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Location Value and Price Leadership in a Product Differentiation Model*

Hyeon-Mo Ku
Managing Director, Head Office of Corperate Strategy, Korea Telecom,
206, Jungja-dong Bundang-gu Sungnam-si Gyeonggi-do 463-711, Korea

Sang-Ho Lee**
Associate Professor, Department of Economics, Chonnam National University,
300, Yongbong-dong, Bukgu, Gwangju, 500-757, Korea

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ABSTRACT

This paper considers the value of location in a linear city model and examines the product differentiation equilibrium of duopoly providing different benefits to consumers. We show that if the value of location is small, symmetric location equilibrium occurs where two firms follow the maximal differentiation principle. However, as the value of location increases, asymmetric location equilibrium occurs where the low-value-location firm moves to the high-value-location firm and thus adjusted maximal differentiation principle holds. We also investigate two different price leadership model and demonstrate the relationship between the value of location and the role of price leadership. In particular, we show that when the location value is high, the price leadership by high-value-location firm will appear as a unique equilibrium.

Keywords: Linear City Model, Maximal Product Differentiation, The Value of Location, Price Leadership.

1. Introduction

Hotelling [7]'s linear city model is a very well known instrument explaining the phe-

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** Corresponding author, E-mail: sangho@chonnam.ac.kr

nomenon of product differentiation in business economics. The assumption of the linear distribution on location can be interpreted as literally representing geographical differentiation, but is designed to be interpreted more generally. For instance, different displays of a commodity in COSTCO and Sam's Club can be regarded as the location difference in linear city. Preference for internet search engines such as Google and Yahoo also can be explained, if we interpret the distance of the linear city as a degree of consumer's needs or preferences on search engines.

While Hotelling [7]'s analysis used linear transportation costs and provided the principle of minimal differentiation, d'Aspremont, *et al.* [3] pointed out the stability problem in the linear model and introduced quadratic transportation costs to derive the principle of maximal differentiation under which each firm locates at the end of line in equilibrium. In the maximal differentiation, firms do not trigger a low price from the rival and thus price competition is softened. Ever since their important contributions, most of literature on spatial competition has assumed a uniform distribution of consumers and quadratic transportation costs.[1]

However, research in marketing has pointed out that consumers' preferences are clustered around some fashionable brands. Similarly, in the urban setting, the distribution of urban amenities is concentrated around the downtown where individuals can jointly consume the social and cultural pleasures. For example, a restaurant at desirable location gives more value than the others at plain location even though they have the same distance. Therefore, the conventional linear city model was less successful in considering non-uniform distributions.

Some researches have incorporated the non-uniform features into the Hotelling's model with several strands in the literature of business economics. For example, Navon *et al.* [10] incorporated spatial network externalities into the model and supported the principle of minimum differentiation. Neven [11] and Tabuchi and Thisse [13] studied the impact of consumers' concentration in the different location on the equilibrium. However, they assumed the distributional function of consumers with unbounded space, and thus they couldn't simplify the relationship between the different value of location and equilibrium location.

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[1] For example, Chang [1] and Friedman and Thisse [5] looked at collusion in which firms maximize joint profit and showed that the maximum differentiation principle does not hold under certain condition. Mai and Peng [9] introduced an element of cooperation between firms in the form of information exchange through communication on R&D and restored the principle of minimum differentiation.

In this paper, we analyze the linear city model with the existence of a distribution of location amenities, where the location of the firm has different value to consumers. That is, instead of a distance function, the position of a firm is expressed by its location in the consumers domain. In this sense, the proposed model is a special version of vertical product differentiation on the basis of horizontal product differentiation.[2] Specifically, we incorporate duopoly firms providing different benefits to consumers according to the location of firm, and reexamine the product differentiation strategies of the firms. We then show that if the value of location is small, there exists a symmetric location equilibrium where two firms follow the maximal differentiation principle of d'Aspremont *et al.* [3], but the low-value-location firm reduces its price while the high-value-location firm provides the same product at higher price. We also show that as the value of location increases, there exists asymmetric location equilibrium where the low-value-location firm moves to the high-value-location firm and thus adjusted maximal differentiation principle holds. And if the low-value-location firm approaches to the location of high-value-location firm, its price also approaches to marginal cost, as in Hotelling [7].

Finally, we consider the price leadership equilibrium and compare the price and profit patterns of each equilibrium, depending on the different types of leadership. We then investigate the role choice game suggested by Dowrick [4] and Lee [8]. We show that when the location value is high, the price leadership by high-value-location firm is a unique equilibrium, which lessens price competition.

The organization of the paper is as follows: In Section 2, we describe the basic model of linear city with different location value, and examine the equilibrium of simultaneous price competition and product differentiation. In Section 3, we extend the model into the sequential case of price leadership. In Section 4, we consider the game where the firm can choose the role of leadership in selecting its price, and compare the equilibrium depending on the different types of leadership. Conclusion is provided in the Section 5.

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[2] Shy [12] addressed the similar model with the location value in a linear city to produce a vertical differentiation model. Choi and Shin [2] considered a simplified duopoly model of vertical differentiation in Tirole [14]'s model, where both firms do not cover the market and found that maximal product differentiation does not hold. A relationship between consumers' willingness to pay for quality and market coverage (or equilibrium) is fully examined by Wauthy [16]. Finally, Wang [15] examined the non-uniform features into the vertical differ-

2. THE BASIC MODEL

2.1 Assumptions

Consider an example where two restaurants sell the same dinner at the same price in the linear city. We assume that the extreme left of the city is an outskirt of downtown meanwhile the extreme right is the center of downtown, that is, the location of the restaurant can give different value to consumers. If consumers are assumed to find more value in having dinner at downtown, the location of the restaurant influences the consumer's benefits. Alternatively, consider another example where there are two farms producing the same products in the linear city. Extreme left of the city is in the downtown and the extreme right is in the unpolluted rural area. Then, consumers will purchase the product from the unpolluted area if generalized prices (a sum of price and transport cost) are the same.

In the following model, we will incorporate the discrepancy in consumer benefits between two locations into the linear city model with quadratic transportation cost.[3] Firm 1 and Firm 2 are located at point a and 1-b, where $a \leq 1\text{-}b$, on the linear city of length 1. Consumers incur a quadratic function of distance cost with t per unit of length, i.e., a consumer at x incurs cost of $t(a\text{-}x)^2$ for firm 1 and a cost of $t(1 - b - x)^2$ for firm 2, where we assume the unit rate of quadratic transportation cost is normalized to one, $t = 1$, without loss of generality.

On the other hand, we consider the different benefits entailed by the location of each firm. In particular, the consumer benefit from purchasing product of firm 1 is denoted by θ a and that of firm 2 is $\theta(1\text{-}b)$, where θ (≥ 0) is introduced to denote the value difference between two locations. That is, two firms have different values depending on their locations, and thus θ can be interpreted as the degree of value of firm's location. Notice that $a \leq 1\text{-}b$ so that the location value of firm 2 is better than or equal to that of firm 1 in the sense of customer benefits. Each consumer buys one unit

entiation model and found out the failure of the high-quality advantage.

3 If the transportation cost is linear, there exists stability problem and thus there might be no unique equilibrium, as shown by D'Aspremont, *et al.* [3] for the original model of Hotelling [7]. In general, if the transportation cost is quadratic in distance, then for any log-concave consumer density function, there exists a unique price equilibrium (Proposition 1, Tabuchi and Thisse [13]).

of the good from the firm having the higher net benefits.

In this section, we consider the standard game rule where firms 1 and 2 determine their locations first and then choose their prices P_1 and P_2 simultaneously. Then, demand, $D_i(P_1, P_2)$, is derived by the location of a consumer who is indifferent between the two firms. That is, as shown in Figure 1, we assume the indifferent consumer is located at $x = D_1(P_1, P_2)$, where x is given by equating net benefits ; i.e.,

$$U + \theta \cdot a - P_1 - (x-a)^2 = U + \theta \cdot (1-b) - P_2 - (1-b-x)^2 \tag{1}$$

where U implies the basic utility from the consumption.[4]

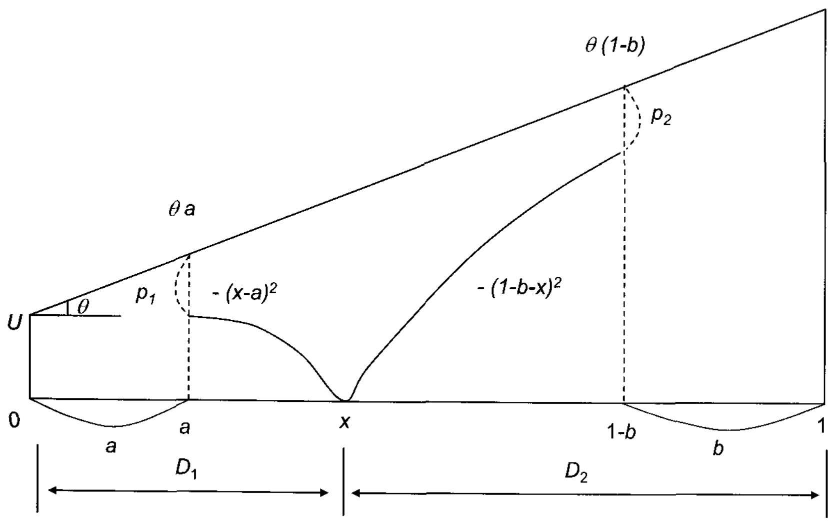

Figure 1. Basic Model Configuration

2.2 Price Competition

By backward induction, we first take the firms' location as given and look for Nash equilibrium in price competition. Since the firms choose their prices P_1 and P_2 at same time, we derive the demand function using equation (1). Then, the firms' respective demands are

4 We assume that U is sufficiently large so that we have an interior solution in the market-covered equilibrium, where every consumer buys one unit of the good from the firm.

$$D_1(P_1, P_2; a, b) = (1 - b + a - \theta)/2 + (P_2 - P_1)/2 \cdot (1 - b - a) \quad (2)$$

And

$$D_2(P_1, P_2; a, b) = 1 - D_1(P_1, P_2) = (1 + b - a + \theta)/2 + (P_1 - P_2)/2 \cdot (1 - b - a) \quad (3)$$

Equation (2) and (3) show that the value difference negatively affects the demand of firm 1, yet positively affects the demand of firm 2. Thus, as θ increases, the demand of firm 1 decreases while the demand of firm 2 increases.

Then, each firm chooses its price so as to maximize its profit given the other firm's price charged by its rival, i.e., Max $\prod_i(P_1, P_2) = (P_i - c)D_i(P_1, P_2)$, where c denotes production cost and i = 1, 2. The Nash equilibrium in prices $P_i(a, b)$ is given by

$$P_1(a, b) = c + (1 - a - b)(1 + (a - b - \theta)/3) \quad (4)$$

and

$$P_2(a, b) = c + (1 - a - b)(1 - (a - b - \theta)/3). \quad (5)$$

Therefore,

$$P_2(a, b) - P_1(a, b) = 2(1 - a - b)(\theta - a + b)/3. \quad (6)$$

Equations in (4) and (5) show that low-value-location firm 1 reduces its price as θ increases while the high-value-location firm raises the price and thus, the price difference between two firms increases.

By substituting (4) and (5) into (2) and (3) we get

$$D_1(a, b) = 1/2 + (a - b - \theta)/6 \quad (7)$$

and

$$D_2(a, b) = 1/2 - (a - b - \theta)/6. \quad (8)$$

In order to get the interior solutions, it should be held that $0 \le D_i(P_1, P_2) \le 1$ and $P_i(a, b) \ge c$, where i = 1, 2. That is, in the model we have the constraint on θ such as $-3 \le a - b - \theta \le 3$. Therefore, since θ should be larger than or equal to 0, we have the following constraint of θ for the interior solutions:

$$0 \le \theta \le a - b + 3 \le 4. \quad (9)$$

2.3 Product Differentiation

Under the two-stage game in which (i) the firms choose their locations (product differentiation) simultaneously and (ii) given the locations, they choose prices simultaneously, each firm understands how its choice of location affects not only its demand functions but also the intensity of price competition.

Then, using the equations in (3), (4), (7), and (8), we can get the following profit functions:

$$\Pi_1(a, b) = (P_1(a, b)-c)\, D_1(a, b, P_1(a, b), P_2(a, b)) \quad (10)$$
$$= (1-a-b)(3 + a-b - \theta)^2 /18$$

and

$$\Pi_2(a,b) = (P_2(a, b) - c)\, D_2(a, b, P_1(a, b), P_2(a, b)) \quad (11)$$
$$= (1 - a - b)(3 - a + b + \theta)^2 /18.$$

Notice that as θ increases, the profit of the firm 1 in (10) decreases while that of the firm 2 in (11) increases, i.e., $\partial\Pi_1/\partial\theta < 0$ and $\partial\Pi_2/\partial\theta > 0$. In addition, as the opponent's location increases, the profit of each firm decreases, i.e., $\partial\Pi_1/\partial b < 0$ and $\partial\Pi_2/\partial a < 0$.

In determining its optimal location, firm 2 first chooses b to maximize its profit function, taking firm 1's location a as given;

$$\partial\Pi_2/\partial b = (3 - a + b + \theta)\,(-1 - 3b - a - \theta)/18, \quad (12)$$

Again, taking firm 2's optimal location as given, firm 1 has the following first-order condition is as follows:

$$\partial\Pi_1/\partial a = (3 + a - b - \theta)\,(-1 - 3a - b + \theta)/18. \quad (13)$$

Proposition 1 : *In a simultaneous price competition game, the equilibrium location yields adjusted maximal differentiation, dependent upon the value of location: (i) if $\theta \le 1$, $a = 0$ and $b = 0$, and (ii) if $\theta > 1$, $a = (\theta-1)/3$ and $b = 0$.*

Proof : First, notice that $\partial\Pi_2/\partial b$ is negative for the required condition of θ in (9), and thus the optimal location for the firm 2 is the right end of the line, i.e., b = 0. Second,

(i) if $\theta \leq 1$, $\partial\prod_1/\partial a < 0$ from (13) and thus a = 0 since $0 \leq a \leq 1$. (ii) if $\theta > 1$, the optimal location for firm 1 is determined at $\partial\prod_1/\partial a = 0$. q.e.d.

This result shows that firm 1, the low-value-location company, has an incentive to deviate from the extreme left and move its location to the right when θ is large. Only when $\theta \leq 1$, then a = b = 0.[5] Therefore, it differs from the maximal differentiation principle of d'Aspremont, *et al.* [3], which does not explain the firm's strategy of asymmetric location.

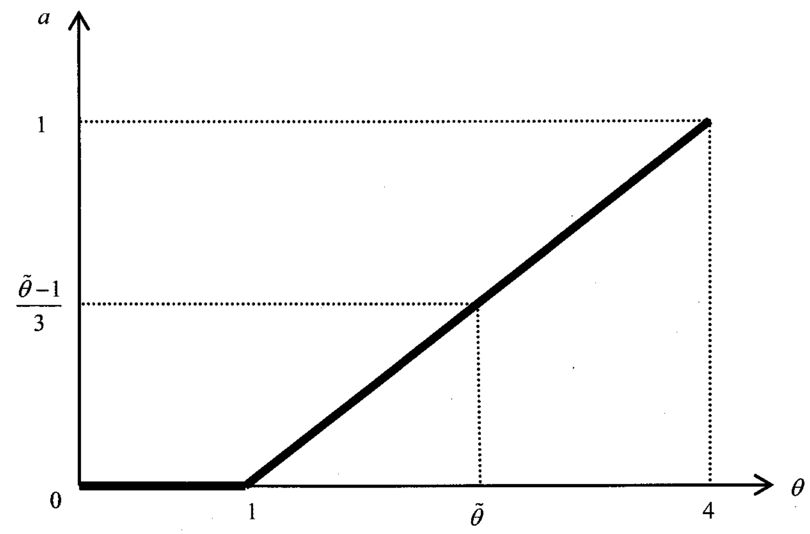

Figure 2. Optimal Location of Firm 1

Examining the feasible values for θ, which should be less than 4 by the required condition in (9), we find that firm 2 always wants to stay at the extreme right point of the linear city and firm 1 to stay at extreme left or move to the right, depending on the size of θ in simultaneous price competition. Figure 1 shows the relationship between θ and the optimal location of the low-value-location firm. Firm 1 follows maximal differentiation principle in d'Aspremont, *et al.* [3] if θ is not larger than 1. However as θ increases, firm 1 begins to move to the right. As θ approaches 4, firm 1

[5] It means that if the degree of value of firm's location, θ is less than a certain number, then firm 1 prefers to stay at the extreme left of linear city because the direct effect of a on its profit (demand effect) is smaller than the indirect effect through the change in firm 2's price (the strategic effect).

gets closer to the location of firm 2. That is, the distance between two firms, 1-b-a, approaches 0 and the price of firm 2 also approaches to marginal cost, which supports the minimal differentiation principle in Hotelling [7].[6]

On the other hand, from equation (4) and (5), we know that equilibrium price is determined by not only the distance between two firms (degree of competition) but also the value of location. Specifically, we have

If $0 \leq \theta \leq 1$, $P_1 = c + (3 - \theta)/3$ and $P_2 = c + (3 + \theta)/3$.
If $1 < \theta \leq 4$, $P_1 = c + 2(4 - \theta)^2/27$ and $P_2 = c + 2(4 - \theta)(5 + \theta)/27$.

Without considering the location value in the model where $\theta = 0$ and thus $a = b = 0$, as in d'Aspremont, *et al.* [3], the equilibrium price of product is the same, $P_1 = P_2$. However, by taking the location value into our model, the equilibrium price of each firm differs and P_1 is always lower than P_2. It accounts for the situation in which high-value-location firm sell at higher price than low-value-location firm even though they provide the same product. Consequently, the equilibrium demand of firm 2 is greater than that of firm 1 and thus, as θ increases, the demand expansion effect for firm 2 by the value of location is increasing, i.e., $D_2 = (3 + \theta)/6 > D_1 = (3 - \theta)/6$ for $0 \leq \theta \leq 1$ and $D_2 = (5 + \theta)/9 > D_1 = (4 - \theta)/9$ for $1 \leq \theta \leq 4$. Finally, the profit of firm 2 is greater than that of firm 1.

3. Price Leadership

Until now, we have focused on the simultaneous game in prices and locations. We now examine the sequential behavior of firms to look for price leadership game. Then, there are two different price leadership models. The first one is price leadership by low-value-location firm 1 and the other is price leadership by high-value-location firm 2. Finally, in the next section, we will compare these equilibria within a larger game structure of choosing roles.

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6 Notice that a marginal-cost pricing with same location is not a Nash equilibrium when $\theta = 4$ since firm 1 would find it profitable to deviate from $a = 1$ to the left. This result is sometimes referred to as Edgeworth Cycles under capacity constraint. See Shy[12, p.112, p.153] within a Hotelling's model.

3.1 Price Leadership by Low-Value-Location Firm (PLL)

Taking the firms' locations as given, we will examine the price leadership competition by firm 1. The story is that firm 1 chooses its optimal price P_1 which maximizes its profit, taking the reaction function of firm 2 as given, and then firm 2 can observe firm 1's price choice and choose its profit-maximizing price level.

Using the backward induction, we can get the following reaction function of firm 2:

$$P_2(P_1; a, b) = (P_1 + c + (1 - a - b)(1 - a + b + \theta))/2. \tag{14}$$

Hotelling's model.

Maximizing the profit function of firm 1 under the constraint of equation (14) yields the following price leadership equilibrium:

$$P_1(a, b) = c + (1 - a - b)(3 + a - b - \theta)/2 \tag{15}$$

and

$$P_2(a, b) = c + (1 - a - b)(5 - a + b + \theta)/4. \tag{16}$$

Therefore,

$$P_2(a, b) - P_1(a, b) = (1 - a - b)(3\theta - 3a + 3b - 1)/4. \tag{17}$$

Equations in (15) and (16) show that low-value-location firm 1 also reduces its price as θ increase while the high-value-location firm raises its price and thus, the price difference between two firms increases.

Substituting (15) and (16) into (2) and (3) we get

$$D_1 (a, b) = 3/8 + (a - b - \theta)/8 \tag{18}$$

and

$$D_2 (a, b) = 5/8 - (a - b - \theta)/8. \tag{19}$$

In order to get the interior solutions, it should be held that $-3 \leq a - b - \theta \leq 5$ and $\theta \geq 0$. Therefore, we have the same constraint of θ in (9) for the interior solutions in the leadership model by low-value-location firm.

We next examine how each firm chooses its optimal location simultaneously.[7]

[7] For more realistic situation, even if we consider the sequential entry game where firm 2 enters the market first and then firm 1 enters the market later, the equilibrium is the same since

Using the equations in (15), (16), (18), and (19), we can get the following profit functions;

$$\Pi_1(a, b) = (1 - a - b)(3 + a - b - \theta)^2 /16 \tag{20}$$

and

$$\Pi_2(a, b) = (1 - a - b)(5 - a + b + \theta)^2 /32. \tag{21}$$

Proposition 2 : *In a sequential price leadership game by low-value-location firm, the equilibrium location yields adjusted maximal differentiation, dependent upon the value of location: (i) if $\theta \leq 1$, $a = 0$ and $b = 0$, and (ii) if $\theta > 1$, $a = (\theta-1)/3$ and $b = 0$.*

Proof : First, in determining the optimal location for firm 2, the first-order condition of firm 2 is negative for the required condition of θ in (9), and thus the optimal location for the firm 2 is the right end of the line, i.e., b = 0. Second, taking firm 2's optimal location as given, the first-order condition for firm 1 yields the following optimal location for the firm 1: (i) if $\theta \leq 1$, $\partial\Pi_1/\partial a < 0$ from (21) and thus a = 0 since $0 \leq a \leq 1$. (ii) if $\theta > 1$, the optimal location for firm 1 is determined at $\partial\Pi_1/\partial a = 0$. q.e.d.

Notice that the choice of location in a price leadership game by low-value-location firm is the same result with the simultaneous price competition. That is, if $\theta \leq 1$, then a = b = 0 and thus, firm 1 prefers to stay at the extreme left of linear city. But, firm 1 moves to the right as θ increases and gets closer to the location of firm 2 as θ approaches 4.

However, the equilibrium prices differ. Specifically, we have

If $0 \leq \theta \leq 1$, $P_1 = c + (3 - \theta)/2$ and $P_2 = c + (5 + \theta)/4$.
If $1 < \theta \leq 4$, $P_1 = c + 2(4 - \theta)^2/9$ and $P_2 = c + 2(4 - \theta)(8 + \theta)/18$.

Notice that the equilibrium prices difference, $P_2 - P_1$, is positive only when $\theta \geq 1/3$. That is, if $0 \leq \theta \leq 1/3$, the price of low-value-location firm is greater than that of high-value-location firm in the equilibrium; high price from leadership exists. But, the equilibrium demand of firm 2 is always greater than that of firm 1, i.e., $D_2 = (5 + \theta)/8 >$

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the firm 2 will always choose the extreme right in the first stage, as described in the following Proposition 2.

$D_1 = (3 - \theta)/8$ for $0 \leq \theta \leq 1$ and $D_2 = (8 + \theta)/12 > D_1 = (4 - \theta)/12$ for $1 \leq \theta \leq 4$. Finally, the profit of high-value-location firm is larger than that of low-value-location firm; the first-mover advantage does not exists.

3.2 Price Leadership by High-Value-Location Firm (PLH)

We will examine the price leadership competition by firm 2, where firm 2 chooses its optimal price P_2, and then firm 1 can observe P_2 and choose P_1. With the same procedure, we can get the following price leadership equilibrium:

$$P_1(a, b) = c + (1 - a - b)(5 + a - b - \theta)/4 \quad (22)$$

and

$$P_2(a, b) = c + (1 - a - b)(3-a + b + \theta)/2. \quad (23)$$

Therefore,

$$P_2(a, b) - P_1(a, b) = (1 - a - b)(3\theta - 3a + 3b + 1)/4. \quad (24)$$

Equations in (22) and (23) also show that low-value-location firm 1 also reduces its price as θ increase while the high-value-location firm raises its price and thus, the price difference between two firms increases.

Again, substituting (22) and (23) into (2) and (3) we get

$$D_1 (a, b) = 5/8 + (a - b - \theta)/8 \quad (25)$$

and

$$D_2 (a, b) = 3/8 - (a - b - \theta)/8. \quad (26)$$

In order to get the interior solutions, it should be held that $-5 \leq a - b - \theta \leq 3$ and $\theta \geq 0$. Therefore, under the constraint of θ in (9), we can get the interior solutions in the leadership competition by high-value-location firm.

To examine the optimal locations, using the equations in (22), (23), (25), and (26), we can get the following profit functions:

$$\Pi_1(a, b) = (1 - a - b)(5 + a - b - \theta)^2 /32 \quad (27)$$

and

$$\Pi_2(a, b) = (1 - a - b)(3 - a + b + \theta)^2 /16. \quad (28)$$

Proposition 3 : *In a sequential price leadership game by high-value-location firm, the equilibrium location yields adjusted maximal differentiation, dependent upon the value of location : (i) if $\theta \le 3$, a = 0 and b = 0, and (ii) if $\theta > 3$, a = $(\theta - 3)/3$ and b = 0.*

Proof : First, the first-order condition of firm 2 is negative for the required condition of θ in (9), and thus b = 0. Second, the first-order condition for firm 1 yields the following optimal location for the firm 1: (i) if $\theta \le 3$, $\partial \prod_1 / \partial a < 0$ from (27) and thus a = 0. (ii) if $\theta > 3$, the optimal location for firm 1 is determined at $\partial \prod_1 / \partial a = 0$. q.e.d.

It implies that if the high-value-location firm has the price leadership, the equilibrium prices and locations differ with the other cases. First, if $\theta \le 3$, then a = b = 0 and thus, firm 1 prefers to stay at the extreme left of linear city. And firm 1 moves to the right as θ increases but stop at the point of 1/3 in the linear line when $\theta = 4$.

Second, in the equilibrium, we have

If $0 \le \theta \le 3$, $P_1 = c + (5 - \theta)/4$ and $P_2 = c + (3 + \theta)/2$.
If $3 < \theta \le 4$, $P_1 = c + (6 - \theta)^2/18$ and $P_2 = c + (6 - \theta)(6 + \theta)/9$.

Thus, the equilibrium prices difference is always positive ; high price from leadership always exists.

Finally, the demand expansion effect depends on the size of location value. If $1 \le \theta \le 4$, the equilibrium demand of firm 2 is always greater than that of firm 1 and thus, the profit of high-value-location firm is larger than that of low-value-location firm; the first-mover advantage exists. However, if $0 \le \theta < 1$, the equilibrium demand of firm 2 is less than that of firm 1 and thus, the profit difference between two firms is ambiguous. Especially, when θ closes to 0, the profit of firm 2 is less than that of firm 1 ; the first-mover disadvantage exists.

4. Role Choice of Leadership

In this section, by considering the role choice game in setting prices, we investigate the market role of each firm within a larger game structure of choosing roles. Specifically, following the game structure by Dowrick [4] and Lee [8], we consider a game where each firm announces independently its choice of role, as price follower or price leader, and then acts in a prescribed manner. Then, there are four possible outcomes to the game. If firms choose adverse roles, two Stackelberg leadership solutions will emerge. One is the price leadership by low-value-location firm (PLL) and the other is the price leadership by high-value-location firm (PLH). However, if both choose to follow, the result is the simultaneous game solution (S). Finally, if both desire to be leaders, the situation is Stackelberg warfare.[8]

We will firstly point out some interesting findings from the previous analysis. First, if the value of location is small, two firms follow the maximal differentiation principle so as to relax price competition. However, because of the value effect of location, the equilibrium prices differ. In general, the low-value-location firm reduces its price while the high-value-location firm provides the same product or service at higher price. Only when the low-value-location firm leads the price under the situation that $0 \leq \theta < 1/3$, $P_1 > P_2$. Therefore, excepting the case when θ is small, the profit of high-value-location firm is in general greater then that of low-value-location firm.

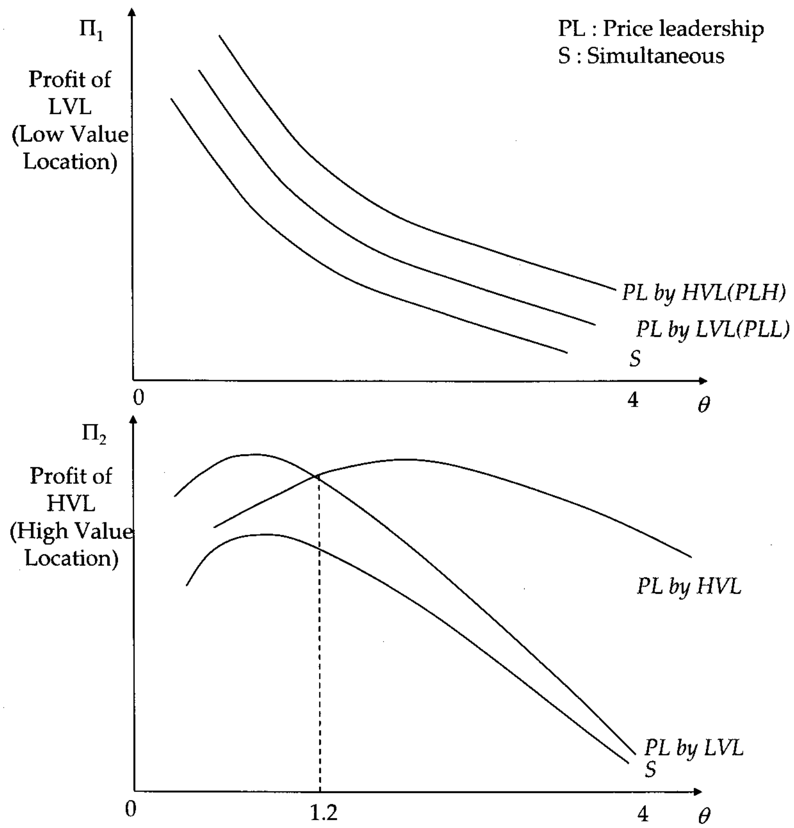
Second, as the value of location increases, the low-value-location firm moves to the high-value-location firm and thus maximal differentiation does not hold. In this case, the value effect of location dominates the distance (demand) effect of location. However, when the high-value-location firm leads the price and the value of location is big, the low-value-location firm stops moving at the point of 1/3 in the linear line at $\theta = 4$.

Third, concerning the profit of the low-value-location firm, we can show that the case when the high-value-location firm leads the price gives the highest profit and the case of simultaneous price competition gives the lowest profit (i.e., the ordering is PLH > PLL > S). However, as the value of location increases, its profit decreases for all three scenarios. Therefore, the best scenario for the low-value-location firm is the case when the high-value-location firm leads the market price and the value of location is small [See the first picture in Figure. 3].

Fourth, concerning the profit of the high-value-location firm, we can show that the case of simultaneous price competition gives the lowest profit among three scenarios. However, the highest profit for the high-value-location firm depends on the location value. When the location value is small, i.e., $\theta \leq 1.2$, the case when the low-

[8] As pointed out by Tirole [14], the Stackelberg warfare will give the least profit to the firms because the war of attrition gives no ex ante rents to the firms, even though the outcomes of

value-location firm leads the price gives the highest profit (i.e., the ordering is PLL > PLH > S). But, when the location value is large, the case when it leads the price gives the highest profit among three scenarios (i.e., PLH > PLL > S). Therefore, the best strategy of role choice for the high-value-location firm depends on the location value [See the first picture in Figure. 3].



Π_1	$PLH > PLL > S$	$PLH > PLL > S$
Π_2	$PLL > PLH > S$	$PLH > PLL > S$
	Reserved Stackelberg Welfare	Fim 2 (HVL) is price leader

Figure 3. The Profit Relationship and Role Choice

Stackelberg warfare are not in general deterministic.

Finally, we will examine the equilibrium of role choice in the price competition game. The profits relationship is described in Figure 3. Then, we can find that the incentive to be leader between two firms differs, depending on the value of location. First, when the location value is high, $1.2 \le \theta \le 4$, price leadership by high-value-location firm yields the higher profit to each firm. Therefore, the price leadership by high-value-location firm will appear as unique subgame perfect equilibrium. As far as concerned on the degree of product differentiation, (i) if $1.2 \le \theta \le 3$, both firms choose the end of the city, i.e., symmetric location equilibrium, and (ii) if $3 \le \theta \le 4$, both firms choose the asymmetric equilibrium of adjusted maximal product differentiation. This implies that when the location value is not so small, the price leadership by high-value-location firm will lessen the price competition and thus adjusted maximal product differentiation still holds.

On the other hand, when the location value is small, i.e., $0 \le \theta \le 1.2$, the subgame perfect equilibrium is ambiguous since each firm wants rival firm to be a price leader. However, we will follow the committed role choice game of Dowrick [4], where each firm announces independently its choice of role and then acts in a prescribed committed manner. Then, we can show that expecting that the other firm is acting as price leader, each firm announces to choose to be follower. Then, the simultaneous game solutions will be subgame perfect equilibria in price choice stage. Therefore, (i) if $0 \le \theta \le 1$, both firms choose the end of the city, i.e., symmetric location equilibrium, and (ii) if $1 < \theta < 1.2$, both firms choose the asymmetric equilibrium of adjusted maximal product differentiation.[9]

Proposition 4 *In a committed role choice game, the equilibrium role depends on the value of location: (i) if $1.2 \le \theta \le 4$, price leadership by high-value-location firm is a unique subgame perfect equilibrium, but (ii) if $0 \le \theta < 1.2$, simultaneous price competition is the subgame perfect equilibrium.*

This also implies that the price of high-value-location firm is always greater than that of low-value-location firm in a committed role choice game.

[9] If we borrow the endogenous role choice game of Hamilton and Slutsky [6], where each firm can change its role after they observe the other firm's announcement, the Stackelberg warfare will emerge since its own price leadership yields higher profit than simultaneous game, which will give the least profit to the firms.

5. Conclusion

This paper examined the principle of product differentiation when duopoly firms provide different benefits to consumers based on their location. We showed that if the value of location is small, two firms follow the maximal differentiation principle since the strategic effect of location distance can relax price competition. As the value of location increases, however, the value effect of location dominates the distance effect of location and thus, the low-value-location firm moves to the high-value-location firm yet its price is less than that of the high-value-location firm. In an extreme case where the value of location is so high, the low-value-location firm approaches to the extreme right and both firms' prices approach to marginal cost.

We have also examined the role of price leadership and shown that when the location value is high, the price leadership by high-value-location firm is unique equilibrium. In this case, the equilibrium prices depend on the value of location, but low-value-location firm always set lower price than that of high-value-location firm. This result suggests that the symmetric equilibrium in the standard linear city model is not well founded, and invites us to pay more attention to asymmetric equilibrium. In addition, this result describes a common situation in which firms implicitly cooperate with competitors to soften price competition. Therefore, if the value of location between two firms differs too much, then the firms may change the type of competition behavior depending on the value of location.

Reference

[1] Chang, M. H., "The Effect of Product Differentiation on Collusive Pricing," *International Journal of Industrial Organization*, 9 (1991), 453-469.

[2] Choi, C. J. and H. S. Shin, "A Comment on a Model of Vertical Product Differentiation," *Journal of Industrial Economics*, 40 (1992), 229-231.

[3] d'Aspremont, C., Gabszewicz, J., and J. F. Thisse, "On Hotelling's 'Stability in Competition'," *Econometrica*, 47 (1979), 1145-1150.

[4] Dowrick, S., "Von Stackelberg and Cournot Duopoly: Choosing Roles," *RAND Journal of Economics*, 17 (1986), 251-260.

[5] Friedman, J. and J. F. Thisse, "Partial Collusion Fosters Minimum Product Differentiation," *RAND Journal of Economics*, 24 (1993), 631-645.

[6] Hamilton, J.H. and S.M. Slutsky, "Endogenous Timing in Duopoly Games : Stackelberg or Cournot Equilibria," *Games and Economic Behavior* 2 (1990), 29-46.

[7] Hotelling, H., "Stability in Competition," *Economic Journal*, 39 (1929), 41-47.

[8] Lee, S. H., "Choosing Roles in a Model of Quality Differentiation," *Seoul Journal of Economics*, 9 (1996), 191-201.

[9] Mai, C. C. and S. Peng, "Cooperation vs. Competition in a Spatial Model," *Regional Science and Urban Economics*, 29 (1999), 463-472.

[10] Navon, A., O. Shy, and J. F. Thisse, "Product Differentiation in the Presence of Positive and Negative Network Effects," CEPR Discussion Papers. DP1306 (1995).

[11] Neven, D. J., "On Hotelling's Competition with Non-uniform Customer Distributions," *Economics Letters*, 21 (1986), 121-126.

[12] Shy, Oz., *Industrial Organization : Theory and Applications*, The MIT Press (1995).

[13] Tabuchi, T. and J. F. Thisse, "Asymmetric Equilibria in Spatial Competition," *International Journal of Industrial Organization*, 13 (1995), 213-227.

[14] Tirole, J., *The Theory of Industrial Organization*, The MIT Press (1988).

[15] Wang, X. Henry, "A Note on the High-Quality Advantage in Vertical Differentiation Models," *Bulletin of Economic Research*, 55 (2003), 91-99.

[16] Wauthy, X., "Quality Choice in Models of Vertical Differentiation," *Journal of Industrial Economics*, 44 (1996), 345-353.