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다층분석법을 이용한 대규모 파라미터 설계 최적화*

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Multi-Level Response Surface Approximation for Large-Scale Robust Design Optimization Problems*

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■ Abstract ■

Robust Design (RD) is a cost-effective methodology to determine the optimal settings of control factors that make a product performance insensitive to the influence of noise factors. To better facilitate the robust design optimization, a dual response surface approach, which models both the process mean and standard deviation as separate response surfaces, has been successfully accepted by researchers and practitioners. However, the construction of response surface approximations has been limited to problems with only a few variables, mainly due to an excessive number of experimental runs necessary to fit sufficiently accurate models. In this regard, an innovative response surface approach has been proposed to investigate robust design optimization problems with larger number of variables. Response surfaces for process mean and standard deviation are partitioned and estimated based on the multi-level approximation method, which may reduce the number of experimental runs necessary for fitting response surface models to a great extent. The applicability and usefulness of proposed approach have been demonstrated through an illustrative example.

Keywords : Robust Design, Response Surface, Multi-Level Analysis, Design of Experiment

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1. Introduction

Off-line quality control has received a lot of attention from researchers as well as practitioners since introduced by Taguchi [6]. The off-line quality control encompasses cost-effective activities designed to improve the product's quality, manufacturability and reliability during the product and process design stages. It basically consists of three phases : system design, parameter design, and tolerance design. The system design is the phase of the product design process where the general product concepts are generated and established. The parameter design is concerned with the optimization of the system identified in the previous stage. Finally, the tolerance design is intended to capture and reduce the variability in the output. Among the three design phases, the parameter design is the most important and crucial phase since it can provide the means for both reducing costs and improving quality. In parameter design, the optimum settings of control factors are determined to minimize the performance sensitivity to noise factors, which is called 'robust design'. The primary goal of Taguchi's concept of robust design is to obtain a target condition on the mean while minimizing the variance. Taguchi advocates the use of signal-to-noise (SN) ratios to achieve this goal. Although the inclusion of noise factors for design optimization has been considered as an innovative concept by researchers, there is a general consensus that several shortcomings are inherent in the Taguchi's approach. First, the Taguchi method lacks a sequential formal investigation for the purpose of optimization. Second, orthogonal arrays are not convincing, particularly when there are high interactions among control and noise factors. Finally, as a heuristic tool to minimize the quality loss, the universal use of SN ratio is not convincing (see Box [1]).

To rectify these problems, there has been such an effort to implement the robust design principle within the framework of well-established statistical analysis. The use of a dual response surface approach, suggested by Myers and Carter [3] and popularized by Vining and Myers [7], has received the most attention due to its greater degree of empirical modeling for both process mean and variance. Separately modeling process mean and variance through experimental data, a dual response surface approach achieves the goal of robust design by minimizing the variance subject to the process mean kept at the target. Let $\hat{\mu}(\boldsymbol{x})$ and $\hat{\sigma}(\boldsymbol{x})$ represent the fitted response functions for the mean and the standard deviation of the quality characteristic, respectively. Assuming a second-order polynomial model for the response functions, the response functions for process mean and standard deviation may be written as

$$\hat{\mu}(\boldsymbol{x}) = \alpha_0 + \sum_{i=1}^{k} \alpha_i x_i + \sum_{i=1}^{k} \alpha_{ii}^2 x_i^2 + \sum_{i=1}^{k} \sum_{i<j} \alpha_{ij} x_i x_j \quad (1)$$

and

$$\hat{\sigma}(\boldsymbol{x}) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii}^2 x_i^2 + \sum_{i=1}^{k} \sum_{i<j} \beta_{ij} x_i x_j, \quad (2)$$

respectively.

However, the construction of response surface approximations has been limited to problems with only a few variables, mainly due to an excessive number of experimental runs necessary to fit sufficiently accurate models. Recently, Koch et al. [2] and Perry et al. [4, 5] proposed an innovative response surface approach to accommodate larger number of variables. Koch et al. [2] partitioned the experimental regions of interest to model complex systems, which usually involve a large number of design variables. It have been demonstrated, through an illustrative example of designing commercial turbofan engine, that the number of experimental runs may significantly be reduced. Perry et al. [4, 5] also suggested the use of partition experimental designs for analyzing a sequential process with a large number of design variables. The reasoning behind the partitioned experimental designs may also be applied to the problem of robust design optimization, where the response surfaces for process mean and standard deviation need to be estimated separately. That is, response surfaces for process mean and standard deviation are partitioned and estimated based on the multi-level approximation method, which may significantly reduce the number of experimental runs necessary for fitting response surface models. The experimental layout may be obtained by combining the concepts of fractional factorial design and central composite design. This article is organized as follows : The concept of multi-level response surface approximation is discussed in Section 2, based on which a large-scale robust design optimization scheme is developed as described in Section 3. The usefulness and applicability of proposed approach to robust design are demonstrated through an illustrative numerical example in Section 4, which is followed by concluding remarks in the last section.

2. Multi-Level Response Surface Approximation

The usefulness of response surface methodology (RSM) for the purpose of robust design optimization has been recognized in a wide variety of industrial applications. One significant limitation of traditional response surface techniques is due to the problem of size for modeling large-scale systems. An excessive amount of expense may be incurred mainly because of the combinatorial explosion in data points necessary for fitting models with a large number of variables. For complex systems, the number of variables affecting the system's performances is often greater than desirable for traditional response surface methodology. Koch et al. [2] proposed an approach for constructing multi-level partitioned response surfaces to overcome the problem of size with large-scale systems. The factors and responses of a complex system are grouped, and the response surface models themselves are partitioned to create multi-level models incorporating the effects of all factors.

The modeling framework of Koch et al. [2] may be summarized as follows : Suppose that there are two responses of interest for a particular system. Let n be the number of factors associated with the system. The factors are to be partitioned into two sets. The first response, denoted by $\hat{y}_1$, is fit as a function of the k factors, and the second response, denoted by $\hat{y}_2$, is then fit as a function of the remaining $(n-k)$ factors. Two separate experiments are designed and concurrently run to fit these two response surfaces as shown in equation (3) and equation (4).

$$\hat{y}_1=\alpha_0+\sum_{i=1}^{k}\alpha_i x_i+\sum_{i=1}^{k}\alpha_{ii}^2 x_i^2+\sum_{i=1}^{k}\sum_{i<j}\alpha_{ij}x_i x_j \tag{3}$$

$$\hat{y}_2=\beta_0+\sum_{i=k+1}^{n}\beta_i x_i+\sum_{i=k+1}^{n}\beta_{ii}^2 x_i^2 + \sum_{i=k+1}^{n}\sum_{i<j}\beta_{ij}x_i x_j, \tag{4}$$

To capture the effects of factors $k+1$ through n on the first response, the intercept term of the first response, α_0, is fit as a function of these factors. The same is done for the second response using factors 1 through k as shown in equation (5) and equation (6).

$$\alpha_0 = \gamma_0 + \sum_{i=k+1}^{n} \gamma_i x_i + \sum_{i=k+1}^{n} \gamma_{ii}^2 x_i^2 + \sum_{i=k+1}^{n} \sum_{i<j} \gamma_{ij} x_i x_j \quad (5)$$

$$\beta_0 = \delta_0 + \sum_{i=1}^{k} \delta_i x_i + \sum_{i=1}^{k} \delta_{ii}^2 x_i^2 + \sum_{i=1}^{k} \sum_{i<j} \delta_{ij} x_i x_j, \quad (6)$$

The effect of modeling the intercept term of each response as a response surface itself is essentially to allow the intercept of the primary response surface to move along the response axis with the secondary effects of the factors used to model intercept term. The defining aspect of this approach is that the model fitting expense associated with experimental runs is reduced tremendously. If, for example, 16 factors are known to be significant to the responses of a system, more than 65,000 experimental runs are required to fit response surfaces using the standard central composite design (CCD). Partitioning these 16 factors into two sets of 8 factors, only 273 experimental runs are required for the standard CCD.

Later and Perry et al. [4, 5] investigated the application of partition experimental designs to obtain the optimum settings of design variables for sequential processes. They recommended the use of small composite design (SCD) instead of standard CCD. In SCD experiments, axial and center point runs are combined with factorial components obtained from fractional factorial design. It is also noted that the experimental data from SCD experiments should be carefully analyzed since the design efficiency may be reduced due to the .alias structure in the fractional portion of experimental runs. Thus, it will be desirable to employ fractional factorial designs with higher resolutions if applicable. This study also employs an SCD experiment to obtain estimated response functions for process mean and standard deviation for the purpose of robust design with a larger number of variables, so-called a large-scale robust design, which is outlined in the following section.

3. Large-Scale Robust Design Optimization

Taguchi's philosophy for robust design emanates from the definition of quality in terms of loss imparted to the society from the time a product is shipped. There is an ideal target value for the quality characteristic from the customer's viewpoint. The main idea is that loss is always incurred when a product's quality characteristic deviates from its target value, regardless of how small the deviation is. From this definition, a loss function is developed to measure the deviation of a product's quality characteristic from its target value in a monetary value. Various loss functions have been discussed in the literature of statistical decision theory. However, a simple quadratic function may be reasonable for many situations. Let $L(y)$ be a measure of losses associated with the quality characteristic Y, whose target value is τ, then the quadratic loss function is given by (Taguchi [6]).

$$L(y) = k(y-\tau)^2,$$

where k is a positive loss coefficient. It is well known that the expected value of the quadratic loss function can be decomposed into bias and variance as follows :

$$E[L(y)] = k[(\mu-\tau)^2 + \sigma^2].$$

This implies that the quality level of a product is determined by both the process mean and standard deviation, so that simultaneous minimization of process bias and variance is desirable. Dual response surface approach (Myers and Carter [3]) suggests that the optimal settings of factors may be obtained by separately modeling the process mean and standard deviation using RSM as shown in equation (1) and equation (2).

As discussed earlier, however, the usual RSM may not be applicable when there exist a number of factors affecting the mean and standard deviation. Multi-level response surface approximations may efficiently be incorporated to overcome this drawback. Replacing $\widehat{y_1}$ in equation (3) and $\widehat{y_2}$ in equation (4) with $\hat{\mu}$ and $\hat{\sigma}$, respectively, one may obtain the response surfaces for process mean and standard deviation even with a large number of factors. It is worth noting that how to partition the factors of interest may be critical to obtain accurate approximations for each response. Arbitrary partitioning may not be desirable since some of the factors mostly affecting the process mean may be allocated to approximate the standard deviation and vice versa. It is not often the case that all the factors of interest have a significant effect on both the process mean and standard deviation. Screening experiments may be conducted to determine which factors have a significant effect on process mean and/or standard deviation. Factors significantly affecting the process mean may be allocated to approximate the mean response $\hat{\mu}$, and remaining factors may be used to estimate the intercept term. On the other hand, factors mostly affecting standard deviation may be assigned to approximate the corresponding response $\hat{\sigma}$, of which the intercept term may be estimated using the factors affecting the process mean.

The partition of experimental design is now demonstrated for the robust design problem with six factors, denoted by A through F. The following simulated response functions for process mean and standard deviation are used :

$$\hat{\mu}(x) = 140.0 - 5.0A + 7.5B - 3.5C + 2.0E - 3.5BD + N(0, 3.0^2) \quad (7)$$

and $$\hat{\sigma}(x) = 1.5 + 0.05B - 0.1C + 0.15E - 0.1F + 0.07EF + N(0.0.1^2) \quad (8)$$

A screening experiment may be conducted to determine which factors affect process mean and/or standard deviation. A 2^{6-2} fractional factorial experiment with resolution IV is conducted. As expected, main effects A, B, C, E, and two factor interaction BD turns out to have a significant effect on process mean, whereas B, C, E, F, and EF are signifiant factors for standard deviation. Based on the results from screening experiments, each factor needs to be divided into two partitions. It is desirable that main effects associated with significant interactions belong to the same partition since the interaction effects between factors from different partitions are not estimable in partitioned experimental designs (Koch et al. [2]). Thus, main effects B and D are assigned to one partition while E and F are to the other partition. For the remaining main effects, it is reasonable to assign A and C to different partitions. Consequently, factors A, B, and D constitute one partition while C, E, and F are allocated to the other partition. Axial and center point runs are now added to each partition to complete the construction of SCD. The value of α is chosen as $\sqrt[4]{F}$ to ensure the design rotatability, where F is the total number of factorial runs across partitions. The number of center points may be determined by condition number, which represents the measure of orthogonality. In this example, three center points for each partition appears to be sufficient. All the design points and corresponding simulated responses are summarized in <Table 1>. It is worth noting that design points in block 1 indicate the screening runs, and axial and center point runs are sequentially conducted and placed in block

〈Table 1〉 Design Points and Simulated Responses

Block	Partition 1			Partition 2			$\hat{\mu}$	$\hat{\sigma}$
	A	B	D	C	E	F		
1	-1	-1	-1	-1	-1	-1	133.113	1.50275
1	1	-1	-1	-1	1	-1	129.300	1.65382
1	-1	1	-1	-1	1	1	159.317	1.94536
1	1	1	-1	-1	-1	1	146.963	1.27450
1	-1	-1	-1	1	1	1	135.833	1.41078
1	1	-1	-1	1	-1	1	120.983	1.09230
1	-1	1	-1	1	-1	-1	149.701	1.38684
1	1	1	-1	1	1	-1	144.142	1.70557
1	-1	-1	1	-1	-1	1	150.989	1.23126
1	1	-1	1	-1	1	1	138.494	1.55425
1	-1	1	1	-1	1	-1	151.255	1.88316
1	1	1	1	-1	-1	-1	142.542	1.64793
1	-1	-1	1	1	1	-1	138.619	1.65875
1	1	-1	1	1	-1	-1	123.729	1.31921
1	-1	1	1	1	-1	1	140.042	1.05699
1	1	1	1	1	1	1	135.915	1.60179
2	-2.0	0	0	-2.0	0	0	161.903	1.66377
2	2.0	0	0	2.0	0	0	119.245	1.35048
2	0	-2.0	0	0	-2.0	0	117.458	0.92487
2	0	2.0	0	0	2.0	0	159.662	2.07900
2	0	0	-2.0	0	0	-2.0	143.773	1.55792
2	0	0	2.0	0	0	2.0	143.281	1.22783
2	0	0	0	0	0	0	140.227	1.44158
2	0	0	0	0	0	0	139.631	1.43214
2	0	0	0	0	0	0	140.093	1.38744
2	0	0	0	0	0	0	141.612	1.58090
2	0	0	0	0	0	0	138.635	1.41994
2	0	0	0	0	0	0	141.793	1.47554

2. It is also pointed out that less than 30 runs are performed in this case whereas as many as 90 experimental runs may be required to run the standard CCD with six factors.

4. An Illustrative Example : Analysis

The results from the statistical analysis for example case presented in Section 3 are further elaborated in this section. The ANOVA tables for screening experiments with respect to process mean and standard deviation are provided in <Table 2> and <Table 3>, respectively. These results are based on the experimental runs from the block 1. All the factors shown in simulated response functions turn out to be significant for each response. In addition, main effect D is also significant with

〈Table 2〉 Analysis of Variance for Screening Experiments with respect to Process Mean

Source	SS	DF	MS	F	P-Value
A	455.33	1	455.33	77.06	< 0.001
B	950.95	1	950.95	160.93	< 0.001
C	283.29	1	283.29	47.94	< 0.001
D	38.78	1	38.78	6.56	0.031
E	64.34	1	64.34	10.89	0.009
BD	179.93	1	179.93	30.45	< 0.001
Error	53.18	9	5.91		
Total	2025.80	15			

〈Table 3〉 Analysis of Variance for Screening Experiments with respect to Standard Deviation

Source	SS	DF	MS	F	P-Value
B	0.06943	1	0.06943	8.37	0.016
C	0.16428	1	0.16428	19.80	0.001
E	0.49753	1	0.49753	59.98	< 0.001
F	0.14143	1	0.14143	17.05	0.002
EF	0.06853	1	0.06853	8.26	0.017
Error	0.08295	10	0.00830		
Total	1.02415	15			

respect to process mean which is not included in the simulated mean response. It is obvious, from <Table 2>, that the main effects B and D should be placed in the same partition along with A, and the remaining main effects C, E, and F go to the other partition.

〈Table 4〉 Estimated Coefficients for Process Mean Using All Runs

Term	Coefficient	SE Coef.	T	P
Constant	140.334	0.4122	340.465	< 0.001
A	-4.844	0.4531	-10.691	< 0.001
B	6.490	0.4531	14.324	< 0.001
C	-3.982	0.4531	-8.788	< 0.001
E	1.865	0.4531	4.117	0.001
BD	-3.936	0.5397	-7.292	< 0.001

Sequentially conducting axial and center point runs as shown in <Table 1>, the process mean and standard deviation responses are analyzed using response surface regression. The estimated coefficients for process mean and standard deviation are summarized in <Table 4> and <Table 5>, respectively. For mean response μ, the values of R^2 and adjusted R^2 are 97.6% and 96.4%, respectively. Sufficiently large values of R^2 and adjusted R^2 (93.0% and 90.1%, respectively) are also examined for standard deviation response σ.

〈Table 5〉 Estimated Coefficients for Standard Deviation Using All Runs

Term	Coefficient	SE Coef.	T	P
Constant	1.47856	0.01565	94.494	< 0.001
B	0.06497	0.01720	3.777	0.001
C	-0.07944	0.01720	-4.168	< 0.001
E	0.17889	0.01720	10.400	< 0.001
F	-0.09004	0.01720	-5.235	< 0.001
EF	0.05078	0.02049	2.479	0.023

Using the coefficients shown in <Table 4> and <Table 5>, the estimated response functions for process mean and standard deviation may be written as follows :

$$\hat{\mu}(x) = 140.334 - 4.844A + 6.490B$$
$$- 3.982C + 1.865E - 3.936BD$$
$$\hat{\sigma}(x) = 1.479 + 0.065B - 0.080C$$
$$+ 0.179E - 0.090F + 0.051EF.$$

Comparing these with equation (7) and equation (8), the above estimated functions closely resembles the simulated functions.

5. Concluding Remarks

The multi-level partitioned response surface modeling approach has been proposed to overcome the problem of size for a complex system, where an excessive number of experimental runs are required. It is demonstrated that a large-scale robust design problem may also be resolved with a moderate number of experimental runs by using the two-level partitioned response surface methodology. Experimental runs from fractional factorial design are combined with axial and center point runs to construct the experimental design for a large-scale robust design optimization. The proposed approach presented in this article may particularly be useful for large-scale robust design problems in which the number of variables prohibits standard response surface experimentation and modeling, and the problem can easily be partitioned. The major assumption in the proposed approach is that the interaction effects between the factors of each partition are negligible or nonexistent. However, many interactions may be shown to be nonexistent by performing screening experiments. The factors can then be partitioned so that only the significant interactions are to be included. The applicability and usefulness of the proposed approach are demonstrated through a numerical example.

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