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A Design and Case Study of a K-Stage BLU Inspection System for Achieving a Target Defective Rate*

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ABSTRACT

In this paper, we address a design problem and a case study of a K-stage back-light-unit (BLU) inspection system, which is composed of K stages, each of which includes an inspection process and a rework process. Assuming the type I, II errors and the inspection-free policy for items classified as good, we determine the smallest integer of K which can achieve a given target defective rate. If K does not exist, holding the current values of the type I, II errors, we search reversely the defective rate of an assembly line and the defective rate of a rework process, to meet the target defective rate. Our formulae and methodology based on a K-stage inspection system could be applied and extended to similar situations with slight modification.

Key words: K-stage Inspection System, Multiple Inspection System, Defective Rate of Lots, BLU

1. Introduction

A back-light unit (BLU), which is attached at the back of a display unit and looks like a rectangle-shaped light, is one of the major components of a Thin-Film-Transistor Liquid Crystal Display (TFT-LCD) as shown in Figure. 1. A BLU consists of several Cold Cathode Fluorescent Lamps (CCFL's), a Light Guide Panel (LGP), and several sheets and so on. It can be made by assembling the above parts sequentially into a mold frame. In general, two CCFL's are located at both right and left sides of a BLU. An LGP and several sheets such as a reflector sheet, a prism sheet, a diffuser sheet, and a protection sheet etc., guide light emitted from two CCFL's to the whole surface

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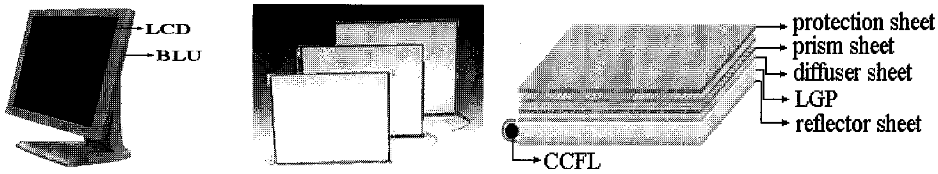
of a BLU in order to attain the aimed uniform intensity of illumination measured at the representative spots on the surface of a BLU.



(a) an LCD and a BLU (b) various BLU's (c) a simplified structure of a BLU

Figure 1. a TFT−LCD, various BLU's, and a simplified structure of a BLU

The inspections of a BLU can be divided into several functional inspections and external appearance inspections; The functional inspections include a brightness test (for example, the measured brightness value of a spot on the surface of a BLU should lie in a specified range, and the mean and variance of the measured brightness values of thirteen spots should lie in an appropriate specified range.), a view angle test, and an electrical conformations test. The external appearance inspections check the surface state of a mold frame, the states of screws and twisted electrical lines, and the surface states of both sheets and an LGP, etc.

One of the current hot issues focused by BLU suppliers is to reduce the outgoing quality rate of BLU's to support the 6σ quality policies by LCD suppliers. If an outgoing quality rate of BLU's is lower than a specified defective rate, a BLU supplier may expect order quantity more than before, the exemption of both a source inspection test (an inspection test for sampled lots by a resident inspector sent by a BLU consumer) and the managerial guides for quality controls. These expectations correspond to economical and managerial savings for BLU suppliers. On the other hand, if an outgoing quality rate becomes higher, a BLU supplier must suffer for various disadvantages and may be even deprived of all its rights as supplier. Hence, the strategy for reducing the outgoing quality rate has been recently one of survival strategies (Kim, 2004).

In order to reduce the outgoing quality rate, BLU suppliers must reduce basically the defective rates of their assembly lines. However, the war against foreign materials (FM's) such as dusts and threads as major factors in defective rates, has an invincible limit. For example, even though a female worker in an assembly line wears a white dustproof uniform covering the whole body except her face, a BLU could become a defective due to the FM's, which result from her scratching on itchy dry skin surface and are dropped on a sheet of a BLU, or due to some dusts coming out of conveyor belts. Since the physiological phenomenon such as scratches can not be prohibited for the purpose of reducing the outgoing quality rate, BLU line foremen or managers train female workers to remove their heavy make-ups on both faces and necks, and check daily whether they observe the rule or not at the entrance of dust-shower rooms.

FM can be a thread occurring from a deteriorated slat-belt conveyor, or a tiny piece of iron occurring from a conveyor motor, or a tiny fragment of plastic occurring from an LGP or sheets and so on. However, BLU suppliers have not collected up to now the defective rates due to various types of FM's. They should have found what types of FM's exist in their lines, how FM's occur and are dropped on a sheet, which paths FM's move along on work floor. In addition, they must introduce an incentive plan for improving the detecting abilities of inspectors, and must reduce the defective rate of a rework process. However, they hardly do those activities because of insufficient investment cost and the absence of experts. Hence the only way to survive and to attain specified outgoing quality rates at the same time has been just the strategy to select good BLU's.

In case of one BLU supplier, a single inspector is assigned at the end of a BLU assembly line. If a finished BLU is accepted as good, it is sent to a packing process for a lot operation. Otherwise, it is sent to a rework process. After rework, it is sent to a packing process without the second inspection. The sampled defective rate just before packing was measured approximately 15,300 PPM and could not attain the goal defective rate, 8,000 PPM, demanded by a BLU consumer. Even though the BLU supplier knew the basic and fundamental activities to attain the outgoing defective rate given by a consumer, a simple and quick method is just to select a good BLU well. Thus, the BLU supplier would like to know how to design and operate their inspection system as well as how to forecast in advance the sampled defective rate at the packing process and the amount of rework.

In this paper, by extending an existing BLU inspection system mentioned above, we address a design problem and a case study of a K-stage BLU inspection system, which consists of K stages in series, each of which includes an inspection process and a rework process. Assuming the type I and II errors, we determine the smallest integer of K which can achieve a given target defective rate demanded by BLU consumers. If K does not exist, assuming that the type I and II errors does not change, we search reversely a new vector : the defective rate of an assembly line and the defective rate of a rework process, which can give the target defective rate.

Since most of papers related to a multiple inspection system assume different designs and operations in addition to limited constraints, and suggest their conclusions, it is not easy to search and utilize the published results from previous papers. However, we summarize some papers slightly related with K-stage inspection system as below. Raz and Thomas [4] presented a branch-and-bound method for determining an optimum sequencing inspection plan for a group of inspectors operating at different skill and cost levels. Production and inspection costs for both accepted and rejected items were considered, and dependencies among successive inspections were permitted. Jaraiedi *et al.* [2] presented a model to determine the average outgoing quality for a product having multiple quality characteristics and subject to multiple 100% inspections where the inspection was subject to both type I and type II inspection errors. Assuming a fixed sequence of unreliable inspection operations with known costs and inspection error probabilities of two types, Avinadav and Raz [3] developed a model for selecting the set of inspections to minimize expected total costs consisting of inspection and penalties, and provided an efficient branch-and-bound algorithm for finding an optimal solution.

In Section 2, we describe our problems for a K-stage inspection system in detail. In Section 3, we derive a formula for the average defective rate just before packing as a function of five factors; (type I error, type II error, the defective rate of an assembly line, the defective rate of a rework process, and K). In addition, we provide a formula for finding a minimum integer of K so that the target defective rate demanded by a BLU consumer is attained. Since the nonexistence of a minimum value of K indicates that the target defective rate can not be attained, we should search new combination of factors in order to attain it. Hence, in Section 4, assuming that the vector, (type I error, type II error), is fixed, we search and find a new vector (the defective rate of an assembly line, the defective rate of a rework process, and K) which can give the target rate. In Section 5, a case study will be given and analyzed.

2. Problems Statement

As shown in Figure 2, our K-stage inspection system consists of K stages, each of which includes an inspection process and a rework process. In the first stage, if an item from an assembly line is classified as good by the first inspector, then it is sent to the packing process. Otherwise, it is sent to the first rework process. After reworking the item by the first reworker, it is sent to the second inspector. If the reworked item is classified as good by the second inspector, then it is sent to the packing process. Otherwise, it is sent to the second rework process and so on. At the last K-th stage, an item classified as good is sent to the packing process and an item classified as bad is reworked and immediately sent to the packing process without inspection.

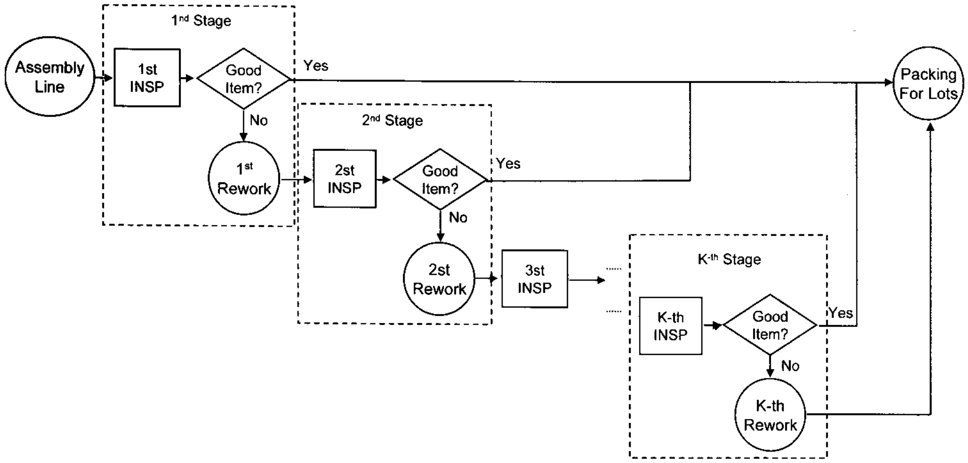


Figure 2. A process diagram of our K−stage inspection system

We assume that inspectors make two kinds of errors of judgment; rejecting a good item (type I error) or accepting a bad item (type II error). Let α and β be the probabilities of a type I and type II errors by an inspector respectively. Let q_0 and q_R be the average defective rate of items from an assembly line and the average defective rate at a rework process respectively, and without loss of generality we assume that $q_0 > q_G$ where q_G is a given target defective rate. Then, the average defective rate of items or the average outgoing quality(AOQ) at the packing process, denoted by q_K, can be expressed as a function of a vector $(\alpha, \beta, q_0, q_R, K)$, i.e., $f(\alpha, \beta, q_0, q_R, K)$. Our objective is to find the smallest integer of K, denoted by K^*, such that $\hat{q}_K = f(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R, K) \leq q_G$ where $\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R$ are the estimates of $(\alpha, \beta, q_0, q_R)$. In other words, our first problem can be stated as follows:

INSP : Given $(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R, q_G)$, find K such that we

Minimize K

subject to $\hat{q}_K = f\left(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R, K\right) \le q_G$

In case that K^* does not exist, it is natural to search new combination of $(\alpha, \beta, q_0, q_R, K)$ in order to achieve q_G. Since finding those values of a new vector is very complicated, assuming that $\left(\hat{\alpha}, \hat{\beta}\right)$ does not change, we search $\left(\tilde{q}_0, \tilde{q}_R, \tilde{K}\right)$ so that the average defective rate of items at the packing process, denoted by $\tilde{q}_{\tilde{K}}$, may achieve q_G. That is,

SP : Given $\left(\hat{\alpha}, \hat{\beta}, q_G\right)$, find $\left(\tilde{q}_0, \tilde{q}_R, \tilde{K}\right)$ such that $\tilde{q}_{\tilde{K}} = f\left(\hat{\alpha}, \hat{\beta}, \tilde{q}_0, \tilde{q}_R, \tilde{K}\right) = q_G$

3. Analysis of Our Inspection Problem

3.1 Derivation of q_K and Some Properties

As shown in Figure 3, for a positive integer k, let G_k and B_k be the numbers of good and bad items given to the (k + 1)-th inspection stage respectively. Let R_0 be the number of items coming into the first stage of our K-stage inspection system and let G_0 and B_0 the numbers of good and bad items given to the first inspection stage respectively. Clearly, $G_0 = (1-q_0)R_0$ and $B_0 = q_0 R_0$. The numbers used in this paper are assumed to be real since R_0 is big enough to justify this assumption.

Let $N_{G/G}(k)$ and $N_{B/G}(k)$ be the numbers of good items that are judged as good and bad by the k-th inspector respectively. Let $N_{G/B}(k)$ and $N_{B/B}(k)$ be the number of bad items that are judged as good and bad by the k-th inspector respectively. Then, for a positive integers k, $N_{G/G}(k) = (1-\alpha)G_{k-1}$, $N_{B/G}(k) = \alpha G_{k-1}$, and $N_{G/B}(k) = \beta B_{k-1}$ and $N_{B/B}(k) = (1-\beta)B_{k-1}$.

Assume that the items classified as good at the k-th inspection are sent to the packing process. Let L_k be the number of items sent to the packing process. Since

L_k is the sum of the number of good items classified as good and the number of bad items classified as good, it can be expressed as

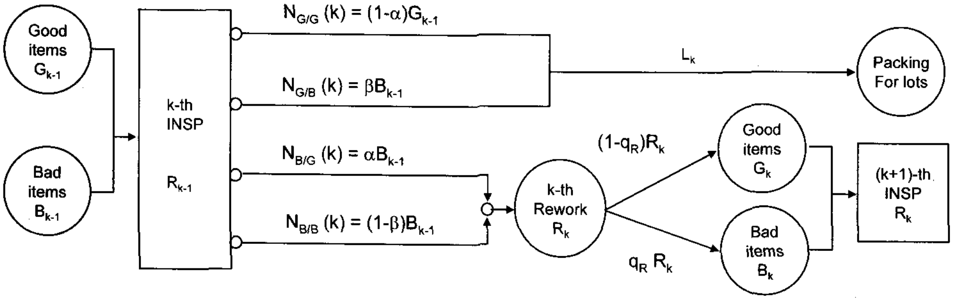


Figure 3. A flow diagram for the number of good and bad items classified after the k−th inspection

$$L_k = N_{G/G}(k) + N_{G/B}(k) = (1-\alpha)G_{k-1} + \beta B_{k-1} \qquad \text{for k = 1, 2,} \cdots \text{, K} \qquad (1)$$

Let R_k be the number of items for the k-th rework. Since R_k is the sum of the number of good items classified as bad and the number of bad items classified as bad, R_k can be expressed as

$$R_k = N_{B/G}(k) + N_{B/B}(k) = \alpha G_{k-1} + (1-\beta)B_{k-1} \qquad \text{for k = 1, 2,} \cdots \text{, K} \qquad (2)$$

After reworking at the k-th rework process, we have

$$G_k = (1-q_R)R_k \qquad \text{for k = 1, 2,} \cdots \text{, K} \qquad (3)$$

$$B_k = q_R R_k \qquad \text{for k = 1, 2,} \cdots \text{, K} \qquad (4)$$

Since we assume that all the items passing through the K-th rework process are sent to the packing process without inspection, the average defective rate at the packing process can be expressed as

$$q_K = \frac{1}{R_0}\left(\sum_{k=1}^{K} N_{G/B}(k) + B_K\right) = \frac{1}{R_0}\left(\beta\sum_{i=0}^{K-1} B_i + q_R R_K\right) \qquad (5)$$

In order to derive an analytical expression for q_K, we need to prove the following properties. For convenience, let γ and ω be $\alpha(1-q_R)+(1-\beta)q_R$ and $\alpha(1-q_0)+(1-\beta)q_0$ respectively. Without loss of generality, we assume that $0<\alpha,\ \beta,\ q_0,\ q_R<1$.

Property 1. If $0<\alpha,\ \beta,\ q_R<1$, then $0<\gamma<1$ and $0<\omega<1$.

Proof : Since $0<\alpha,\ \beta,\ q_R<1$, it follows that $0<\gamma$.

If $\alpha\geq 1-\beta$, $\gamma=\alpha(1-q_R)+(1-\beta)q_R\leq\alpha(1-q_R)+\alpha q_R=\alpha<1$

If $\alpha<1-\beta$, $\gamma=\alpha(1-q_R)+(1-\beta)q_R<(1-\beta)(1-q_R)+(1-\beta)q_R=1-\beta<1$.

Therefore we have, $0<\gamma<1$. Using the similar manner above, clearly $0<\omega<1$. □

Property 2. If $0<\alpha,\ \beta,\ q_R<1$, then

(1) $R_k=\ R_0\omega\gamma^{k-1}$, for k = 1, 2,⋯, K

(2) $G_k=\ R_0\omega(1-q_R)\gamma^{k-1}$, for k = 1, 2,⋯, K

(3) $B_k=\ R_0\omega q_R\gamma^{k-1}$, for k = 1, 2,⋯, K

(4) $L_k=\ R_0(1-\omega)$ for k = 1

$\quad R_0\omega(1-\gamma)\gamma^{k-2}$, for k = 2, 3,⋯, K

Proof : (1) For k = 1, 2, ..., K, we have,

$$
\begin{aligned}
R_k &= \alpha G_{k-1}+(1-\beta)B_{k-1} && \text{(From Eq. (2))}\\
&= \alpha(1-q_R)R_{k-1}+(1-\beta)q_R R_{k-1} && \text{(Using Eq. (3) and Eq. (4))}\\
&= \gamma R_{k-1}\\
&= R_1\gamma^{k-1} && (\because \text{Property 1, } \gamma\neq 1)
\end{aligned}
$$

Since $G_0=(1-q_0)R_0$ and $B_0=q_0R_0$, using Eq. (2), R_1 can be expressed as

$$R_1=\{\alpha(1-q_0)+(1-\beta)q_0\}R_0=R_0\omega \tag{6}$$

It follows that $R_k=R_1\gamma^{k-1}=R_0\omega\gamma^{k-1}$.

(2) and (3): They hold true clearly from Eq. (3) and Eq. (4).

$$
\begin{aligned}
\text{(4)}\quad L_1 &= (1-\alpha)G_0+\beta B_0 && \text{(Using Eq. (1))}\\
&= R_0\{(1-\alpha)(1-q_0)+\beta q_0\} && (\because G_0=(1-q_0)R_0 \text{ and } B_0=q_0R_0)\\
&= R_0(1-\omega)
\end{aligned}
$$

For k = 2,⋯, K, we have,

$$
\begin{aligned}
L_k &= (1-\alpha)G_{k-1} + \beta B_{k-1} && \text{(Using Eq. (1))} \\
&= (1-\alpha)R_0\omega(1-q_R)\gamma^{k-2} + \beta R_0\omega q_R\gamma^{k-2} && \text{(Using Property 2 - (2) and (3))} \\
&= R_0\omega(1-\gamma)\gamma^{k-2} && (\because (1-\alpha)(1-q_R)+\beta q_R = 1-\gamma) \quad \square
\end{aligned}
$$

Proposition 3. If $0<\alpha,\ \beta,\ q_R<1$, then for a positive integer K,

(1) $q_K = \beta q_0 + \frac{\omega q_R}{1-\gamma}\{\beta + (1-\beta-\gamma)\gamma^{K-1}\}$

(2) $\lim_{K\to\infty} q_K = \frac{\beta}{1-\gamma}\{(1-\alpha)q_0 + \alpha q_R\}$

Proof: (1) Using Eq. (5) and Eq. (6), we have

$$q_1 = \frac{1}{R_0}(\beta B_0 + q_R R_1) = \beta q_0 + \omega q_R \tag{7}$$

For a positive integer K greater than one, we have

$$
\begin{aligned}
q_K &= \frac{1}{R_0}\left(\beta\sum_{i=0}^{K-1} B_i + q_R R_K\right) && \text{(Using Eq. (5))} \\
&= \frac{1}{R_0}\left(\beta B_0 + \beta R_0\omega q_R\sum_{i=1}^{K-1}\gamma^{i-1} + R_0\omega q_R\gamma^{K-1}\right) && \text{(Using Property 2-(1) and (3))} \\
&= \beta q_0 + \omega q_R\left\{\frac{\beta(1-\gamma^{K-1})}{1-\gamma} + \gamma^{K-1}\right\} && \text{(Using Eq. (6))} \\
&= \beta q_0 + \frac{\omega q_R}{1-\gamma}\{\beta + (1-\beta-\gamma)\gamma^{K-1}\}
\end{aligned}
$$

Since replacing K in the above equation with one gives Eq. (7), the above equation holds true when K = 1.

(2) Since $0<\gamma<1$, $\lim_{K\to\infty}\gamma^{K-1} = 0$. It follows that $\lim_{K\to\infty} q_K = \frac{\beta}{1-\gamma}\{(1-\alpha)q_0 + \alpha q_R\}$. □

If inspectors are perfect, that is, $(\alpha,\ \beta)$ = (0, 0), then using Proposition 3, $q_K = q_0 q_R^K$ and q_K converges to zero. On the other hand, when inspectors are imperfect, that is $(\alpha,\ \beta) = (1, 1)$, $q_K = 1-(1-q_0)(1-q_R)^K$ and q_K converges to one. It follows that the feasible region of q_K can be theoretically represented as the light dark area as shown in

Figure 4. It can be observed that our inspection system does not always guarantee that q_K will decreases as K increases. Hence, it will be useful to derive some conditions that guarantee $q_K \le q_G$. Before deriving the conditions, we need to prove the following properties. For convenience, let q_{0L} be $\alpha q_R \{(1-\beta)(1-q_R)+\alpha q_R\}^{-1}$.

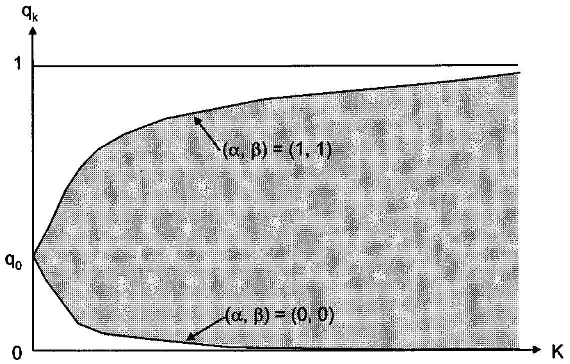


Figure 4. The feasible region of q_K

Property 4. If $0<\alpha,\ \beta,\ q_R<1$, then

(1) If $0<q_0 \le q_{0L}$, then $q_1 \ge q_0$.

(2) If $q_{0L}<q_0<1$, then $q_1<q_0$.

Proof : Using Eq. (7), (q_0-q_1) can be expressed as a first-order linear function of q_0, that is, $(q_0-q_1)=g(q_0)=\{(1-\beta)(1-q_R)+\alpha q_R\}q_0-\alpha q_R$. Since $0<\alpha,\ \beta,\ q_R<1$, we have, $\{(1-\beta)(1-q_R)+\alpha q_R\}>0$. It follows that g(0) becomes an infimum and g(1) becomes a supremum. Since g(0) = $-\alpha q_R<0$ and g(1) = $(1-\beta)(1-q_R)>0$, there exists q_{0L} such that $g(q_{0L})=0$. It follows that $q_{0L}=\alpha q_R\{(1-\beta)(1-q_R)+\alpha q_R\}^{-1}$. Therefore Property 4 holds true. □

Property 5. If $0<\alpha,\ \beta,\ q_R<1$, then for a positive integer K,

(1) If $0<\alpha+\beta<1$, then $q_{K+1}<q_K$.

(2) If $1\le\alpha+\beta<2$, then $q_{K+1}\ge q_K$.

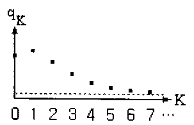
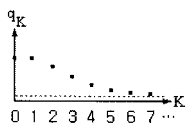
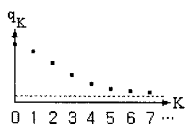
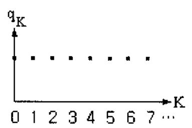
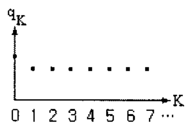
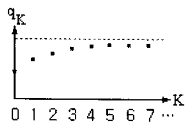
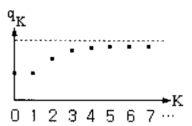
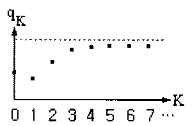
Proof: (1) Using Proposition 3 - (1), $q_{K+1}-q_K=-\omega q_R(1-q_R)(1-\alpha-\beta)\gamma^{K-1}$. Since $0<\omega<1$, $0<q_R(1-q_R)<1$, and $\gamma^{K-1}>0$, we have, $q_{K+1}<q_K$ if $0<\alpha+\beta<1$. (2) holds also true. □

Proposition 6. If $0 < \alpha,\ \beta,\ q_R < 1$, $q_{0L} < q_0 < 1$, and $0 < \alpha + \beta < 1$, then for a nonnegative integer K, q_K is a strictly decreasing function of K. That is, $q_{K+1} < q_K$.

Proof: Trivial from Proposition 3, Property 4 and 5. □

From Proposition 3, Property 4 and 5, the shape of q_K, depending on both q_0 and $(\alpha + \beta)$, will become one of nine shapes as shown in Table 1. The case that $1 \le \alpha + \beta < 2$ is meaningless since q_K will not decrease as K increases. The case that $0 < \alpha + \beta < 1$ and $0 < q_0 \le q_{0L}$ is also meaningless even though q_K may finally decrease as K increases since the value of q_{0L} is so small that our inspection system will be of no use at all. The case that $0 < \alpha + \beta < 1$ and $q_{0L} < q_0$ always guarantees that q_K will decrease as K increases.

Table 1. Nine shapes of q_k depending on the range of q_0 and $(\alpha + \beta)$

conditions	$0 < q_0 \le q_{0L}$	$q_{0L} = q_0$	$q_{0L} < q_0 < 1$
$0 < \alpha + \beta < 1$			
$\alpha + \beta = 1$			
$1 < \alpha + \beta < 2$			

3.2 Determination of K^*

It can be observed that the case that $0 < \alpha + \beta < 1$ and $q_{0L} < q_0$ does not always guarantee that $q_K \le q_G$ since there is a case that $\lim_{K \to \infty} q_K \ge q_G$. Thus, if $0 < \alpha,\ \beta,\ q_R < 1$, $q_{0L} < q_0 < 1$, $0 < \alpha + \beta < 1$, and $\lim_{K \to \infty} q_K < q_G$, then from Proposition 3 and Proposition 6, there exists a positive integer K^* such that $q_{K^*} \le q_G$. Now, K^* can be derived as follows. Define $\lceil x \rceil$ to be the smallest integer greater than or equal to x. Let $\hat{\gamma} = \hat{\alpha}(1-\hat{q}_R)+(1-\hat{\beta})\hat{q}_R$ and $\hat{\omega} = \hat{\alpha}(1-\hat{q}_0)+(1-\hat{\beta})\hat{q}_0$, where $\left(\hat{\alpha},\ \hat{\beta},\ \hat{q}_0,\ \hat{q}_R\right)$ are estimates of $\left(\alpha,\ \beta,\ q_0,\ q_R\right)$.

Theorem 7. If $0<\hat{\alpha},\ \hat{\beta},\ \hat{q}_R<1$, $q_{0L}<\hat{q}_0<1$, $0<\hat{\alpha}+\hat{\beta}<1$, and $\lim_{K\to\infty} q_K < q_G$, then

$$K^* = \left\lceil 1+\frac{\ln\hat{\tau}}{\ln\hat{\gamma}} \right\rceil \quad \text{where } \hat{\tau} = \frac{1}{\hat{\omega}(1-\hat{\beta}-\hat{\gamma})\hat{q}_R}\left\{(q_G-\hat{\beta}\hat{q}_0)(1-\hat{\gamma})-\hat{\beta}\hat{\omega}\hat{q}_R\right\},$$

Proof : Given $\hat{\alpha},\ \hat{\beta},\ \hat{q}_0,\ \hat{q}_R,\ q_G$, we need to obtain the smallest value of K such that $\hat{q}_K = f(\hat{\alpha},\ \hat{\beta},\ \hat{q}_0,\ \hat{q}_R,\ K) \le q_G$. Let K_E be the real number such that $f(\hat{\alpha},\ \hat{\beta},\ \hat{q}_0,\ \hat{q}_R,\ K_E) = q_G$. That is, using Proposition 3, we have

$$f\left(\hat{\alpha},\ \hat{\beta},\ \hat{q}_0,\ \hat{q}_R,\ K_E\right) = \hat{\beta}\hat{q}_0+\frac{\hat{\omega}\hat{q}_R}{1-\hat{\gamma}}\left\{\hat{\beta}+\left(1-\hat{\beta}-\hat{\gamma}\right)\hat{\gamma}^{K_E-1}\right\} = q_G \tag{8}$$

Solving the above equation for K_E gives

$$K_E = 1+\frac{\ln\hat{\tau}}{\ln\hat{\gamma}} \quad \text{where } \hat{\tau} = \frac{1}{\hat{\omega}(1-\hat{\beta}-\hat{\gamma})\hat{q}_R}\left\{(q_G-\hat{\beta}\hat{q}_0)(1-\hat{\gamma})-\hat{\beta}\hat{\omega}\hat{q}_R\right\}$$

Since $\lim_{K\to\infty} q_K < q_G$ and q_K is a strictly decreasing function of K from Proposition 6, K^* will be the smallest integer greater than or equal to K_E. We can express K^* as

$$K^* = \lceil K_E \rceil = \left\lceil 1+\frac{\ln\hat{\tau}}{\ln\hat{\gamma}} \right\rceil$$

3.3 Estimation of $(\alpha,\ \beta)$

There might be various methods for estimating $(\alpha,\ \beta)$. In this paper, we suggest an indirect method for estimating $(\alpha,\ \beta)$. Two equations can be obtained from Eq. (6) and Eq. (7) as follows:

$$(1-q_0)\alpha - q_0\beta = \frac{R_1}{R_0} - q_0 \tag{9}$$

$$(1-q_0)q_R\alpha + q_0(1-q_R)\beta = q_1 - q_0 q_R \tag{10}$$

Since it is practically possible to estimate both (q_0, q_1, q_R) and (R_0, R_1), the right hand sides of Eq. (9) and Eq. (10) will be constant. Thus, we can obtain the following estimators by solving simultaneously the above two equations.

$$\hat{\alpha} = \frac{1}{(1-\hat{q}_0)}\left\{\hat{q}_1 - \hat{q}_0 + \frac{\hat{R}_1}{\hat{R}_0}(1-\hat{q}_R)\right\} \tag{11}$$

$$\hat{\beta} = \frac{1}{\hat{q}_0}\left\{\hat{q}_1 - \frac{\hat{R}_1}{\hat{R}_0}\hat{q}_R\right\} \tag{12}$$

4. A Procedure For Our Search Problem

Nonexistence of K^* indicates that our K-stage inspection system with the current vector, $(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R)$, can not achieve q_G. There might be several ways in order to meet q_G. In this paper, we assume that (α, β) can not be controlled and that we can control (q_0, q_R) from $(\hat{q}_0, \hat{q}_R)$ to $(\tilde{q}_0, \tilde{q}_R)$ such that $f(\hat{\alpha}, \hat{\beta}, \tilde{q}_0, \tilde{q}_R, \tilde{K}) = \tilde{q}_{\tilde{K}} = q_G$ for a positive integer $\tilde{K}$.

In our search problem, $(\tilde{q}_0, \tilde{q}_R)$ can be easily determined by holding either $\tilde{q}_0$ or $\tilde{q}_R$. Without loss of generality, we hold $\tilde{q}_R$. Now, using Proposition 3-(1), we have

$$\tilde{q}_{\tilde{K}} = \hat{\beta}\tilde{q}_0 + \frac{\tilde{\omega}\tilde{q}_R}{1-\tilde{\gamma}}\left\{\hat{\beta} + \left(1-\hat{\beta}-\tilde{\gamma}\right)\tilde{\gamma}^{\tilde{K}-1}\right\} = q_G \tag{13}$$

where $\tilde{\gamma} = \hat{\alpha}(1-\tilde{q}_R) + (1-\hat{\beta})\tilde{q}_R$ and $\tilde{\omega} = \hat{\alpha}(1-\tilde{q}_0) + (1-\hat{\beta})\tilde{q}_0$. Replacing $\tilde{\omega}$ in Eq. (13) with $\hat{\alpha}(1-\tilde{q}_0) + (1-\hat{\beta})\tilde{q}_0$, we can express $\tilde{q}_0$ as

$$\tilde{q}_0 = \frac{(1-\tilde{\gamma})q_G - \hat{\alpha}\tilde{q}_R\left\{\hat{\beta} + \left(1-\hat{\beta}-\tilde{\gamma}\right)\tilde{\gamma}^{\tilde{K}-1}\right\}}{(1-\tilde{\gamma})\hat{\beta} + \tilde{q}_R\left(1-\hat{\alpha}-\hat{\beta}\right)\left\{\hat{\beta} + \left(1-\hat{\beta}-\tilde{\gamma}\right)\tilde{\gamma}^{\tilde{K}-1}\right\}} \tag{14}$$

Since $(1-\tilde{\gamma})\hat{\beta} + (1-\hat{\alpha}-\hat{\beta})\hat{\beta}\tilde{q}_R = (1-\hat{\alpha})\hat{\beta}$, and $(1-\hat{\alpha}-\hat{\beta})(1-\tilde{q}_R) = (1-\hat{\beta}-\tilde{\gamma})$, $\tilde{q}_0$ can be expressed as a function of $(\tilde{q}_R, \tilde{K})$ as follows.

$$\tilde{q}_0 = \frac{(1-\tilde{\gamma})q_G - \hat{\alpha}\tilde{q}_R\left\{\hat{\beta}+\left(1-\hat{\beta}-\tilde{\gamma}\right)\tilde{\gamma}^{\tilde{K}-1}\right\}}{\left(1-\hat{\alpha}\right)\hat{\beta}+\left(1-\hat{\alpha}-\hat{\beta}\right)\left(1-\hat{\beta}-\tilde{\gamma}\right)\tilde{q}_R\tilde{\gamma}^{\tilde{K}-1}} \qquad (15)$$

Hence, using Eq. (15), we can obtain $\left(\tilde{q}_0, \tilde{q}_R, \tilde{K}\right)$ when $(\tilde{q}_R, \tilde{K})$ is given.

5. A Case Study

A BLU supplier was selected and one of the major products produced by the supplier was selected. Whenever a batch of the selected product were produced, both the number of good items and the number of bad items were recorded by an inspector at the end of a production line. In addition, after reworking the bad items sent by the inspector, the number of bad items was also recorded by an inspector at the end of a rework place. After collecting the accumulated data for six months, we estimated $(q_0, q_1, q_R, R_0, R_1)$ as (16.1%, 1.53%, 5.0%, 1,200,000 units, 193,000 units).

Using Eq. (11) and Eq. (12), we estimated (α, β) as (0.8453%, 4.5083%). Since the target defective rate of the BLU supplier in study was 8,000 PPM and the values of $\hat{q}_K$ for K = 2, 3, 4, 5 were computed sequentially as 8,069 PPM, 7,666 PPM, 7,644 PPM, 7,642 PPM as shown in Figure 5, clearly we have $K^* = 3$. The same result can be obtained from Theorem 7. Note that K^* exists in this case since $\lim_{K\to\infty} q_K$ was computed using Proposition 1-(2) as 7,642 PPM < 8,000 PPM = q_G. It can be observed that the value of $\hat{q}_K$ falls down with the biggest slope when K = 1, and that the difference between two values of $\hat{q}_K$ and $\hat{q}_{K+1}$ could be ignored if K becomes greater than 2.

Suppose that q_G = 7,000 PPM. Since $\lim_{K\to\infty} q_K$ = 7,642 PPM, we can not achieve 7,000 PPM at all using the current values (0.8453%, 4.5083%, 16.1%, 5.0%) of $\left(\hat{\alpha}, \hat{\beta}, \hat{q}_0, \hat{q}_R\right)$ regardless of any value of K. As discussed, assuming that $\left(\hat{\alpha}, \hat{\beta}\right)$ does not change, we search the curve of $\left(\tilde{q}_0, \tilde{q}_R\right)$, all the points on which can achieve 7,000

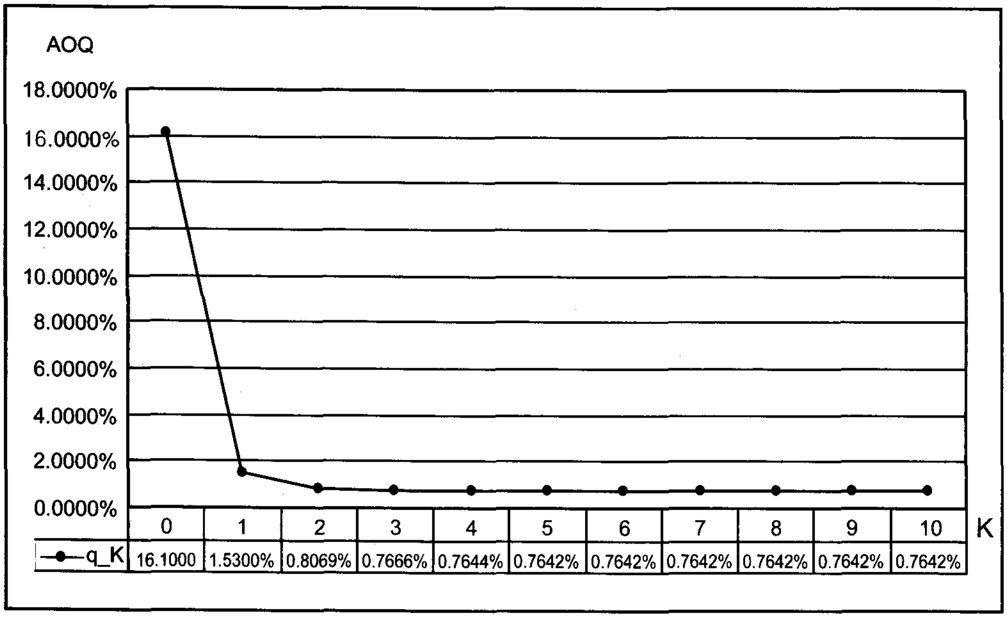


Figure 5. The curve of $\hat{q}_K$ as a function of the number of stages (K)

PPM. By setting some practical combination of $(\tilde{q}_R, \tilde{K})$ for $\tilde{K}$ = 1, 2, 3 and $\tilde{q}_R$ = 0.0%, 0.2%, ⋯, 5.0%, using Eq. (15), we compute and summarize $\tilde{q}_0$'s in Table 2. For example, if the company improves (q_0, q_R) from $(\hat{q}_0, \hat{q}_R)$ = (16.1%, 5.0%) to either $(\tilde{q}_0, \tilde{q}_R)$ = (14.3566%, 4%) or $(\tilde{q}_0, \tilde{q}_R)$ = (13.9550%, 5%), then the given target defective rate of 7,000 PPM can be theoretically achieved by a 2-Stage inspection system(i.e., $\tilde{K}$ = 2). Suppose that a 1-stage inspection system must be used and that the current value of $\hat{q}_R$ (= 5%) can not be reduced. In this case, the reduction of q_0 to 7.1179% guarantees the target defective rate of 7,000 PPM. An appropriate choice among many alternatives may be selected depending upon circumstances of a company. Note that when $\tilde{q}_R$ = 0.0%, the values of $\tilde{q}_0$'s for $\tilde{K}$ = 1, 2, 3 happen to be computed as the same value as 15.5270% due to the round-off at the fifth digit below the decimal point. From Table 2, given $(\tilde{q}_R, \tilde{K})$, the curve of $\tilde{q}_0$ satisfying $\tilde{q}_G$ = 7,000 PPM is drawn in Figure 6. It can be observed that, as $\tilde{q}_R$ increases, the values of $\tilde{q}_0$ with $\tilde{K}$ = 1 go down more rapidly than the values of $\tilde{q}_0$ with $\tilde{K}$ = 2 or 3. In addition, it can be observed that the variation of $\tilde{q}_0$ to attain $\tilde{q}_G$ becomes smaller than the variation of $\tilde{q}_R$ to attain

$\tilde{q}_G$ as $\tilde{K}$ increases.

Table 2. $(\tilde{q}_0, \tilde{q}_R, \tilde{K})$ when $(\hat{\alpha}, \hat{\beta}, q_G)$ = (0.8453%, 4.5083%, 7,000 PPM) (Entries = $\tilde{q}_0$ %)

$\tilde{q}_R$ (%)	$\tilde{K}=1$	$\tilde{K}=2$	$\tilde{K}=3$	$\tilde{q}_R$ (%)	$\tilde{K}=1$	$\tilde{K}=2$	$\tilde{K}=3$
0.0	15.5270	15.5270	15.5270	2.6	9.7290	14.8553	15.1106
0.2	14.8653	15.4889	15.4956	2.8	9.4481	14.7892	15.0775
0.4	14.2549	15.4483	15.4641	3.0	9.1817	14.7213	15.0442
0.6	13.6900	15.4054	15.4326	3.2	8.9287	14.6516	15.0106
0.8	13.1658	15.3602	15.4009	3.4	8.6880	14.5803	14.9768
1.0	12.6779	15.3126	15.3692	3.6	8.4589	14.5073	14.9428
1.2	12.2228	15.2629	15.3374	3.8	8.2405	14.4327	14.9086
1.4	11.7971	15.2109	15.3054	4.0	8.0321	14.3566	14.8740
1.6	11.3983	15.1567	15.2734	4.2	7.8329	14.2790	14.8392
1.8	11.0237	15.1005	15.2411	4.4	7.6425	14.2000	14.8041
2.0	10.6713	15.0421	15.2088	4.6	7.4601	14.1196	14.7688
2.2	10.3392	14.9818	15.1762	4.8	7.2854	14.0379	14.7331
2.4	10.0256	14.9195	15.1435	5.0	7.1179	13.9550	14.6971

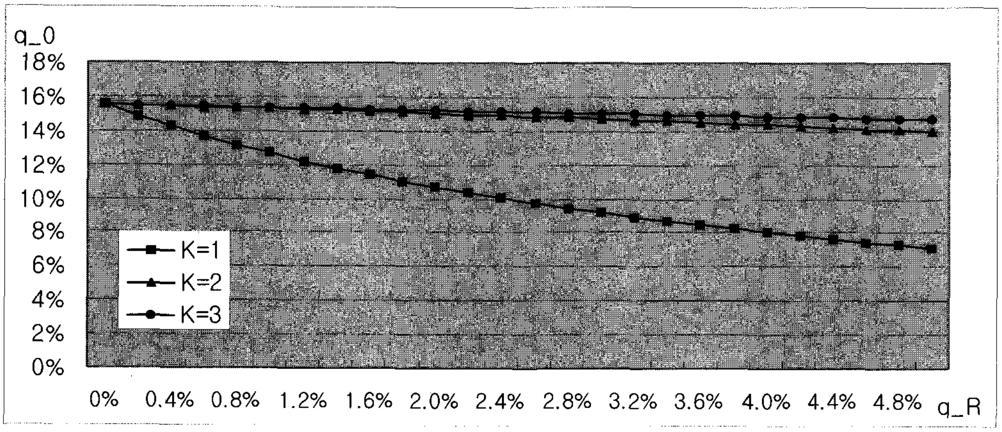


Figure 6. Three curves of $(\tilde{q}_0, \tilde{q}_R)$ where all the points on the lines can achieve the target defective rate of 7,000 PPM

6. Concluding Remarks

As mentioned in our case study, we have adopted and operated a 3-stage inspection system for the purpose of achieving the given target defective rate of 8,000 PPM. However, as there always exists a real gap between theory and practice, we could not achieve the theoretical defective rate of 7,666 PPM, estimated by our formula. In the early period of operations of our 3-stage inspection system, the realized defective rates collected every day have fluctuated from twice to ten times of the theoretical defective rate.

After discussing abnormal phenomenon with the related field managers, we made a provisional conclusion that the realized rates had been due to the psychological warfare and/or discord among inspectors. In other words, we inferred that the first inspector belonging to a production department, while recognizing that there was the second inspection after her inspection, tended to delay her work to the second inspector, who belonged to a quality department. Also the resistive attitude by inspectors against a new system without some incentive plans was also discussed as one of the various reasons. It was, however, not easy to find the exact reasons.

Further research may be concentrated on the study for the selecting ability of an inspector due to the psychological warfare or variation, and the study for determining an optimal defective rate in terms of costs and benefits due to the reduction of defective rate.

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