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A Fuzzy-Goal Programming Approach For Bilevel Linear Multiple Objective Decision Making Problem*

S.R. Arora
Department of Mathematics, Hans Raj College University of Delhi, Delhi

Ritu Gupta**
Department of Mathematics, University of Delhi, Delhi

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ABSTRACT

This paper presents a fuzzy-goal programming (FGP) approach for Bi-Level Linear Multiple Objective Decision Making (BLL-MODM) problem in a large hierarchical decision making and planning organization. The proposed approach combines the attractive features of both fuzzy set theory and goal programming (GP) for MODM problem. The GP problem has been developed by fixing the weights and aspiration levels for generating pareto-optimal (satisfactory) solution at each level for BLL-MODM problem. The higher level decision maker (HLDM) provides the preferred values of decision vector under his control and bounds of his objective function to direct the lower level decision maker (LLDM) to search for his solution in the right direction. Illustrative numerical example is provided to demonstrate the proposed approach.

Key words: Bi-level Programming, Multiple Objective Decision Making, Fuzzy Set Theory, Goal Programming, Pareto-Optimal Solution, Aspiration Levels, Membership function

1. Introduction

Hierarchical optimization or multi-level programming (MLP) problems are the characterization of mathematical programming to solve decentralized planning problems

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** Corresponding author, E-mail: ritu_pv@yahoo.com

with multiple DMs where each unit seeks its own interest.

Bilevel programming (BLP) is the simplest case of MLP problem, with a structure of two levels in a hierarchical decision system. BLP is a sequence of two optimization problems in which the constraint region of one is determined implicitly by the solution of the second. BLP structure is used for central economic planning at the regional or national level such as to model problems concerning organizational design, facility location, signal optimization and traffic assignment. In BLPP, two decision makers (DMs) located at two different hierarchical levels, each independently controls a set of decision variables with conflicting objectives. The lower level DM, called the follower, executes its policies after the decision of the higher level DM, called the leader and then the leader optimizes its objective independently but may be affected by the reaction of the follower.

The formal formulation of the BLPP was first defined by Candler and Townsley [3] as well as Fortuny-Amat and Mc Carl [8]. During the past two decades of last century, several approaches for solving BLPPs have been deeply studied such as vertex enumeration approach [3, 20], the Kuhn-Tucker approach [3, 6] and multiple objective linear programming techniques [1]. For methods developed for linear BLPP, vertex enumeration and Kuhn-Tucker solution approaches have been used widely in searching for the optimal solutions [21]. A few studies have proposed methods for bilevel multiple objective decision making problem and planning organization.

Now in a hierarchical decision making context, it has been realized that each DM should have a motivation to cooperate with each other and minimum level of satisfaction of a DM at a lower level must be considered for the over all benefit of the organization. In this sequel, the concept of fuzzy set theory has been introduced by Lai to solve BLPPs as well as MLPPs in 1996 [10]. Lai's solution concept has been further extended by Shih *et al* [11]. In these approaches, the problem is revaluated again and again by redefining the membership functions. So the computational load is inherently associated with these approaches. To overcome such a situation, fuzzy goal programming (FGP) approach to BLPP has been recently studied by Moitra and Pal [14]. They have presented a fuzzy goal programming procedure for solving linear bilevel programming problems. In their approach, they have used the concept of tolerance membership function for measuring the degree of satisfaction of the objectives of the decision makers at both the levels and the vector of decision variables controlled by the leader. They developed a linear programming model by using distance function to minimize the group regret of degree of satisfaction of both the DMs. In their decision process, they have transformed the linear programming model into an equivalent fuzzy goal programming model to achieve the highest degree (unity) of each of the defined membership goals to the extent possible by minimizing their deviational variables and thereby obtaining the most satisfactory solution for both the decision makers.

This paper extends the FGP approach to solve the proposed bi-level linear programming problems with multiple objectives at both the levels. In most of the multi-objective decision making problems, the objectives are competitive, incommensurable and often conflicting in nature. Such problems involve trade-off relations among the objectives to get the "optimal compromise solution." Goal Programming introduced by Charnes and Cooper in 1961 [4] appeared as a robust tool to solve these type of problems. However, its main disadvantage is that both the aspiration level of goals and their priorities must be specified priori by the Decision Maker (DM). Most often the DM has no knowledge about fixing the priorities and goals to the objectives. This may lead to wrong results. The proposed method uses the concept of conflict among the objectives and theory of fuzzy sets to obtain the optimal compromise solution of the DMs at both the levels. The objectives should have the goals and weights according to its degree of conflict with the other objectives. This would improve the overall satisfaction of the system.

In the model formulation of the problem, the fuzzy goals of the objectives as well as the decision vector controlled by the leader are defined first. The fuzzy goals are then characterized by the associated membership functions. In the solution process, the weights and aspiration levels of each objective is obtained by using the concept of conflict between the objectives. Then the objective functions are defined as flexible goals by introducing under-and over-deviational variables and assigning the aspiration level to each of them. Then in the achievement function, minimization of under-deviational variables for achieving the objective goals to the assigned aspiration levels is taken into account. The problem so formed is solved to obtain a satisfactory solution for both the decision makers. Generally combined with the set of control variables and objectives with tolerances, the satisfactory solution obtained by FGP model are pareto-optimal solution for both DMs. A numerical example is given to illustrate the proposed algorithm.

2. Problem Definition Of Bll-Modm

Let the decision problem be such that there is a cooperative relationship between the leader and the follower and each of them is interested in optimizing his/her own objective function paying serious attention to the interest of the other.

Let the vector of decision variables $X_1 \in R^{n_1}$ and $X_2 \in R^{n_2}$ be under the control of leader and follower respectively, $n_1, n_2 \geq 1$, $n = n_1 + n_2$. Let $F_1 : R^{n_1} \times R^{n_2} \rightarrow R^{N_1}$ and $F_2 : R^{n_1} \times R^{n_2} \rightarrow R^{N_2}$ be the leader's and the follower's objective functions respectively, $N_1, N_2 \geq 2$. Let the leader and the follower have N_1 and N_2 objective functions respectively. Let S be the set of feasible choices $\{(X_1, X_2)\}$.

So the BLL-MODM may be formulated as follows [17] :

(P1) $\underset{X_1}{Max}\ F1(X_1, X_2) = \underset{X_1}{Max}\ \{f1\,1(X_1, X_2), f1\,2(X_1, X_2), \cdots, f_{1N_1}(X_1, X_2)\}$

where for a given X_1, X_2 solves

$\underset{X_2}{Max}\ F2(X_1, X_2) = \underset{X_2}{Max}\ \{f2\,1(X_1, X_2), f2\,2(X_1, X_2), \cdots, f_{2N_2}(X_1, X_2)\}$

subject to $(X_1, X_2) \in S = \{A_1 X_1 + A_2 X_2 \leq b, X_1, X_2 \geq 0\}$

where it is assumed that $S\ (\neq \phi)$ is bounded and $A = [A_1, A_2]$ is an $m \times n$ matrix and $b \in R^m$.

The decision mechanism of BLL-MODM is that the two DMs adopt the leader - follower Stackelberg game [6, 18]. Stackelberg games were first proposed by economist Von Stackelberg(1934) to solve non-cooperative decision problems where in one of the players (the leader) has the ability to enforce his/her decision on the other player(s) (the follower(s)).That is the leader announces his/her decision in advance and then the follower(s) responds in a way of maximizing his/her own objective. According to the leader – follower Stackelberg game [6, 18] and mathematical programming, the definitions of the solution for the model BLL-MODM are:

Definition 1 [17] : For any X_1 ($X_1 \in S_0 = \{X_1 \mid (X_1, X_2) \in S\}$) given by the leader, if the decision making variable X_2 ($X_2 \in S_1 = \{X_2 \mid (X_1, X_2) \in S\}$) at the lower level is the pareto-optimal solution of the follower then (X_1, X_2) is a feasible solution of the model BLL-MODM.

Definition 2 [17] : If (X_1^*, X_2^*) is a feasible solution of BLL-MODM and there does not exist any other feasible solution $(X_1, X_2) \in S$ such that $f_{1j}(X_1^*, X_2^*) \leq f_{1j}(X_1, X_2)$; atleast one j (j = 1, ⋯, N1) is strict inequality then (X_1^*, X_2^*) is the pareto-optimal solution of the model BLL-MODM.

It is proved that if the follower's response to every decision of the leader is unique, the leader's Stackelberg solution will not be worse than his/her nash solution, but the follower might be worse off because of his/her position in the decision making process.

3. Review Of Existing Fgp Methods

FGP is an extension of conventional goal programming introduced by Charnes and Cooper [4] in 1961. As a robust tool for MODM problems, GP has been studied extensively for the last 35 years. In the recent past FGP in the form of classical GP has been introduced by Mohamed [12] and further studied by B.B. Pal and Moitra [16]. Recently Moitra and Pal [14] have applied FGP to solve bilevel linear programming problems of the following form:

(P2)

$$\max_{X_1} f_1(X_1, X_2) = C_{11}X_1 + C_{12}X_2$$

where for a given X_1, X_2 solves

$$\max_{X_2} f_2(X_1, X_2) = C_{21}X_1 + C_{22}X_2$$

$$\text{subject to} \quad A_1 X_1 + A_2 X_2 \leq b$$

$$X_1, X_2 \geq 0$$

where C_{11}, C_{12}, C_{21}, C_{22} and b are constant vectors and A_1 and A_2 are constant matrices. The functions f_1 and f_2 are assumed to be linear and bounded.

To formulate the FGP model of the problem (P2), they have converted the objective functions f_1 and f_2 and the decision vector X_1 of the leader into fuzzy goals by assigning an aspiration level to each of them. These goals are characterized by the membership functions, defined as

$$\mu_{f_1} = \begin{cases} 1 & \text{if} \quad f_1(X_1, X_2) > f_1^U \\ \dfrac{f_1(X_1, X_2) - f_1^L}{f_1^U - f_1^L} & \text{if} \quad f_1^L \leq f_1(X_1, X_2) \leq f_1^U \\ 0 & \text{if} \quad f_1(X_1, X_2) < f_1^L \end{cases}$$

$$\mu_{f_2} = \begin{cases} 1 & \text{if} \quad f_2(X_1, X_2) > f_2^L \\ \dfrac{f_2(X_1, X_2) - f_2^U}{f_2^L - f_2^U} & \text{if} \quad f_2^U \le f_2(X_1, X_2) \le f_2^L \\ 0 & \text{if} \quad f_2(X_1, X_2) < f_2^U \end{cases}$$

$$\mu_{X_1} = \begin{cases} 1 & \text{if} \quad X_1 > X_1^U \\ \dfrac{X_1 - X_1^m}{X_1^U - X_1^m} & \text{if} \quad X_1^m \le X_1 \le X_1^U \\ 0 & \text{if} \quad X_1 < X_1^m \end{cases}$$

where (X_1^U, X_2^U ; f_1^U) and (X_1^L, X_2^L ; f_2^L) are the optimal solutions of the leader and follower respectively when calculated in isolation. X_1^m ($X_1^L \le X_1^m \le X_1^U$) is the tolerance limit of the fuzzy decision goal of the leader.

The membership goals are assigned the highest aspiration level 1. Under the framework of minsum GP, the FGP model of the problem (P2) is formulated as:

(P3) Minimize $z = w_1 d_1^- + w_2 d_2^- + w_3 d_3^-$

subject to $\dfrac{f_1(X_1, X_2) - f_1^L}{f_1^U - f_1^L} + d_1^- - d_1^+ = 1$

$\dfrac{f_2(X_1, X_2) - f_2^U}{f_2^L - f_2^U} + d_2^- - d_2^+ = 1$

$\dfrac{X_1 - X_1^m}{X_1^U - X_1^m} + d_3^- - d_3^+ = I_1$

$A_1 X_1 + A_2 X_2 \le b$

$X_1, X_2 \ge 0$

$d_i^-, d_i^+ \ge 0$ *with* $d_i^- \cdot d_i^+ = 0;\ i = 1, 2, 3.$

Where d_i^- and d_i^+ represent the vectors of deviational variables from the aspired levels of the respective fuzzy goals and z represents the fuzzy achievement function consisting of the weighted under-deviational variables. I_1 is the row vector with all elements equal to 1 and the dimension of it depends on μ_{X1} or X_1. The numerical weights w_i (≥ 0) associated with d_i^-, I = 1, 2, 3 represent the relative importance of achieving the aspired levels of the respective fuzzy goals subject to the given set of constraint.

The values of w_i are determined as

$$w_1 = \frac{1}{f_1^U - f_1^L}, \qquad w_2 = \frac{1}{f_2^L - f_2^U}, \qquad w_3 = \frac{1}{X_1^U - X_1^m}$$

The FGP model (P3) provides the most satisfactory solution.

This paper studies the FGP approach to solve the bilevel linear programming problem with multiple objectives at both the levels.

4. Proposed Fgp Method

To solve the BLL-MODM by adopting the leader-follower Stackelberg game and GP model, one first obtains the satisfactory solution that is acceptable to leader and then gives the leader's goal and decision vector some relaxation to the follower to seek the satisfactory solution and to arrive at the solution which is closest to the satisfactory solution of the leader. Due to this the follower not only optimize the objective functions but also tries to satisfy the leader's goal and preferences as much as possible.

In this way, the solution method simplifies a BLL-MODM problem by transforming it into separate MODM problem at higher and lower levels, by that means the difficulty associated with non-convexity to arrive at an optimal solution is avoided.

4.1 Basic Idea (On The Weighting System And Aspiration Level) : Concept Of Conflict And Membership Function

In real life problems, the decision environment and the situations are often imprecisely defined. The technique based on the theory of fuzzy sets reflect the true realities. Fuzzy programming based techniques for solving the MODM problems are given in [23]. Also in most of the multiple objective decision making problems, the objectives are competitive, incommensurable and often conflicting in nature. To deal quantitatively with these types of qualitative situations and hence to arrive at a satisfactory solution very often it requires an appropriate compromise among the objectives. That is, trade-off relations among the fuzzily defined objectives are viable to get the "optimal compromise" solution. GP method is a robust tool to deal with such situations As the objectives functions are fuzzy it will be more realistic if we can establish this trade-off relation in the form of fuzzy sets. FGP method based on weights and aspiration levels [13, 19] is employed to convert the MODM problem into equivalent GP problem by appropriately fixing the weights and goals to the objectives. The method uses the concept of conflict among the objectives and Zimmermann's membership function approach to achieve this trade-off relation in the form of fuzzy sets. This membership function helps in linearly expressing the degrees of conflict among the objectives. It is also numerically explicit in the implicitly defined conflicts among the objectives. This approach is more compatible with Cohon's [5] gradient method.

The membership functions for the objectives give an insight with regard to fixing the weights and aspiration levels of the objectives. The angle θ between the gradients of a pair of objectives characterizes the degree of conflict between the two objectives. The degree of conflict is 0 when $\theta = 0$ and maximum when $\theta = \pi$. As a result of this approach, a function of conflict between pairs of objectives which uses the theory of fuzzy sets can be constructed. These functions always incorporate a pair of objectives at a time and the conflict functions are defined for the objectives pairwise.

A symmetric matrix is then constructed where the entries of the matrix indicate a numerical measure of the degrees of non-conflict among the objectives. This matrix enables to derive the weights and the aspiration level of the objectives.

4.2 Detailed Description on the Proposed Method

To formulate the decision making model of the problem (P1), the problem under consideration is solved in two phases : (i) at the first phase the FGP model of the MODM problem for the leader and the follower are formulated and an efficient solution at each level is obtained. (ii) at the second phase, the leader and the follower discloses their solution and the FGP model of the bilevel MODM problem is considered to obtain the most satisfactory solution.

4.2.1 First Phase : Fgp Model of The Modm Problem

To formulate the FGP model of the MODM problem, the system constraints are treated here as defining a conventional (crisp) feasible solution set over which the achievement of the fuzzy goals to their aspired levels is determined. Weights and the aspiration level of the objectives can be defined by using the concept of membership function introduced by Zimmermann [23] and the concept of conflict among the objectives.

Consider the linear MODM problem

(P4) $\underset{X}{Max} \{ f_1(X) = C_1X, f_2(X) = C_2X, \cdots, f_K(X) = C_KX\}$

subject to $X \in S = \{ A_1 X_1 + A_2 X_2 \leq b, X_1, X_2 \geq 0\}$

where :

a) the C_kX's, k = 1, 2,$\cdots$, K are the linear criterion functions; and

b) the f_k's, k = 1, 2,$\cdots$, K are the values of the criterion functions.

Construction of Membership Function

Using the concept of fuzzy sets, the membership functions can be defined based on the following steps given by Zimmermann [23].

Step1 : Find the individual best solutions (f_k^{max}) for each of the objective, where

$f_k^{max} = \max_{X \in S} f_k(X)$, k = 1, 2,$\cdots$, K.

Let X_k^* (k = 1, 2$\cdots$,K) be the optimal solution to the objectives fk(X).

Step2 : Find $f_k^{min} = \min_k f_k(X_k^*)$ $\forall$ k = 1, 2,$\cdots$, K

Step3 : Define the membership function for the objective fk(X) as

$$\mu_k(f_k) = \begin{cases} 1 & \text{if } f_k(X) > f_k^{max} \\ \dfrac{f_k(X) - f_k^{min}}{f_k^{max} - f_k^{min}} & \text{if } f_k^{min} \leq f_k(X) \leq f_k^{max} \\ 0 & \text{if } f_k(X) < f_k^{min} \end{cases} \qquad (1)$$

The membership function for the objectives which are to be minimized can be obtained in the similar fashion.

Concept of Conflict and Non-Conflict

Let $(c_{r1}, c_{r2},\cdots,c_{rk})$ and $(c_{s1}, c_{s2},\cdots c_{sk})$ be the gradients of the two objectives f_r and f_s respectively. The region of conflict can be measured by determining the angle θ_{rs} which lies between the gradients of f_r and f_s as follows:

$$Cos\theta_{rs} = \frac{\sum_{k=1}^{K} c_{rk}c_{sk}}{\sqrt{\sum_{k=1}^{K} c_{rk}^2 \sum_{k=1}^{K} c_{sk}^2}} \tag{2}$$

From Cohon's procedure [5], the simultaneous achievement of objectives f_r and f_s is possible if $\theta_{rs} = 0$, in this case the gradients of both the objectives f_r and f_s are simultaneously in the same increasing direction and there is no conflict between them and the situation of conflict arises when $\theta_{rs} \neq 0$, i.e. when the gradients of f_r and f_s are not coincident. The degree of conflict increases when $\theta_{rs} \in [\pi/2, \pi]$. This becomes maximum when $\theta_{rs} = \pi$. In this

case the gradients of increasing directions for both the objectives f_r and f_s are opposite to each other.

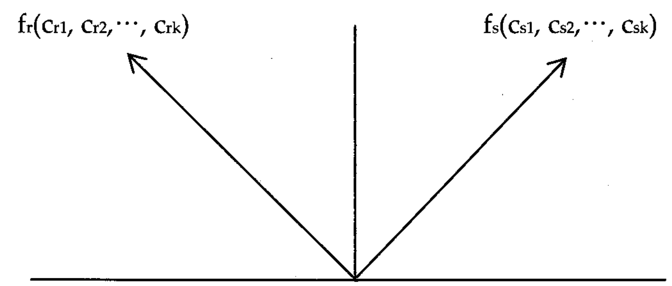


Figure 1. Degree of conflict between objectives using local gradients

The function of non-conflict between f_r and f_s is defined as follows:

$$\eta_{f_r f_s} = \begin{cases} 1 & \text{if } \theta_{rs} = 0 \\ \dfrac{\pi - \theta_{rs}}{\pi} & \text{if } 0 \leq \theta_{rs} \leq \pi \\ 0 & \text{if } \theta_{rs} = \pi \end{cases} \tag{3}$$

Using the concept of non-conflict between the objectives, a symmetric matrix ($\eta_{f_r f_s} = \eta_{f_s f_r}$), where the entries of the matrix indicate a numerical measure of the degrees of non-conflict among the objectives and the extent $\eta_{f_r f_s}$ to which the objective is non-conflicting with the other objectives, is then constructed as follows:

$$\Delta = \eta_{f_r f_s} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_K \end{array} \begin{pmatrix} f_1 & f_2 & \cdots & f_K \\ 1 & \eta_{f_1 f_2} & \cdots & \eta_{f_1 f_K} \\ \eta_{f_2 f_1} & 1 & \cdots & \eta_{f_2 f_K} \\ \vdots & \vdots & \vdots & \vdots \\ \eta_{f_K f_1} & \eta_{f_K f_2} & \cdots & 1 \end{pmatrix} \tag{4}$$

This matrix facilitates in obtaining the weights and the aspiration levels of the objectives.

Determination of Weights and Aspiration Level

Step1 : Using the concept of symmetric matrix, the total amount of support, the objective f_k gets from all the objectives is defined as follows:

$$wk = \frac{\sum_{s=1}^{K} \eta_{f_k f_s}}{K}, \ k = 1, 2, \cdots, K. \tag{5}$$

where K is used as denominator for the purpose of normalizing. w_k can be interpreted as an aggregate measure to denote how far the objective f_k has a non-conflict with the other objectives. Thus, the measure of importance w_k may be associated with the objective f_k, k = 1, 2,⋯, K.

Step2 : Depending on the values of w_k associated with the objectives, the aspiration level can be reasonably defined by using the membership function in equation (5). The extent of non-conflict w_k of a particular objective is incorporated in the inverse of membership function, to obtain the aspiration level. It is defined as

$$\mu_{f_k^{-1}}(wk) = Gk \quad k = 1, 2, \cdots, K \tag{6}$$

This is because an objective should have an aspiration level depending on its degree of non-interaction with the other objectives.

Now, for the weights and the aspiration levels obtained in (5) and (6) the FGP model of the MODM problem (P4) can be presented as

$$\textbf{(P5)}\quad \text{Minimize } Z = \sum_{k=1}^{K} w_k d_k^-$$

$$\text{subject to}\quad f_k(X) + d_k^- - d_k^+ = G_k,\ k = 1, 2, \cdots, K \tag{7}$$

$$X \in S$$

$$d_k^+ . d_k^- = 0, \qquad d_k^+,\ d_k^- \geq 0, \qquad k = 1, \cdots, K$$

where d_k^+, d_k^- represent the vectors of deviational variables from the aspired levels of the respective fuzzy goals.

The solution procedure of this section is summarized as follows:

Algorithm (I) :

Step 1 : Define the membership functions of the objectives $f_k(X)$ as given in equation (1)

Step 2 : Determine the angle θ_{rs} between the objectives f_r and f_s as given in equation (2).

Step 3 : Define the function of non-conflict between f_r and f_s as given by equation (3).

Step 4 : Arrange the values of $\eta_{f_r f_s}$ which results from step 3 in the form ofsymmetric matrix Δ as given in equation (4).

Step 5 : Determine the weights associated with each of the objectives f_k, $k = 1, \cdots, K$ as given by equation (5).

Step 6 : Determine the aspiration level of each of the objectives f_k, $k = 1, \cdots, K$ as given by equation (6).

Step 7 : Convert the objective functions into goals by introducing under-and over-deviational variables.

Step 8 : Formulate the FGP model of the problem (P4) as in (7).

Formulate the FGP model of the MODM problem for the leader and the follower by using the above algorithm and obtain the satisfactory solution for each of them.

4.2.2 Second Phase : Fgp Approach to Bll-Modm Problem

To formulate the FGP model of the BLL-MODM problem under consideration, the objective functions at both the levels and the decision vector X1 are required to be transformed into fuzzy goals by means of assigning an aspiration level to each of them. Then they are characterized by defining the tolerance limits for the achievement of the aspired levels of the goals.

Construction of Membership functions

Since the DM's of both the levels are interested in optimizing their individual benefits over the same feasible region, the satisfactory solution for them, when calculated in isolation, can be considered as the aspiration level for the associated fuzzy goal.

Let (X_1^H, X_2^H, f_{1k}^H ; k = 1,⋯, N_1) and (X_1^L, X_2^L, f_{2k}^L ; k = 1,⋯, N_2) be the satisfactory solutions of the leader's and the follower's MODM problem respectively obtained by using algorithm (I).

Now the above solutions of the leader and the follower are disclosed. If their individual satisfactory solutions are same then the satisfactory solution of BLL-MODM has been reached. But this rarely happens due to the conflict in nature between two levels objective functions. The leader knows that using the optimal decision X_1^H as a control factor for the follower is not practical. It is more reasonable to have some tolerance that gives the follower a feasible region to search for his optimal solution and also reduce searching time or interaction. In this way the range of the decision variable X_1 should be "around X_1^H with its maximum tolerance t_1" and the following membership function can be stated as

$$\mu_{X_1} = \begin{cases} \dfrac{X_1 - (X_1^H - t_1)}{t_1} & \text{if } X_1^H - t_1 \le X_1 \le X_1^H \\ \dfrac{(X_1^H + t_1) - X_1}{t_1} & \text{if } X_1^H \le X_1 \le X_1^H + t_1 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where X_1^H is the most preferred solution; ($X_1^H - t$) and ($X_1^H + t$) are the worst acceptable decision and the satisfaction is linearly increasing within the interval [$X_1^H - t$, X_1^H] and linearly decreasing within the interval [X_1^H, $X_1^H + t$] and the other decisions are not acceptable. The membership function is a triangular fuzzy number.

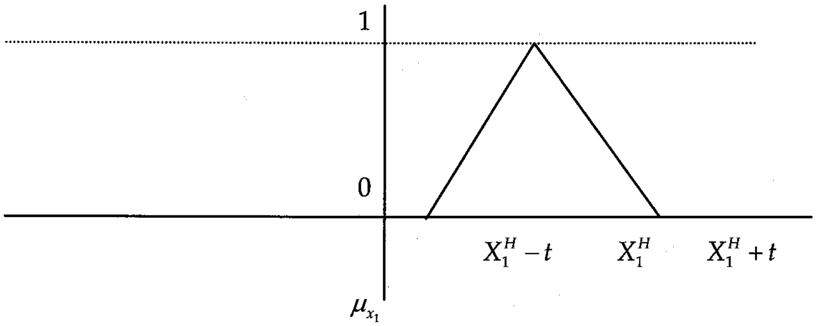


Figure 2. Degree of satisfaction of the vector controlled by the leader

The leader must specify his objective functions within the stipulated bounds to the follower to direct/supervise him to search for his solution in the correct direction. The upper bound (f_{1k}^u) and the lower bound (f_{1k}^l) on the objective functions f_{1k} ; k = 1, ⋯, N_1 can be obtained from the satisfactory solutions of the leader and the follower as

$$f_{1k}^u = \text{Max}\,(f_{1k}^H,\ f_{1k}^L) \text{ and } \quad f_{1k}^l = \text{Min}\,(f_{1k}^H,\ f_{1k}^L) \tag{9}$$

This is due to the fact that at each level we have MODM problem which have conflicting objectives, so it may happen that the solution of the second level may give a better value of f_{1k}; k = 1,⋯, N_1 than the solution of the higher level. We use linear membership function to model this information. Diagramatically we illustrate the membership function of f_{1k}; k = 1,⋯, N_1 (see fig. 3).

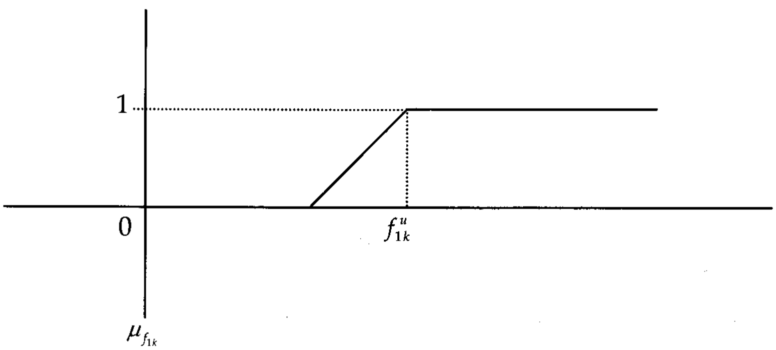


Figure 3. Degree of satisfaction of f1k

Therefore the linear membership function is

$$\mu_{f_{1k}} = \begin{cases} 1 & \text{if } f_{1k}(X) > f_{1k}^{u} \\ \dfrac{f_{1k}(X) - f_{1k}^{l}}{f_{1k}^{u} - f_{1k}^{l}} & \text{if } f_{1k}^{l} \le f_{1k}(X) \le f_{1k}^{u} \ ; \quad k = 1, \cdots, N_1 \\ 0 & \text{if } f_{1k}(X) < f_{1k}^{u} \end{cases} \tag{10}$$

For each possible solution available to the leader, the follower may be willing to build the membership functions for his/her objective functions so that he/she can rate the satisfaction of each potential solution. In this way, the follower has the following membership functions for his/her goals:

$$\mu_{f_{2k}} = \begin{cases} 1 & \text{if } f_{2k}(X) > f_{2k}^{u} \\ \dfrac{f_{2k}(X) - f_{2k}^{l}}{f_{2k}^{u} - f_{2k}^{l}} & \text{if } f_{2k}^{l} \le f_{2k}(X) \le f_{2k}^{u} \ ; k = 1, \cdots, N_2 \\ 0 & \text{if } f_{2k}(X) < f_{2k}^{u} \end{cases} \tag{11}$$

Where $f_{2k}^{u} = \text{Max}\,(f_{2k}^{H},\ f_{2k}^{L})$ and $f_{2k}^{l} = \text{Min}\,(f_{2k}^{H},\ f_{2k}^{L})$.

Now in a decision making situation , the aim of each of the DM is to achieve the highest membership value (unity) of the associated fuzzy goals. But in actual practice, the achievement of all membership value to the highest degree is not possible due to the limitation of the resources. In such a case, the FGP solution technique for solving multi-objective decision analysis, is used for solving achievement problem of the membership functions and thereby obtaining the most satisfactory solution.

The minsum FGP formulation of the problem can be presented as

(P6) Minimize $Z = \sum_{k=1}^{N_1} w_{1k} d_{1k}^{-} + \sum_{k=1}^{N_2} w_{2k} d_{2k}^{-} + w_3 d_3^{-} + w_3 d_3^{+}$

subject to $\dfrac{f_{1k}(X) - f_{1k}^{l}}{f_{1k}^{u} - f_{1k}^{l}} + d_{1k}^{-} - d_{1k}^{+} = 1,\ k = 1, \cdots, N_1$

$$\frac{f_{2k}(X) - f_{2k}^{l}}{f_{2k}^{u} - f_{2k}^{l}} + d_{2k}^{-} - d_{2k}^{+} = 1, \quad k = 1, \cdots, N_2 \tag{12}$$

$$\frac{X_1 - X_1^t}{X_1^H - X_1^t} + d_3^- - d_3^+ = I_1$$

$$X \in S$$

$$d_{1k}^-,\ d_{1k}^+ \geq 0 \text{ with } d_{1k}^- \cdot d_{1k}^+ = 0, \quad k = 1, \cdots, N_1$$

$$d_{2k}^-,\ d_{2k}^+ \geq 0 \text{ with } d_{2k}^- \cdot d_{2k}^+ = 0, \quad k = 1, \cdots, N_2$$

$$d_3^-,\ d_3^+ \geq 0 \text{ with } d_3^- \cdot d_3^+ = 0$$

where d_{1k}^-, d_{1k}^+ ; d_{2k}^-, d_{2k}^+ represent under and over-deviational variables respectively from the aspired levels of the respective fuzzy goals, d_3^-, d_3^+ represent the vectors of deviational variables associated with the membership goals for the decision vector X_1, I_1 is the column vector with all elements equal to 1 and the dimension of it depends on X_1, w_{1k} $(k=1,\cdots, N_1)$, w_{2k} $(k=1,\cdots, N_2)$ and w_{3j} $(j=1,\cdots, n_1)$ are numerical weights associated with the deviational variables d_{1k}^- $(k=1,\cdots, N_1)$, d_{2k}^- $(k=1, \cdots, N_1)$, and d_{3j}^+ and d_{3j}^- $(j=1,\cdots, n_1)$ respectively.

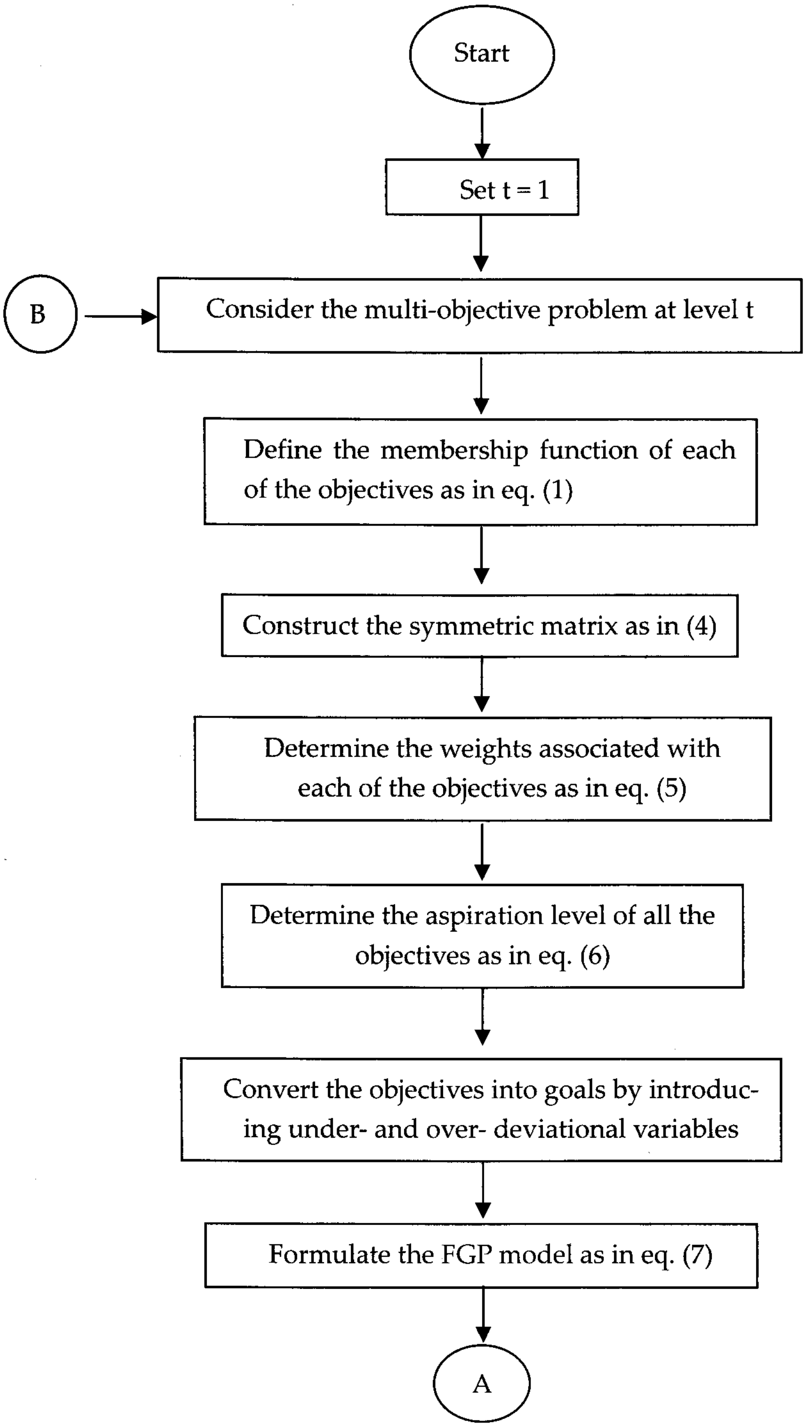
The values of vector of weights w_1, w_2 and w_3 are determined as

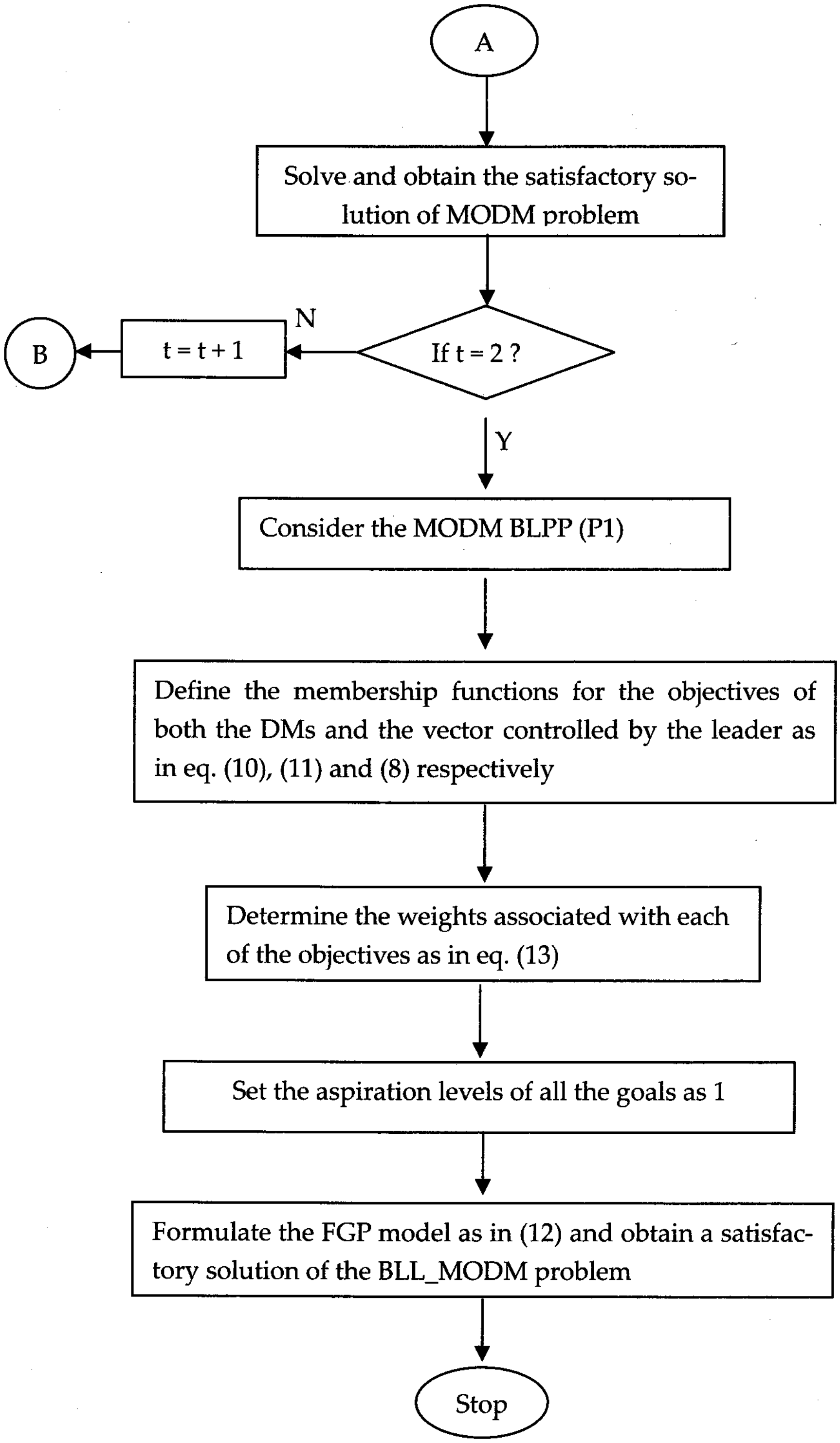
$$\begin{aligned} w_{1k} &= \frac{1}{f_{1k}^u - f_{1k}^l} \quad ; k = 1, 2, \cdots, N_1 \\ w_{2k} &= \frac{1}{f_{2k}^u - f_{2k}^l} \quad ; k = 1, 2, \cdots, N_2 \\ w_{3j} &= \frac{1}{X_{1j}^H - X_{1j}^t} \quad ; j = 1, 2, \cdots, n_1 \end{aligned} \tag{13}$$

The FGP model (12) provides the most satisfactory decision by achieving the aspired levels of the membership goals to the extent possible in the decision-making environment.

5. The Flowchart

The flowchart of the proposed method is given below:


Start
Set t = 1
B
Consider the multi-objective problem at level t
Define the membership function of each of the objectives as in eq. (1)
Construct the symmetric matrix as in (4)
Determine the weights associated with each of the objectives as in eq. (5)
Determine the aspiration level of all the objectives as in eq. (6)
Convert the objectives into goals by introducing under- and over- deviational variables
Formulate the FGP model as in eq. (7)
A


A
Solve and obtain the satisfactory solution of MODM problem
N
B
t = t + 1
If t = 2 ?
Y
Consider the MODM BLPP (P1)
Define the membership functions for the objectives of both the DMs and the vector controlled by the leader as in eq. (10), (11) and (8) respectively
Determine the weights associated with each of the objectives as in eq. (13)
Set the aspiration levels of all the goals as 1
Formulate the FGP model as in (12) and obtain a satisfactory solution of the BLL_MODM problem
Stop

6. Illustrative Example

To demonstrate the solution method for BLL-MODM, let us consider the following example:

Consider the problem of transporting three types of products from a factory to three different retailers. Products are transferred either directly from the factory to the retailers or via the outlet. Let x_{11}, x_{12}, x_{13} be the three different types of products produced at the factory and x_{21}, x_{22}, x_{23} be the old stock for the products at the outlet. The objectives that are considered at the first level are:

(i) maximizing the total expected profit f_{11} of the factory when the products are supplied to the first retailer

(ii) maximizing the total expected profit f_{12} of the factory when the products are supplied to the second retailer

(iii) maximizing the total expected profit f_{13} of the factory when the products are supplied to the third retailer

The objectives considered at the second level are

(i) maximizing the profit f_{21} for the the first retailer

(ii) maximizing the profit f_{22} for the the second retailer

(iii) maximizing the profit f_{23} for the the third retailer

The functional form of the problem is

$f_{11}(X) = 15x_{11} + 10x_{12} + 10x_{13}$

$f_{12}(X) = 5x_{11} + 4x_{12} + 8x_{13}$

$f_{13}(X) = 2x_{11} + 3x_{12} + 3x_{13}$

$f_{21}(X) = 10x_{11} + 15x_{12} + 20x_{13} + 2x_{21} + 4x_{22} + 5x_{23}$

$f_{22}(X) = 8x_{11} + 10x_{12} + 20x_{13} + 4x_{21} + 2x_{22} + 3x_{23}$

$f_{23}(X) = 20x_{11} + 10x_{12} + 15x_{13} + 10x_{21} + 15x_{22} + 10x_{23}$

The set S of feasible choices is

$2x_{11} + 3x_{12} + 2x_{13} \leq 100$ (availability at factory)

$x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} \leq 150$ (storage capacity at the outlet)

$10 \leq x_{11} + x_{21} \leq 40$

$20 \leq x_{12} + x_{22} \leq 45$ (policy constraints)

$15 \leq x_{13} + x_{23} \leq 30$

$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0.$

First the satisfactory solution for the leader and the follower are obtained by using algorithm(I)

Consider the MODM leader's problem

Max $f_{11}(X) = 15x_{11} + 10x_{12} + 10x_{13}$

Max $f_{12}(X) = 5x_{11} + 4x_{12} + 8x_{13}$

Max $f_{13}(X) = 2x_{11} + 3x_{12} + 3x_{13}$

subject to $X \in S$.

Table 1. Objective function values for different f_{1i}'s

X_i^*	$f_{11}(X_1^*)$	$f_{12}(X_2^*)$	$f_{13}(X_3^*)$
(40, 0, 10, 0, 20, 5)	700	280	110
(20, 0, 30, 0, 20, 0)	600	340	130
(10, 6.67, 30, 0, 13.33, 0)	516.7	316.68	130

The membership functions of the objectives $f_{11}(X)$, $f_{12}(X)$ and $f_{13}(X)$ are defined as follows:

$$\mu_{f_{11}(x)} = \begin{cases} 1 & \text{if } f_{11}(X) \\ \dfrac{f_{11}(X) - 516.7}{700 - 516.7} & \text{if } 516.7 < f_{11}(X) < 700 \\ 0 & \text{iff }_{11}(X) \end{cases} \tag{14}$$

$$\mu_{f_{12}(X)} = \begin{cases} 1 & if\ f_{11}(X) \geq 700 \\ \dfrac{f_{12}(X) - 280}{340 - 280} & if\ \ 280 < f_{12}(X) < 340 \\ 0 & if\ f_{12}(x) \leq 280 \end{cases} \tag{15}$$

$$\mu_{f_{13}(X)} = \begin{cases} 1 & if\ f_{11}(X) \geq 700 \\ \dfrac{f_{13}(X) - 110}{130 - 110} & ,\ if\ \ 110 < f_{13}(X) < 130 \\ 0 & ,\ if\ f_{13}(x) \leq 110 \end{cases} \tag{16}$$

The angles between the gradients of the objectives f_{11}, f_{12} and f_{13} are given by

$\cos\theta_{11,12} = 0.9231$ and $\theta_{11,12} = 22.616$

$\cos\theta_{11,13} = 0.9308$ and $\theta_{11,13} = 21.44$

$\cos\theta_{12,13} = 0.857$ and $\theta_{12,13} = 16.86$

The symmetric matrix which represents the non-conflict degree is arranged as

$$\Delta = \eta_{f_r f_s} = \begin{array}{c} f_{11} \\ f_{12} \\ f_{13} \end{array}\begin{array}{c} \begin{array}{ccc} f_{11} & f_{12} & f_{13} \end{array} \\ \begin{bmatrix} 1 & 0.87435 & 0.881 \\ 0.87435 & 1 & 0.906 \\ 0.881 & 0.906 & 1 \end{bmatrix} \end{array}$$

The weights associated with each objectives are obtained as

$w_{11} = 0.91845 \qquad w_{12} = 0.92678 \qquad w_{13} = 0.929$

The aspiration level of these objectives are obtained by equating the values of W_{11}, W_{12} and W_{13} to the membership functions as defined in equations (14), (15) and (16). The aspiration level for the objectives are

$G_{11} = 685.05 \qquad G_{12} = 335.61 \qquad G_{13} = 128.58$

Now for the obtained weights and aspiration levels, the equivalent FGP problem for the leader's MODM problem can be presented as

$$\text{Min } Z = 0.91845\, d_1^- + 0.92678\, d_2^- + 0.929\, d_3^-$$

subject to

$$15x_{11} + 10x_{12} + 10x_{13} + d_1^- - d_1^+ = 685.05$$
$$5x_{11} + 4x_{12} + 8x_{13} + d_2^- - d_2^+ = 335.61$$
$$2x_{11} + 3x_{12} + 3x_{13} + d_3^- - d_3^+ = 128.58$$
$$X \in S$$
$$d_i^-, d_i^+ \geq 0 \text{ with } d_i^-.d_i^+ = 0;\ i = 1, 2, 3.$$

Solving the problem, a satisfactory solution of the leader's MODM problem is obtained as (37.01, 0, 12.99, 0, 20, 2.01) with $f_{11} = 685.05$, $f_{12} = 288.97$, $f_{13} = 112.99$.

Secondly considering the follower's MODM problem

$$\text{Max } f_{21} = 10\, x_{11} + 15x_{12} + 20x_{13} + 2x_{21} + 4x_{22} + 5x_{23}$$
$$\text{Max } f_{22} = 8\, x_{11} + 10x_{12} + 20x_{13} + 4x_{21} + 2x_{22} + 3x_{23}$$
$$\text{Max } f_{23} = 20\, x_{11} + 10x_{12} + 15x_{13} + 10x_{21} + 15x_{22} + 10x_{23}$$

subject to $\quad X \in S$

Table 2. Objective function values for different f_{2i}'s

X_i^*	$f_{21}(X_1^*)$	$f_{22}(X_2^*)$	$f_{23}(X_3^*)$
(20, 0, 30, 20, 45, 0)	1020	930	1725
(0, 13.33, 30,40, 31.67, 0)	1006.63	956.67	1458.35
(40, 0, 10, 0, 45, 20)	880	670	1825

The membership functions of the objectives $f_{21}(X)$, $f_{22}(X)$ and $f_{23}(X)$ are defined as follows:

$$\mu_{f_{21}(x)} = \begin{cases} 1 & \text{if } f_{21}(X) \geq 1020 \\ \dfrac{f_{21}(X) - 880}{1020 - 880} & \text{if } 880 < f_{21}(X) < 1020 \\ 0 & \text{if } f_{21}(X) \leq 880 \end{cases}$$

$$\mu_{f_{22}(X)} = \begin{cases} 1 & \text{if } f_{22}(X) \geq 956.67 \\ \dfrac{f_{22}(X) - 670}{956.67 - 670} & \text{if } 670 < f_{22}(X) < 956.67 \\ 0 & \text{if } f_{22}(x) \leq 670 \end{cases}$$

$$\mu_{f_{23}(X)} = \begin{cases} 1 & \text{if } f_{22}(X) \geq 1825 \\ \dfrac{f_{23}(X) - 1458.35}{1825 - 1458.35} & \text{if } 1458.35 < f_{23}(X) < 1825 \\ 0 & \text{if } f_{23}(x) \leq 1458.35 \end{cases}$$

The angles between the gradients of the objectives f_{21}, f_{22} and f_{23} are given by

$\theta_{21,22} = 11.9856$ $\theta_{21,23} = 34.014$ $\theta_{22,23} = 36.946$

The symmetric matrix is arranged as

$$\Delta = \eta_{f_r f_s} = \begin{matrix} & f_{21} & f_{22} & f_{23} \\ f_{21} & 1 & 0.9334 & 0.8110 \\ f_{22} & 0.9334 & 1 & 0.7947 \\ f_{23} & 0.8110 & 0.7947 & 1 \end{matrix}$$

The weights associated with the objectives are obtained as

$w_{21} = 0.9148$ $w_{22} = 0.9096$ $w_{23} = 0.8685$

The aspiration level of these objectives are determined as

$G_{21} = 1008.072 \qquad G_{22} = 930.75 \qquad G_{23} = 1776.78$

The equivalent FGP model for the followers MODM problem can be presented as

$$\text{Min } Z = 0.9148\, d_1^- + 0.9096\, d_2^- + 0.8685\, d_3^-$$

subject to

$$10x_{11} + 15x_{12} + 20x_{13} + 2x_{21} + 4x_{22} + 5x_{23} + d_1^- - d_1^+ = 1008.072$$

$$8x_{11} + 10x_{12} + 20x_{13} + 4x_{21} + 2x_{22} + 3x_{23} + d_2^- - d_2^+ = 930.75$$

$$20x_{11} + 10x_{12} + 15x_{13} + 10x_{21} + 15x_{22} + 10x_{23} + d_3^- - d_3^+ = 1776.78$$

$$X \in S$$

$$d_i^-, d_i^+ \geq 0 \text{ with } d_i^- . d_i^+ = 0,\ i = 1, 2, 3.$$

A satisfactory solution is obtained as (20, 0, 30, 20, 45, 0) with

$f_{21} = 1020, \qquad f_{22} = 930,\ f_{23} = 1725$

Note : The solution of the MODM problem for the leader and the follower obtained by using the weights and aspiration levels given by Moitra and Pal are respectively (20, 0, 30,0,20,0) and (20, 0, 30, 20, 45, 0) with

$f_{11} = 600, \quad f_{12} = 340, \quad f_{13} = 130, \quad f_{21} = 1020, \quad f_{22} = 930, \quad f_{23} = 1725.$

and that obtained by taking the aspiration level as unity and weights as

$$w_k = \frac{1}{w_k^{\max} - w_k^{\min}} \quad \forall\ k$$

where $w_k^{\max}$ and $w_k^{\min}$ are the best and worst values of the k^{th} objective function, are respectively (20, 0, 30, 0, 20, 0) and (20, 0, 30, 20, 20, 0) with $f_{11} = 600$, $f_{12} = 340$, $f_{13} = 130$, $f_{21} = 920$, $f_{22} = 880$, $f_{23} = 1350$.

A comparison shows that the solution obtained by the proposed method is better achieved here in terms of achievement of objectives for both the DMS.

Now the leader and the follower discloses their satisfactory solution. Let the leader decides that the control variable x_{11} and x_{13} can be relaxed upto 15 and 5 and not beyond that.

Now by building the membership functions and determining the weights the FGP model of the BLL-MODM can be formulated as

$$\text{Min} z = 0.01176 d_1^- + 0.019596 d_2^- + 0.05879 d_3^- + .0033327 d_4^- + 0.0030479 d_5^-$$

$+ 0.0021283d_6^- + 0.08326d_7^- + 0.08326d_7^+ + 0.1252\, d_8^- + 0.1252d_8^\circ$

subject to

$$\frac{f_{11}(X)-600}{685.05-600} + d_1^- - d_1^+ = 1$$

$$\frac{f_{12}(X)-288.97}{340-288.97} + d_2^- - d_2^+ = 1$$

$$\frac{f_{13}(X)-112.99}{130-112.99} + d_3^- - d_3^+ = 1$$

$$\frac{f_{21}(X)-719.95}{1020-719.95} + d_4^- - d_4^+ = 1$$

$$\frac{f_{22}(X)-601.91}{930-601.91} + d_5^- - d_5^+ = 1$$

$$\frac{f_{23}(X)-1255.15}{1725-1255.15} + d_6^- - d_6^+ = 1$$

$$\frac{x_{11}-15}{37.01-15} + d_7^- - d_7^+ = 1$$

$$\frac{x_{13}-5}{12.99-5} + d_8^- - d_8^+ = 1$$

$X \in S$

$d_i^-, d_i^+ \geq 0$ with $d_i^- . d_i^+ = 0 \quad i = 1, 2, \cdots, 8.$

Solving the above problem a satisfactory solution is obtained as (37.01, 0, 12.99, 2.99, 45, 17.01) with the objective functions value as

$f_{11} = 685.05$ $f_{12} = 288.97$ $f_{13} = 112.99$

$f_{21} = 900.93$ $f_{22} = 708.87$ $f_{23} = 1810.05$

7. Summary and concluding Remarks

This paper has proposed a two-planner bi-level linear multi objective decision making model and a solution method for solving this problem. Graphically the problem considered can be positioned in the Figure 4.

The solution method uses the concepts of conflict among the objectives, tolerance membership functions and multi-objective optimization at each level to develop a fuzzy-goal programming model for generating pareto-optimal solution for BLL-MODM. The main advantage of the proposed method is that it uses the concept of conflict among the objectives to generate weights and aspiration levels. In MODM this can provide a more realistic framework to account for incommensurable and conflicting nature of the objectives. This is especially the case in a group decision situation. Also by adopting the leader-follower Stackelberg game, the solution method simplifies a BLL-MODM by transforming into separate MODM problems at each level. Thus the non-convex mathematical programming difficulty for optimal solution of BLL-MODM has been avoided.

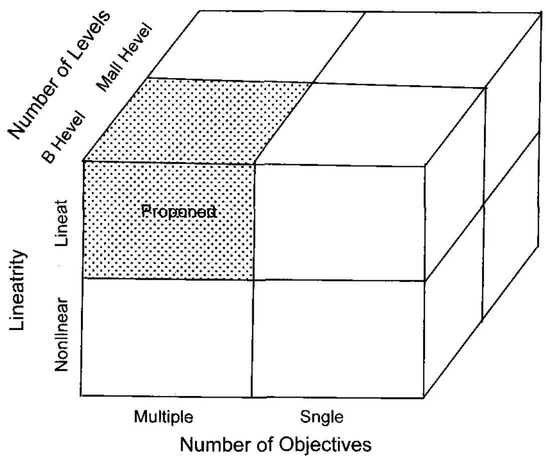


Figure 4

The proposed FGP method gives a satisfactory solution for BLL-MODM keeping the hierarchy intact. The leader provides the preferred values of decision variables under his control and the bounds of his objective function to the follower. The information then constraints the leader's feasible region. An illustrative numerical example has been provided to demonstrate the proposed solution method.

However, the proposed approach should be explored and extended to
the area of multi-level optimization, such as:

- Fuzzy approach is needed for dealing with multi-level, multi-objectives and mutli decision maker's problem.
- In most of the real world situations, the input data or parameters are often imprecise or fuzzy in nature, so models and algorithms for BLL-MODM and MLL-

MODM with fuzzy parameters in the objective functions and in the constraints will be required in the near future.

- On the basis of the proposed approach, other membership functions such as piecewise linear, exponential, hyperbolic or some specific power functions may be needed for practical and interaction reasons. However, in such cases the problem becomes non-linear programming problem. So the proposed method should be extended to non-linear multi-level MODM programming problems.

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