A Design of Free Velocity Bias for GPS Receiver

Phi-Long Nguyen†, Hyunsoo Kim and Hansil Kim***

Abstract – This paper proposes a 2-step Kalman filter model for land vehicle navigation using civilian-band GPS measurements. The velocity bias caused by the Earth's rotation would be removed completely when applying this model. Because the linearization of velocity equations in this model is not necessary, the error is significantly reduced. The experiment reveals that estimated position error with stationary data is about 5m during a 15-20 minute interval. The other benefit of this model is that it can be feasibly applied as a GPS receiver module thanks to the small sizes of the necessary manipulating matrices.

Keywords : Bias, Earth rotation, GPS, Kalman filter

1. Introduction

GPS works on the principle that 3-D position solution can be estimated with knowledge of the coordinates and distance to any three points in space [2]. The principle of the GPS has been applied to develop several indoor positioning systems, such as the Bat system [8], which makes use of ultrasonic signals to measure distance, or the RADAR system [9, 10] which uses the signal strength and an appropriate data base to estimate position. The combination of the Bat and RADAR systems was also proposed to develop a self positioning system for MANET [11]. Some other methods were also proposed using beacon recognition [5, 6].

Although much research effort has been devoted to indoor navigation problems in recent years, little has been undertaken for the outdoor equivalent. The outdoor navigation directly using the Global Positioning System (GPS) is much more difficult because of problems relating to environmental conditions and Earth rotation affection.

For the GPS civilian user, the SPS (Standard Positioning Service) is available at L1 frequency (1575.42MHz), which carries and broadcasts the 1.023MHz pseudorandom (C/A) code. The GPS receiver needs to track the L1 signal and demodulated C/A code to obtain the necessary data for navigation [2]. The 8-state Kalman filter has been introduced [1] for estimating the position and velocity of a GPS receiver. The position error of this method is about 20m in case of fixed position estimation [3].

There are a number of error sources inherent in the GPS which degrade system accuracy. The major one behaves as 'low-frequency quasi biases' [4], which can be seen as slow drifts in the position solution for a stationary receiver. An effort to reduce the drift error has been proposed in [3]. Eight more states were chosen for first-order Gauss-Markov process shaping filter, leaving the system with a total of 16 states to solve. The Gauss-Markov process shaping filter helps to reduce the bias with non-zero mean system noise. The solution error in this scenario is reduced to about 10m with stationary data. The trade-off with the proposed method is the large size of matrix operation, which is not easy to apply for a GPS receiver module.

One other proposal [7] involves using the CV model to estimate receiver acceleration. It then will be fedback to adjust velocity estimated value. The proposal used an available GPS receiver output as measurement data, so it needed a filter outside to re-estimate all states.

Another approach proposed in this paper is to deal with the bias, which is mainly caused by the Earth's rotation. Because the bias just affects the receiver velocity, we only need one more state variable that will lead the number of receiver velocity state equations to 5 states.

2. Navigation using Global Positioning System (GPS)

2.1 Navigation measurement equations

We consider the noiseless pseudorange equation used to estimate GPS receiver position:

$$p_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} + c\delta t = r_i + c\delta t \quad (1)$$

where

c = 299,792,458m/s is the speed of light

p_i: pseudorange or measurement range, $p_i = c\Delta t_i$ with

† Corresponding Author: School of Electrical Engineering, University of Ulsan, Mukeo-Dong Nam-Ku, Ulsan, Korea

* School of Electrical Engineering, University of Ulsan, Mukeo-Dong Nam-Ku, Ulsan, Korea (hskim@ulsan.ac.kr)

** Railway System Team, R&D Center, POSCON Corporation, Anam-Dong, Seongbuk-Gu, Seoul, Korea (hskim@poscon.co.kr)

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Δt_i is the measured transmission interval of magnetic wave from satellite i to receiver, Δt_i is measured by correlation operation.[2]

$r_i = \sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}$: real range from receiver coordinates (x,y,z) to satellite i coordinates (x_i,y_i,z_i)

δt : receiver clock bias, i.e. the offset between clock receiver and GPS time [2]

To estimate the GPS receiver velocity, noiseless Doppler equation [2] is used

$$D_i = \frac{(x-x_i)(vx-vx_i)+(y-y_i)(vy-vy_i)+(z-z_i)(vz-vz_i)}{r_i}+c\delta f \quad (2)$$

where

(vx,vy,vz) : receiver velocity

(vx_i,vy_i,vz_i): satellite i velocity

D_i is pseudorange rate in m/s of satellite i, $D_i = f_{di}\frac{c}{f_{L1}}$,

f_{L1}=1575.42MHz is the nominal frequency transmitted from satellites; f_{di} is Doppler frequency, derived from tracking loop result, it is the difference between the nominal frequency L1 with the actual tracking frequency in the channel i PLL.[2]

δf : receiver clock drift, i.e. the clock bias (δt) rate

Once satellite i is tracked, f_{di} is determined and eventually D_i is derived, D_i then will be the measurement data to calculate receiver velocity.

2.2 Linearization

(1) and (2) play roles of measurement models for Kalman filter, and because they are nonlinear equations, they should be linearized before applying Kalman filter [1].

We define state vectors:

$P = [x \quad y \quad z \quad c\delta t]^T$: Position state vector

$V = [vx \quad vy \quad vz \quad c\delta f]^T$: Velocity state vector

The position linearization equation

$$\Delta p_i = p_i - p_i^* = \left.\frac{\partial p_i}{\partial P}\right|_{P=P^*} \Delta P + H.O.T \approx \left.\frac{\partial p_i}{\partial P}\right|_{P=P^*} \Delta P \quad (3)$$

For velocity, we rearrange (2) to use without the need of linearization:

$$D_i = \frac{(x-x_i)(vx-vx_i)+(y-y_i)(vy-vy_i)+(z-z_i)(vz-vz_i)}{r_i}+c\delta f$$

$$D_i + \left(\frac{(x-x_i)}{r_i}vx_i + \frac{(y-y_i)}{r_i}vy_i + \frac{(z-z_i)}{r_i}vz_i\right) = \frac{(x-x_i)}{r_i}vx + \frac{(y-y_i)}{r_i}vy + \frac{(z-z_i)}{r_i}vz + c\delta f \quad (4)$$

Denote:

$$Dv_i = \frac{(x-x_i)}{r_i}vx_i + \frac{(y-y_i)}{r_i}vy_i + \frac{(z-z_i)}{r_i}vz_i$$

(4) becomes

$$D_i + Dv_i = \frac{(x-x_i)}{r_i}vx + \frac{(y-y_i)}{r_i}vy + \frac{(z-z_i)}{r_i}vz + c\delta f \quad (5)$$

$$Z_i = D_i + Dv_i = \begin{bmatrix} \frac{(x-x_i)}{r_i} & \frac{(y-y_i)}{r_i} & \frac{(z-z_i)}{r_i} & 1 \end{bmatrix} * [vx \quad vy \quad vz \quad c\delta f]^T \quad (6)$$

$$Z_i = H_i * V \quad (7)$$

Where

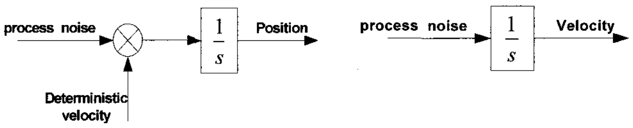
$$H_i = \begin{bmatrix} \frac{(x-x_i)}{r_i} & \frac{(y-y_i)}{r_i} & \frac{(z-z_i)}{r_i} & 1 \end{bmatrix} \quad (8)$$

is the measurement matrix

Once receiver position is estimated, D_i will be available and (6) is a linear equation which does not need to be linearized anymore.

3. The 2-Step Kalman filter

3.1 System models



(a) position model (b) velocity model

Fig. 1. System model

Two consequent system models are used, one for calculating velocity and the other for calculating position. To deal with the dynamic of the receiver, the deterministic velocity which is the previous estimated velocity state of the receiver is integrated in the position model.

The respective model equation for position and velocity:

$$P_k = \Phi_p * P_{k-1} + V_{k-1} * \tau + w_{pk} = P_{k-1} + V_{k-1} * \tau + w_{pk} \quad (9)$$

$$V_k = \Phi_v * V_{k-1} + w_{vk} = V_{k-1} + w_{vk} \quad (10)$$

where

P_k, V_k are position and velocity vector at state k

w_{pk} is process noise vector of position model at state k

w_{vk} is process noise vector of velocity model at state k

τ is sampling time

$\Phi_p = \Phi_v = I\,(identity)$ is the transition matrix of two models.

3.2 Measurement models

First we consider the measurement model without bias state aiding:

Position measurement model

$$\underbrace{\begin{bmatrix}\Delta p_1\\ \Delta p_2\\ \cdots\\ \Delta p_n\end{bmatrix}_k}_{\Delta Z_k}=\underbrace{\begin{bmatrix}\left.\frac{x-x_1}{r_1}\right|_{x=x_k^-} & \left.\frac{y-y_1}{r_1}\right|_{y=y_k^-} & \left.\frac{z-z_1}{r_1}\right|_{z=z_k^-} & 1\\ \left.\frac{x-x_2}{r_2}\right|_{x=x_k^-} & \left.\frac{y-y_2}{r_2}\right|_{y=y_k^-} & \left.\frac{z-z_2}{r_2}\right|_{z=z_k^-} & 1\\ \cdots & & & \\ \left.\frac{x-x_n}{r_n}\right|_{x=x_k^-} & \left.\frac{y-y_n}{r_n}\right|_{y=y_k^-} & \left.\frac{z-z_n}{r_n}\right|_{z=z_k^-} & 1\end{bmatrix}}_{H_k}\underbrace{\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\\ \Delta(c\delta t)\end{bmatrix}_k}_{\Delta P_k}+v_{pk} \qquad (11)$$

In short-hand note:

$$\Delta Z_k = H_k * \Delta P_k + v_{pk} \qquad (12)$$

where v_{pk} is the position measurement noise vector at state *k*.

Velocity measurement model

The first step Kalman filter gives estimated position $\hat{P}_k$, the velocity measurement model then is established from (6):

$$\underbrace{\begin{bmatrix}Z_1\\ Z_2\\ \cdots\\ Z_n\end{bmatrix}_k}_{Zv_k}=\underbrace{\begin{bmatrix}\left.\frac{x-x_1}{r_1}\right|_{x=\hat{x}_k} & \left.\frac{y-y_1}{r_1}\right|_{y=y_k} & \left.\frac{z-z_1}{r_1}\right|_{z=\hat{z}_k} & 1\\ \left.\frac{x-x_2}{r_2}\right|_{x=\hat{x}_k} & \left.\frac{y-y_2}{r_2}\right|_{y=y_k} & \left.\frac{z-z_2}{r_2}\right|_{z=\hat{z}_k} & 1\\ \cdots & & & \\ \left.\frac{x-x_n}{r_n}\right|_{x=\hat{x}_k} & \left.\frac{y-y_n}{r_n}\right|_{y=y_k} & \left.\frac{z-z_n}{r_n}\right|_{z=\hat{z}_k} & 1\end{bmatrix}}_{H_k}\underbrace{\begin{bmatrix}vx\\ vy\\ vz\\ c\delta f\end{bmatrix}_k}_{V_k}+v_{vk} \qquad (13)$$

In short-hand note:

$$Zv_k = H_k * V_k + v_{vk} \qquad (14)$$

where v_{vk} is the velocity measurement noise vector at state *k*

The noise covariance matrices need to be chosen to apply Kalman filter

$$Q_{kp}=\begin{bmatrix}Sp*\tau & 0 & 0 & 0\\ 0 & Sp*\tau & 0 & 0\\ 0 & 0 & Sp*\tau & 0\\ 0 & 0 & 0 & S_f*\tau+S_g\frac{\tau^3}{3}\end{bmatrix}\qquad R_{kp}=\begin{bmatrix}r_p & 0 & 0 & 0\\ 0 & r_p & 0 & 0\\ \cdots & & & \\ 0 & 0 & 0 & r_p\end{bmatrix}$$

$$Q_{kv}=\begin{bmatrix}Sv*\tau & 0 & 0 & 0\\ 0 & Sv*\tau & 0 & 0\\ 0 & 0 & Sv*\tau & 0\\ 0 & 0 & 0 & Sg*\tau\end{bmatrix}\qquad R_{kv}=\begin{bmatrix}r_v & 0 & 0 & 0\\ 0 & r_v & 0 & 0\\ \cdots & & & \\ 0 & 0 & 0 & r_v\end{bmatrix}$$

Sp, Sv, Sg, Sf: white noise spectral amplitudes affect for each system model[1], respectively.

r_p : expected position measurement error covariance, e.g. r_p=255 for expected 15m measurement error.

r_v: expected velocity measurement error covariance, e.g. r_v=100 for expected 10m/s measurement error.

3.3 Kalman filter

The procedure to estimate position and velocity is depicted in Figure 2.

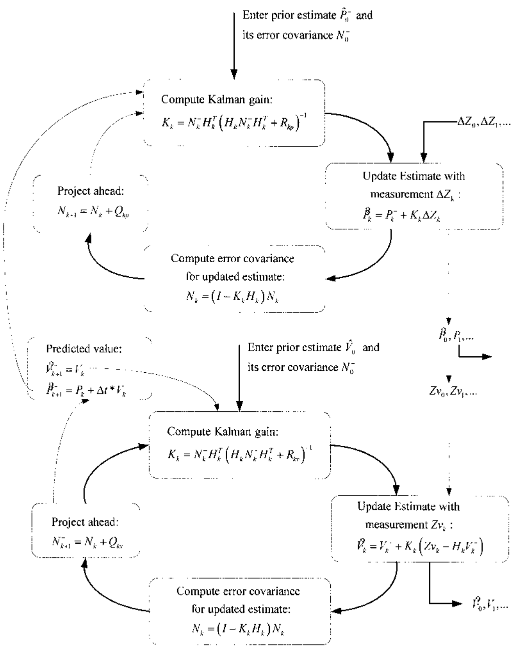
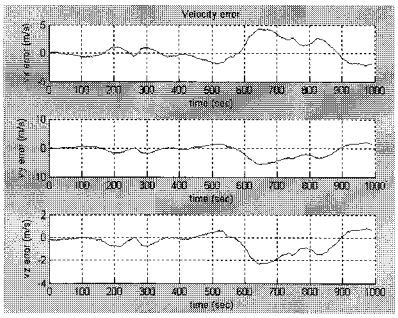
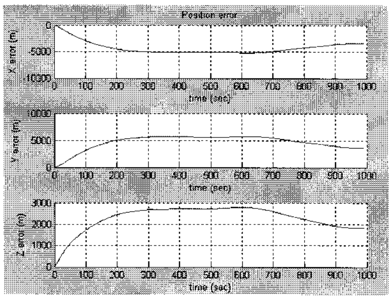


Fig. 2. 2-step Kalman filter loop



(a) Velocity error



(b) Position error

Fig. 3. Error when calculated without removal of Earth rotation bias

Fig. 2 shows that the Kalman loop is executed as 2 steps to estimate position and velocity in sequence. At first, the position model given by (9) and (12) is used for Kalman loop to estimate receiver position, with linearization in (11). From the position estimated, the measurement data for velocity estimation are calculated as (6). In the second step, Kalman filter uses the velocity model given by (10) and (14) to estimate receiver velocity, without the need of linearization as in (6). The final result will update the predicted values for the next coming estimation.

The following experiments are implemented by collecting measurement data including pseudoranges and Doppler, then using the Kalman filters in Fig. 2 written with MATLAB to estimate position and velocity.

The result when calculated with this method:

We can see that although velocity errors are small, because of the integration, position errors diverge to very large values.

The velocity error is a low frequency component and has a form of a bias. Actually in (2), we have removed the affection of the Earth rotation to D_i, and this affection depends on receiver position [2]. So a drift of receiver position [4] will cause an error on the compensated value for the Earth's rotation, and somehow, eventually (9) and (12) will cause the receiver velocity bias error. The error is integrated and accumulated because of a large drift of position.

To deal with this problem, we propose one more variable for velocity model, i.e. the velocity bias b caused by error on Earth rotation compensation.

Measurement model with bias considered:

The velocity state vector:

$V=[vx \;\; vy \;\; vz \;\; c\delta f \;\; b]^T$

where b is the velocity bias error mentioned above.

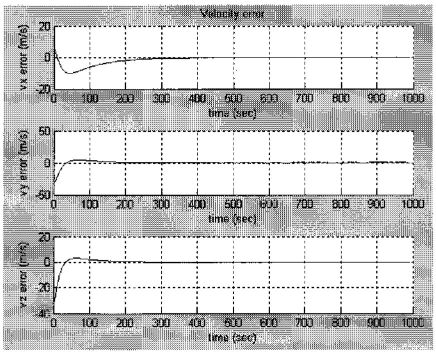
(6) becomes

$$Z_i = D_i + Dv_i = \left[\frac{(x-x_i)}{r_i} \;\; \frac{(y-y_i)}{r_i} \;\; \frac{(z-z_i)}{r_i} \;\; 1 \;\; k_v\right] * [vx \;\; vy \;\; vz \;\; c\delta f \;\; b]^T \quad (15)$$

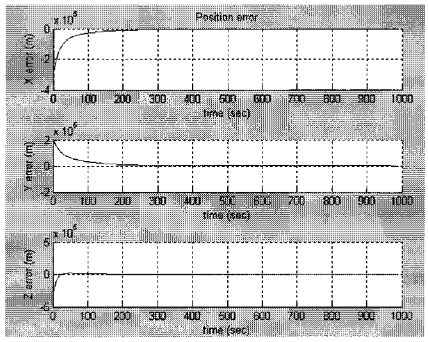
We introduce k_v as the dynamic coefficient of the receiver. If the dynamic of receiver is high, the affection of bias in the model is small, then k_v should be small and vice versa.

The measurement matrix H given in (13) and (14) now will have 5 columns

$$H_k = \begin{bmatrix} \left.\frac{x-x_1}{r_1}\right|_{x=\hat{x}_k} & \left.\frac{y-y_1}{r_1}\right|_{y=y_k} & \left.\frac{z-z_1}{r_1}\right|_{z=\hat{z}_k} & 1 & k_v \\ \left.\frac{x-x_2}{r_2}\right|_{x=\hat{x}_k} & \left.\frac{y-y_2}{r_2}\right|_{y=y_k} & \left.\frac{z-z_2}{r_2}\right|_{z=\hat{z}_k} & 1 & k_v \\ \cdots & & & & \\ \left.\frac{x-x_n}{r_n}\right|_{x=\hat{x}_k} & \left.\frac{y-y_n}{r_n}\right|_{y=y_k} & \left.\frac{z-z_n}{r_n}\right|_{z=\hat{z}_k} & 1 & k_v \end{bmatrix} \quad (16)$$



(a)Velocity error



(b)Position error

Fig. 4. Error when calculated with removal of Earth rotation bias, unknown initial position

The velocity noise covariance matrix also changes

$$Q_{kv} = \begin{bmatrix} Sv*\tau & 0 & 0 & 0 & 0 \\ 0 & Sv*\tau & 0 & 0 & 0 \\ 0 & 0 & Sv*\tau & 0 & 0 \\ 0 & 0 & 0 & Sv*\tau & 0 \\ 0 & 0 & 0 & 0 & (Sv+Sp)*\tau \end{bmatrix}$$

The bias is caused by both position and velocity error, so its noise covariance is assumed to be a sum of Sp and Sv.

All other matrices are kept unchanged. Applying the same procedure in Fig. 2, we check the convergence of the estimated model with an unknown condition:

The position errors now converge well to zero with time, confirming that the assumed bias b is right.

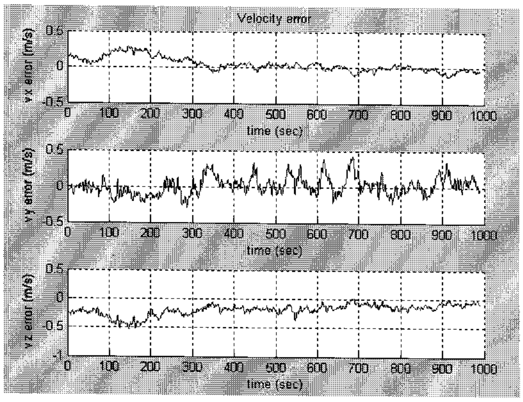
Tuning filter:

By analyzing system performance, we can tune the filter by adjusting all parameters, which are position, velocity clock model, and bias standard deviations of both system model and measurement model. In the other words, we need to tweak Q_k and R_k to get better performance. By "trial and error" testing, together with the DOP (Dilution of Precision) and URA (user range accuracy) standard [2], the final values for the tuned filter are:

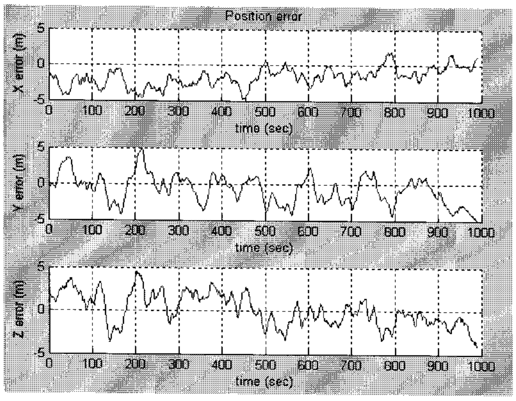
$Sp = 0.32\ m^2$: position process noise boundary is set about 0.56*m*.

$Sv = 0.3\ m^2/s^2$: velocity process noise boundary is set about 0.54*m/s*.

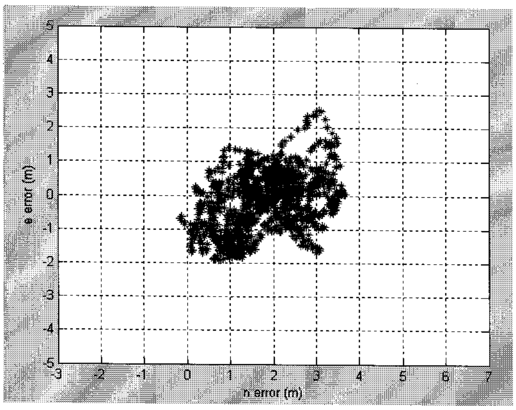
$Sg = 0.035$; $Sf=0.009$: the clock model error chosen in [1].



(a) Velocity error



(b) Position error



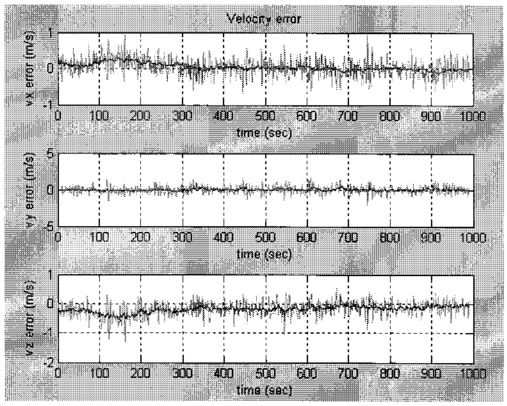
(c)North and East error

Fig. 5. Experiment result on October 20, 2006

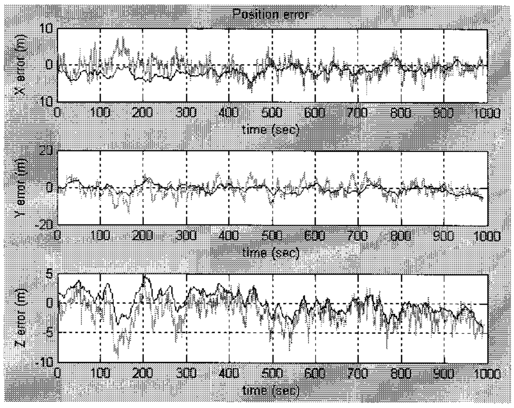
$rp = 100\ m^2$: position measurement noise boundary is set at 10m because SA (Selective Available) is turned off and also because of affection of DOP [2].

$rv = 30\ m^2/s^2$: velocity measurement noise boundary is set about 5.4m/s, rather large to enclose error bias affection.

The following result is estimated with correct initial condition. The error in the North and East direction is about 5m, with slight drift in the North direction. We compare the propose Kalman filter with Least Square method estimation to confirm the result. The dark (blue) line depicts Kalman filter error while the dim (green) one depicts Least square method error result. We can see that the



(a) Velocity error



(b) Position error

Fig. 6. Least square method used to confirm proposed Kalman filter with bias aiding

shapes depicting the two methods are quite similar, but the error range when using the Kalman filter is smaller and smother than that using the Least square method. This confirms the validation of the proposed Kalman filter model.

4. Conclusion

The 2-step Kalman filter with Earth rotation bias removed applied for estimating position and velocity is introduced and calculated. The Earth rotation bias has been successfully removed by introducing one more state variable. The experiment indicates that the position error is less than 5m during the interval of about 15-20 minutes. By using this model, the necessary manipulating matrices sizes are small, a factor that is meaningful to apply for the microprocessor to build an accurate GPS receiver module.

Acknowledgments

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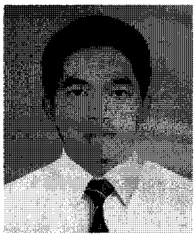
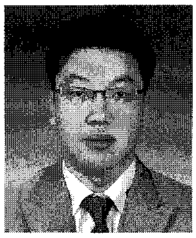


Phi-Long Nguyen

He received his B.S. degree in Electrical Engineering from Hochiminh City National University of Technology, Vietnam in 2004, and his M.S. degree in Electrical Engineering from the University of Ulsan, Korea in 2007. He is currently pursuing a Ph.D. degree at the University of Ulsan. His research interests are GPS middle ware, robust control, filtering, and signal processing.



Hyunsoo Kim

He received his B.S. degree in Electrical Engineering from Ulsan University in Korea, in 2005, and his M.S. degree in Electrical Engineering from the University of Ulsan, Korea in 2007. He is currently working at the R&D center in the POSCON Cooperation. His research interests are GPS middle ware, robust control, trajectory control, railway system control, and signal processing.

Hansil Kim

He received his B.S. and M.S. degrees in Electrical Engineering from Seoul National University, and his Ph.D. in Electrical and Computer Engineering from West Virginia University. He is currently working at the School of Electrical Engineering, University of Ulsan, Korea. His research interests are stochastic process, robust control, and signal processing.