash equilibrium is best response analysis. Let $q^{-k} = \left[q^{\kappa}\right]_{\kappa=1,\kappa\neq k}^{M}$ denote the quantity profile for generators other than those owned by firm-k. Then, the best response function $(q^k)^{Br}$ of firm-k is defined as:

$$q^{k,Br}(\cdot) = q^{k,Br}\left(q^{-k}\right) = \arg\max_{q^k}\left[\pi^k\left(q^k, q^{-k}\right)\right] \tag{9}$$

Nash equilibrium strategy $q^{k,Ne}$ of generating firm-k is obtained by the fixed point of the best response functions.

5. Illustrative Example

Consider a power system consisting of 4 buses, 2 demands, 2 generating firms, and 4 transmission lines as shown in Fig. 1. In the figure, the electrical properties of each line are also demonstrated. Two generators are identical and the cost functions of generating firms are of the same form:

$$\begin{aligned} C_3^{1,1}\left(q_3^{1,1}\right) &= 0.01\left(q_3^{1,1}\right)^2 + 20 q_3^{1,1}, \\ C_4^{1,2}\left(q_4^{1,2}\right) &= 0.01\left(q_4^{1,2}\right)^2 + 20 q_4^{1,2} \end{aligned} \tag{10}$$

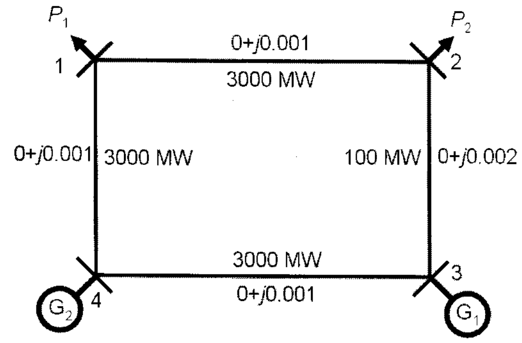

Fig. 1. Sample System Configuration

The inverse demand function at each demand node is:

$$\begin{aligned} P_1(q) &= -0.01q + 50, \\ P_2(q) &= -0.015q + 60. \end{aligned} \tag{11}$$

Given the generators' outputs $q_3^{1,1}$ and $q_4^{1,2}$, the optimization problem for market clearing is:

$$\max_{q^D} B\left(q^D\right) = -0.0005\left(q_1\right)^2 + 50q_1 - 0.00075\left(q_2\right)^2 + 60q_2$$

$$s.t.\ q_1 + q_2 = q_3^{1,1} + q_4^{1,2} \tag{12}$$

$$abs\left\{\begin{bmatrix} 0.8 & 0.6 & 0.2 \\ -0.2 & 0.6 & 0.2 \\ 0.2 & 0.4 & -0.2 \\ 0.2 & 0.4 & 0.8 \end{bmatrix}\begin{bmatrix} -q_1 \\ -q_2 \\ q_3^{1,1} \end{bmatrix}\right\} \le \begin{bmatrix} 3000 \\ 3000 \\ 100 \\ 3000 \end{bmatrix}$$

Note that the line capacity constraints in the above formulation are approximated by DC power flow assumptions and the left hand side of the inequality constraints should be the absolute values in order to consider direction of the flow. As discussed in Section 4.2, the above formulation can be rewritten as:

$$\min_{q^D} \tilde{C}\left(q^D\right) = 0.0005\left(q_1\right)^2 - 50q_1 + 0.00075\left(q_2\right)^2 - 60q_2$$

$$s.t.\ q_1 + q_2 = q_3^{1,1} + q_4^{1,2} \tag{13}$$

$$abs\left\{\begin{bmatrix} 0.8 & 0.6 & 0.2 \\ -0.2 & 0.6 & 0.2 \\ 0.2 & 0.4 & -0.2 \\ 0.2 & 0.4 & 0.8 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ -q_3^{1,1} \end{bmatrix}\right\} \le \begin{bmatrix} 3000 \\ 3000 \\ 100 \\ 3000 \end{bmatrix}$$

This can be interpreted as an OPF formulation where both demands are regarded as generators whose total costs are represented by the function $\tilde{C}$ and both generators are modeled as demands $q_3^{1,1}$ and $q_4^{1,2}$ respectively.

Note that, even though all four lines have capacity limits, only the capacity constraint of the line from node 2 to node 3 will be binding for any market clearing. That is, we need to consider only one line constraint when the market is cleared. Here, suppose that there is no binding transmission line constraint first. Then, by solving the OPF problem (13) with energy balancing constraint only, q_1 and q_2 are determined as:

$$\begin{aligned} q_1 &= 0.6\left(q_3^{1,1} + q_4^{1,2}\right) - 400, \\ q_2 &= 0.4\left(q_3^{1,1} + q_4^{1,2}\right) + 400. \end{aligned} \tag{14}$$

Since there is no congestion, all nodal prices are the same and represented with respect to $q_3^{1,1}$ and $q_4^{1,2}$ as:

$$p_1 = p_2 = p_3 = p_4 = -0.006\left(q_3^{1,1} + q_4^{1,2}\right) + 54 \tag{15}$$

Therefore, the profits π_1 and π_2 of generating firms are determined as:

$$\begin{aligned} \pi_1 &= -0.016\left(q_3^{1,1}\right)^2 - \left(0.006q_4^{1,2} - 34\right)q_3^{1,1} \\ \pi_2 &= -0.016\left(q_4^{1,2}\right)^2 - \left(0.006q_3^{1,1} - 34\right)q_4^{1,2} \end{aligned} \tag{16}$$

The best responses of generating firms are the same as:

$$\begin{aligned} q_3^{1,1,Br}\left(q_4^{1,2}\right) &= -0.1875q_4^{1,2} + 1062.5 \\ q_4^{1,2,BR}\left(q_3^{1,1}\right) &= -0.1875q_3^{1,1} + 1062.5 \end{aligned} \tag{17}$$

The fixed point of (17) would be a Nash equilibrium. However, the resulting outcome will violate the transmission line constraint and it contradicts our assumption of no binding line constraint.

Now, assume that the line connecting node 2 and node 3 is congested. Then, the market clearing problem is solved from the energy balancing and the line capacity constraints and q_1 and q_2 are determined as:

$$\begin{aligned} q_1 &= 3q_3^{1,1} + 2q_4^{1,2} - 500 \\ q_2 &= -2q_3^{1,1} - q_4^{1,2} + 500 \end{aligned} \tag{18}$$

From the market clearing outcome, the inverse price curve for each generator node is determined as:

$$\begin{aligned} p_3 &= 3p_1 - 2p_2 \\ p_4 &= 2p_1 - p_2 \end{aligned} \tag{19}$$

Using (10) and (18), (19) is rewritten as:

$$\begin{aligned} p_1\left(q_3^{1,1}, q_4^{1,2}\right) &= -0.03q_3^{1,1} - 0.03q_4^{1,2} + 30 \\ p_2\left(q_3^{1,1}, q_4^{1,2}\right) &= -0.09q_3^{1,1} - 0.055q_4^{1,2} + 57.5 \end{aligned} \tag{20}$$

Therefore, the payoff functions of the generating firms, π_1 and π_2, are determined as:

$$\begin{aligned}\pi_1 &= -0.04\left(q_3^{1,1}\right)^2 - \left(0.03q_4^{1,2} - 10\right)q_3^{1,1} \\ \pi_2 &= -0.065\left(q_4^{1,2}\right)^2 - \left(0.009q_3^{1,1} - 37.5\right)q_4^{1,2}\end{aligned} \tag{21}$$

The best responses of generating firms are:

$$\begin{aligned}q_3^{1,1,Br}\left(q_4^{1,2}\right) &= -0.375q_4^{1,2} + 125 \\ q_4^{1,2,Br}\left(q_3^{1,1}\right) &= -0.692q_3^{1,1} + 288.462\end{aligned} \tag{22}$$

The Nash equilibrium is obtained by the fixed point of (22) as:

$$\begin{aligned}q_3^{1,1,Ne} &= 22.727 \\ q_4^{1,2,Ne} &= 272.727\end{aligned} \tag{23}$$

By checking the assumption and constraints, we can verify that (23) is truly the Nash equilibrium. The result shows that, even though the generating firms are identical, the location difference results in very contrasting equilibrium quantities due to the binding line constraint.

6. Conclusion

This paper provides an electricity market equilibrium model considering power system network constraints. A game model is set up under Cournot competition assumption and, in the model, transmission line capacity constraints are explicitly represented. Based on locational marginal pricing principle, market clearing is formulated as a total consumers' benefit maximization problem. Without major modification, market clearing can be converted to a conventional optimal power flow formulation. Best response analysis is also designed, based on which Nash equilibrium is formulated. Finally, a numerical study for a four node system with two generating firms and two loads are presented as an illustration. The proposed equilibrium model may enable the Cournot model framework to consider power system network constraints which are one of the most unique and crucial characteristics of electricity markets.

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Manho Joung
He received his B.S. and M.S. degrees in Electrical Engineering from Seoul National University in 1995 and 1997, respectively. He is currently studying at the University of Texas, Austin, as a Ph.D. candidate. His research interests are electricity market design and power system economics.

Jin-Ho Kim
He received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Seoul National University in 1995, 1997, and 2001, respectively. He worked at Pusan National University as an Assistant Professor from 2004 to 2007 and is currently working at Kyungwon University as an Assistant Professor. His research interests are electricity market analysis, demand response, and distributed energy resources.