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WEIGHTED COMPOSITION OPERATORS ON LORENTZ SPACES

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Abstract. In this paper we characterize the boundedness, compactness and closedness of the range of the weighted composition operators on Lorentz spaces $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$.

1. Introduction

Let f be a complex-valued measurable function defined on a σ-finite measure space $(X, \mathcal{A}, \mu)$. For $s \geq 0$, define μ_f the distribution function of f as

$$\mu_f(s) = \mu\{x \in X : |f(x)| > s\}.$$

By f^* we mean the non-increasing rearrangement of f given as

$$f^*(t) = \inf\{s > 0 : \mu_f(s) \leq t\}, \quad t \geq 0.$$

For $t > 0$, let

$$f^{**}(t) = \frac{1}{t}\int_0^t f^*(s)ds.$$

For a measurable function f on X, define $||f||_{pq}$ as

$$||f||_{pq} = \begin{cases} \left\{\frac{q}{p}\int_0^\infty (t^{1/p}f^{**}(t))^q \ \frac{dt}{t}\right\}^{1/q}, & 1 < p < \infty, \ 1 \leq q < \infty \\ \sup\limits_{t>0} t^{1/p}f^{**}(t), \quad 1 < p \leq \infty, \ q = \infty. \end{cases}$$

The *Lorentz space* $L(p,q)$ consists of those complex-valued measurable functions f on X such that $||f||_{pq} < \infty$. Also $||\cdot||_{pq}$ is a norm and $L(p,q)$ is a Banach space with respect to this norm ([14], Theorem 3.22, pp. 204). The L^p- spaces for $1 < p \leq \infty$ are equivalent to the spaces $L(p,p)$. For more on Lorentz spaces one can refer to [2, 5, 11, 12 and 14]. Let $T : X \to X$ be a measurable ($T^{-1}(E) \in \mathcal{A}$, for $E \in \mathcal{A}$) non-singular transformation ($\mu(T^{-1}(E)) = 0$ whenever $\mu(E)$=0) and u a complex-valued measurable function defined on X.

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We define a linear transformation $W = W_{u,T}$ on the Lorentz space $L(p,q)$ into the linear space of all complex-valued measurable functions by

$$W_{u,T}(f)(x) = u(T(x))f(T(x)),\ x \in X,\ f \in L(p,q).$$

If W is bounded with range in $L(p,q)$, then it is called a ***weighted composition operator*** on $L(p,q)$. If $u \equiv 1$, then $W \equiv C_T : f \mapsto f \circ T$ is called a ***composition operator*** induced by T. If T is identity mapping, then $W \equiv M_u : f \mapsto u \cdot f$, a ***multiplication operator*** induced by u. The study of these operators on L^p-spaces has been made in [3, 6, 7, 13 and 15] and references therein. Composition and multiplication operators on the Lorentz spaces were studied in ([9], [10]) and [1] respectively. In this paper a characterization of the non-singular measurable transformations T from X into itself and complex-valued measurable functions u on X inducing weighted composition operators is obtained and subsequently their compactness and closedness of the range on the Lorentz spaces $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$ are completely identified.

2. Characterizations

Theorem 2.1. *Let $(X,\ \mathcal{A},\ \mu)$ be a σ-finite measure space and $u : X \to \mathbb{C}$ a measurable function. Let $T : X \to X$ be a non-singular measurable transformation such that the Radon-Nikodym derivative $f_T = d(\mu T^{-1})/d\mu$ is in $L_\infty(\mu)$. Then*

$$W_{u,T} : f \mapsto u \circ T \cdot f \circ T$$

is bounded on $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$ if $u \in L_\infty(\mu)$.

Proof. Suppose $b = \|f_T\|_\infty$. Then for f in $L(p,q)$, the distribution function of Wf satisfies, where $Wf = W_{u,T}(f) = u \circ T \cdot f \circ T$,

$$\begin{aligned} \mu_{Wf}(s) &= \mu\{x \in X : |u(T(x))f(T(x))| > s\} \\ &= \mu T^{-1}\{x \in X : |u(x)||f(x)| > s\} \\ &\leq \mu T^{-1}\{x \in X : \|u\|_\infty |f(x)| > s\} \\ &\leq b\ \mu\{x \in X : \|u\|_\infty |f(x)| > s\} = b\ \mu_{\|u\|_\infty f}(s). \end{aligned}$$

Therefore, for each $t \geq 0$, we have

$$\{s > 0 : \mu_{\|u\|_\infty f}(s) \leq t/b\} \subseteq \{s > 0 : \mu_{Wf}(s) \leq t\}.$$

This gives

$$\begin{aligned} (Wf)^*(t) &= \inf\{s > 0 : \mu_{Wf}(s) \leq t\} \\ &\leq \inf\{s > 0 : \mu_{\|u\|_\infty f}(s) \leq t/b\} \\ &= \inf\{s > 0 : \mu\{x \in X : \|u\|_\infty |f(x)| > s\} \leq t/b\} \\ &= \inf\{\|u\|_\infty s > 0 : \mu\{x \in X : |f(x)| > s\} \leq t/b\} \\ &= \|u\|_\infty f^*(t/b) \end{aligned}$$

and consequently, we have

$$(Wf)^{**}(t) \leq \|u\|_\infty f^{**}(t/b).$$

Therefore, for $1 < p < \infty$, $1 \leq q < \infty$,

$$\begin{aligned} ||Wf||_{pq}^q &= \frac{q}{p}\int_0^\infty (t^{1/p}(Wf)^{**}(t))^q \frac{dt}{t} \\ &\leq ||u||_\infty^q \frac{q}{p}\int_0^\infty (t^{1/p}f^{**}(t/b))^q \frac{dt}{t} \\ &= (b^{q/p})||u||_\infty^q \; ||f||_{pq}^q. \end{aligned}$$

For $q = \infty$, $1 < p \leq \infty$, we have

$$\begin{aligned} ||Wf||_{p\infty} &= \sup_{t>0} t^{1/p}(Wf)^{**}(t) \\ &\leq ||u||_\infty \sup_{t>0} t^{1/p} f^{**}(t/b) = b^{1/p}||u||_\infty \; ||f||_{p\infty}. \end{aligned}$$

Thus, W is a bounded operator on $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$ and

$$||W||_{pq} \leq b^{1/p}||u||_\infty.$$

The result follows. □

The above theorem also holds if $u \in L_\infty(\mu T^{-1})$, that is $u \circ T \in L_\infty(\mu)$.

Theorem 2.2. *Let u be a complex-valued measurable function and $T : X \to X$ be a non-singular measurable transformation such that $T(E_\varepsilon) \subseteq E_\varepsilon$ for each $\varepsilon > 0$, where $E_\varepsilon = \{x : |u(x)| > \varepsilon\}$. If $W_{u,T}$ is bounded on $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$, then $u \in L_\infty(\mu)$.*

Proof. If possible u be not in $L_\infty(\mu)$. Then for each natural number n, the set $E_n = \{x \in X : |u(x)| > n\}$ has a positive measure. Now using the fact that $T(E_n) \subseteq E_n$ equivalently $\chi_{E_n} \leq \chi_{T^{-1}(E_n)}$, we find that

$$\begin{aligned} \{x : |\chi_{E_n}(x)| > s\} &\subseteq \{x : |\chi_{T^{-1}(E_n)}(x)| > s\} \\ &\subseteq \{x : |u(T(x))\chi_{T^{-1}(E_n)}(x)| > ns\}. \end{aligned}$$

Therefore

$$\begin{aligned} (W\chi_{E_n})^*(t) &= \inf\{s > 0 : \mu\{x : |u \circ T(x)\chi_{E_n} \circ T(x)| > s\} \leq t\} \\ &= n \inf\{s > 0 : \mu\{x : |u(T(x))\chi_{T^{-1}(E_n)}(x)| > ns\} \leq t\} \\ &\geq n \inf\{s > 0 : \mu\{x : |\chi_{E_n}(x)| > s\} \leq t\} \\ &= n\chi_{E_n}^*(t). \end{aligned}$$

Thus we have

$$\begin{aligned} (W\chi_{E_n})^{**}(t) &= \frac{1}{t}\int_0^t (W\chi_{E_n})^*(s)ds \\ &\geq n\chi_{E_n}^{**}(t). \end{aligned}$$

This gives

$$||(W\chi_{E_n})||_{pq} \geq n||\chi_{E_n}||_{pq}$$

which contradicts our assumption. Hence the result. □

Now by combining Theorem 2.1 and Theorem 2.2 we have the following:

Theorem 2.3. *Let $u : X \to \mathbb{C}$ be a measurable function and $T : X \to X$ be a non-singular measurable transformation such that the Radon-Nikodym derivative $f_T = d(\mu T^{-1})/d\mu$ is in $L_\infty(\mu)$ and $T(E_\varepsilon) \subseteq E_\varepsilon$ for each $\varepsilon > 0$, where $E_\varepsilon = \{x : |u(x)| > \varepsilon\}$. Then $W_{u,T}$ is bounded on $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$ if and only if $u \in L_\infty(\mu)$.*

Example 2.4. Let $X = [0,1]$, $\mu =$ Lebesgue measure. Consider $T(x) = \sqrt{x}$ for all $x \in X$. Then $f_T \in L_\infty(\mu)$ (by Example 5.1 and Theorem 2.3 [9]). Let u be defined as

$$u(x) = \begin{cases} 1/2, & 0 \leq x < 1/2 \\ 1, & 1/2 \leq x \leq 1. \end{cases}$$

Then $T(E_\varepsilon) \subseteq E_\varepsilon$ for each $\varepsilon > 0$, where

$$E_\varepsilon = \begin{cases} [0,1], & 0 < \varepsilon < 1/2 \\ [1/2,1], & 1/2 \leq \varepsilon < 1 \\ \phi, & \varepsilon \geq 1. \end{cases}$$

Thus $W_{u,T}$ is a weighted composition operator on $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$.

Example 2.5. Let $X = [0,1]$, $\mu =$ Lebesgue measure. Consider $T(x) = \sqrt{x}$ for all $x \in X$, and for any n, $u_n(x) = x^n$. Then we find $f_T \in L_\infty(\mu)$, $T(E_\varepsilon) \subseteq E_\varepsilon$ and $u_n \in L_\infty(\mu)$. Hence $W_{u,T}$ is a weighted composition operator on $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$.

Example 2.6. Let $X = \mathbb{R}$ and $\mathcal{A}$ be σ-algebra of Borel subsets and μ be Lebesgue measure. For $a \in \mathbb{R}, a > 0$ fixed, if we define $T : X \to X$ as $T(x) = x + a$ and $u(x) = x$ for each $x \in X$. Then $\mu T^{-1} \ll \mu$ and $f_T \in L_\infty(\mu)$. Also $T(E_\varepsilon) \subseteq E_\varepsilon$ for all $\varepsilon > 0$. But $W_{u,T}$ is not bounded as u does not belong to $L_\infty(\mu)$.

3. Compactness and closed range

In this section an effort has been made to discuss the compactness, the closed range and the injectiveness of the weighted composition operator $W = W_{u,T} : f \mapsto u \circ T \cdot f \circ T$ on the Lorentz spaces $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$.

Let $T : X \to X$ be a non-singular measurable transformation with $f_T = d(\mu T^{-1})/d\mu$, the Radon-Nikodym derivative. If $f_T \in L_\infty(\mu)$ and $b = \|f_T\|_\infty$, then for each $f \in L(p,q)$, we have for $t > 0$,

$$\begin{aligned} (Wf)^*(bt) &= \inf\{s > 0 : \mu\{x : |u(T(x))f(T(x))| > s\} \leq bt\} \\ &= \inf\{s > 0 : \mu T^{-1}\{x : |(u \cdot f)(x)| > s\} \leq bt\} \\ &\leq \inf\{s > 0 : \mu\{x : |(u \cdot f)(x)| > s\} \leq t\} = (M_u f)^*(t), \end{aligned}$$

and so we obtain

$$\|Wf\|_{pq} \leq b^{1/p}\|M_u f\|_{pq}. \tag{3.1}$$

Now if f_T is bounded away from zero on S, i.e., $f_T > \delta$ *a.e.* for some $\delta > 0$, then

$$\mu T^{-1}(E) = \int_E f_T d\mu \geq \delta\mu(E)$$

for all $E \in \mathcal{A}$, $E \subseteq S$, where $S = \{x : u(x) \neq 0\}$. Thus we find that

$$\|Wf\|_{pq} \geq \delta^{1/p}\|M_u f\|_{pq}. \tag{3.2}$$

Hence for each $f \in L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$, we have

$$\alpha\|M_u f\|_{pq} \leq \|Wf\|_{pq} \leq \beta\|M_u f\|_{pq} \tag{3.3}$$

for some α, $\beta > 0$, whenever $f_T \in L_\infty(\mu)$ and is bounded away from zero. From this observation (3.3) and the results of ([1], Theorem 3.1) or ([4], Theorem 2.4), we have the following:

Theorem 3.1. *Let $T : X \to X$ be a non-singular measurable transformation such that $f_T \in L_\infty(\mu)$ and is bounded away from zero. Let $u : X \to \mathbb{C}$ be a measurable function such that $W_{u,T}$ is bounded on the Lorentz space $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$. Then the following are equivalent:*

(i) *$W_{u,T}$ is compact,*

(ii) *M_u is compact,*

(iii) *$L^{pq}(u,\varepsilon)$ are finite dimensional for each $\varepsilon > 0$, where*

$$L^{pq}(u,\varepsilon) = \{f\chi_{(u,\varepsilon)} : f \in L(p,q)\} \text{ and } (u,\varepsilon) = \{x \in X : |u(x)| \geq \varepsilon\}.$$

Since $W_{u,T} = C_T M_u$, a sufficient condition for the compactness of the weighted composition operator $W_{u,T}$ on $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$ follows using ([9], Theorem 3.1).

Theorem 3.2. *Let $T : X \to X$ be a non-singular measurable transformation with $f_T \in L_\infty(\mu)$ and $u : X \to \mathbb{C}$ be a measurable function such that $u \in L_\infty(\mu)$. Let $\{A_n\}$ be all the atoms of X with $\mu(A_n) > 0$, for each n. Then $W_{u,T}$ is compact on the Lorentz space $L(p,q)$, $1 < p < \infty$, $1 \leq q \leq \infty$ if μ is purely atomic and*

$$b_n = \frac{\mu T^{-1}(A_n)}{\mu(A_n)} \to 0.$$

Theorem 3.3. *If μ is non-atomic measure and $W_{u,T}$ is bounded on the Lorentz space $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$. Then $W_{u,T}$ is compact if and only if $u \cdot f_T = 0$ a.e.*

Proof. Suppose $W = W_{u,T}$ is compact. If $u \cdot f_T \neq 0$, then there exists some $k \geq 1$ such that the set

$$E = \{x \in X : |u(x)| \text{ and } f_T(x) > 1/k\}$$

has positive measure. Since μ is non-atomic we can find a sequence $\{E_n\}$ of measurable subsets of E such that $E_{n+1} \subseteq E_n$,

$$\mu(E_n) = \frac{a}{2^n},\ 0 < a < \mu(E).$$

Let $e_n = \chi_{E_n}/||\chi_{E_n}||_{pq}$. Then $\{e_n\}$ is a bounded sequence in $L(p,q)$. For $m, n \in \mathbb{N}$, let $m = 2n$, then for $t \geq 0$,

$$\begin{aligned}
&(We_n - We_m)^*(t/k) \\
= \ & \inf\{s > 0 : \mu\{x \in X : |u(Tx)e_n(Tx) - u(Tx)e_m(Tx)| > s\} \leq t/k\} \\
= \ & \inf\{s > 0 : \mu T^{-1}\{y \in E_n : |u(y)||e_n(y) - e_m(y)| > s\} \leq t/k\} \\
\geq \ & \inf\{s > 0 : \mu\{y \in E_n : |e_n(y) - e_m(y)| > sk\} \leq t\} \\
= \ & (1/k)\inf\{s > 0 : \mu\{y \in E_n : |e_n(y) - e_m(y)| > s\} \leq t\} \\
\geq \ & (1/k)\inf\{s > 0 : \mu\{y \in E_n \setminus E_m : |e_n(y) - e_m(y)| > s\} \leq t\}.
\end{aligned}$$

This gives

$$(We_n - We_m)^*(t/k) \ \geq \ \frac{1}{k}\frac{(\chi_{E_n \setminus E_m})^*(t)}{||\chi_{E_n}||_{pq}}.$$

Hence

$$||We_n - We_m||_{pq} \ \geq \ \frac{1}{k^2}\left(\frac{\mu(E_n \setminus E_m)}{\mu(E_n)}\right)^{1/p} \geq \varepsilon,$$

for some $\varepsilon > 0$ and large values of n.

Thus $\{We_n\}$ does not admit a convergent subsequence, which contradicts the compactness of W. Hence $u \cdot f_T = 0$ *a.e.*

The converse is obvious. □

The following theorem establishes a characterization for a weighted composition operator to have a closed range on $L(p,q)$.

Theorem 3.4. *Let $T : X \to X$ be a non-singular measurable transformation with f_T in $L_\infty(\mu)$ and is bounded away from zero. Let $u : X \to \mathbb{C}$ be a measurable function such that $W_{u,T}$ is bounded on the Lorentz space $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$. Then $W_{u,T}$ has closed range if and only if there exists a $\delta > 0$ such that*

$$|u(x)| \geq \delta \ a.e.$$

on $S = \{x \in X : u(x) \neq 0\}$, the support of u.

Proof. Suppose that $W = W_{u,T}$ has closed range, then there exists an $\varepsilon > 0$ such that

$$||Wf||_{pq} \geq \varepsilon||f||_{pq},$$

for all $f \in L^{p,q}(S)$, where

$$L^{p,q}(S) = \{f\chi_S : f \in L(p,q)\}.$$

Choose $\delta > 0 :$ $b^{1/p}\delta < \varepsilon$, where $b = ||f_T||_\infty$.

If possible $E = \{x \in X : |u(x)| < \delta\}$ has positive measure, that is $0 < \mu(E) < \infty$ and so $\chi_E \in L^{pq}(S)$. Therefore using (3.1) we obtain

$$\begin{aligned}
||W\chi_E||_{pq} \ &\leq \ b^{1/p}||u \cdot \chi_E||_{pq} \\
&\leq \ b^{1/p}\delta||\chi_E||_{pq} < \varepsilon||\chi_E||_{pq},
\end{aligned}$$

which is a contradiction. Therefore $|u(x)| \geq \delta$ *a.e.* on S.

Conversely if $|u(x)| \geq \delta$ *a.e.* on S, then using $f_T > k$ *a.e.* for some $k > 0$ and (3.2) we have, for $f \in L^{pq}(S)$,

$$\begin{aligned} \|Wf\|_{pq} &\geq k^{1/p}\|u \cdot f\|_{pq} \\ &\geq k^{1/p}\delta\|f\|_{pq}. \end{aligned}$$

Therefore W has closed range being $\ker(W) = L^{pq}(X \setminus S)$. □

Corollary 3.5. *If $T^{-1}(E_\varepsilon) \subseteq E_\varepsilon$ for each $\varepsilon > 0$ and $W_{u,T}$ has closed range, then $|u(x)| \geq \delta$ a.e. on $S = \{x \in X : u(x) \neq 0\}$ for some $\delta > 0$.*

The following Theorem follows directly using the observation (3.3) and ([1], Theorem 4.1).

Theorem 3.6. *Let $T : X \to X$ be a non-singular measurable transformation such that $f_T \in L_\infty(\mu)$ and is bounded away from zero. Let $u : X \to \mathbb{C}$ be a measurable function such that $W_{u,T}$ is bounded on the Lorentz space $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$. Then the following are equivalent:*

(1) *$W_{u,T}$ has closed range,*
(2) *M_u has closed range,*
(3) *$|u(x)| \geq \delta$ a.e. for some $\delta > 0$ on $S = \{x : u(x) \neq 0\}$.*

Theorem 3.7. *If μ is non-atomic measure and $W_{u,T}$ is bounded on Lorentz space $L(p,q)$, $1 < p \leq \infty$, $1 \leq q \leq \infty$, then $W_{u,T}$ is injective if and only if $u \circ T \neq 0$ a.e. and T is surjective.*

Proof. Suppose that $W_{u,T}$ is injective. If T is not surjective then there exists a measurable set $F \subseteq X \setminus T(X)$ such that $0 \neq \chi_F \in L(p,q)$, then $W_{u,T}\chi_F = 0$ so that $W_{u,T}$ is not injective, a contradiction. Further, suppose there exists a measurable set $E = \{x \in X : |u \circ T(x)| = 0\}$ such that $\mu(E) > 0$. Then we can find a measurable set A such that $T^{-1}(A) \subseteq E$ and $\mu(A) < \infty$. Then $\chi_A \in L(p,q)$. Also $(u \circ T \cdot \chi_A \circ T)^*(t) = 0$, for all t. This gives a non-trivial kernel of $W_{u,T}$, which is a contradiction. Hence $u \circ T \neq 0$ *a.e.*

The converse part is obvious. □

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