經營科學
第24卷 第2號
2007年 11月

한국 주식 수익률에 대한 Extreme 분포의 적용 가능성에 관하여*

김 명 석**

On the Applicability of the Extreme Distributions to Korean Stock Returns*

Myung Suk Kim**

■ Abstract ■

Weekly minima of daily log returns of Korean composite stock price index 200 and its five industry-based business divisions over the period from January 1990 to December 2005 are fitted using two block-based extreme distributions : Generalized Extreme Value (GEV) and Generalized Logistic (GLO). Parameters are estimated using the probability weighted moments. Applicability of two distributions is investigated using the Monte Carlo simulation based empirical p-values of Anderson Darling test. Our empirical results indicate that both the GLO and GEV models seem to be comparably applicable to the weekly minima. These findings are against the evidences in Gettinby et al. [7], who claimed that the GEV model was not valid in many cases, and supported the significant superiority of the GLO model.

Keywords : Anderson Darling Test, Extreme Distribution, International Monetary Fund, Korean Composite Stock Price Index, Probability Weighted Moment, Value at Risk

논문접수일 : 2007년 01월 15일 논문게재확정일 : 2007년 10월 04일
* The author is grateful to the Associate Editor and two referees for their helpful comments and suggestions. This work was supported by the Sogang University Research Grand of 2007.
** 서강대학교 경영학과

1. Introduction

Modeling the extreme loss of returns on the stock markets has been a particular interest to many academic authors as well as to the market practitioners due to its critical impact to the whole capital markets. In the beginning, normal distributions were applied for modeling the extreme losses. However, the failures of the normal distribution assumption have been reported in many papers, and the direct extreme distribution based modeling has been suggested. Furthermore, the extreme distributions have been applied to the value at risk (VaR) computation (De Vries and Danielsson [4]). Login [16, 17] suggested applying a Frechet distribution to modeling the extreme stock returns in US refuting the normal assumption. Nam and Kim [18] showed invalidity of applying the normal distribution assumption to measuring the VaR for the Korean composite stock price index (KOSPI) 200. Similar study was in Gettinby et al. [7]. They considered several distributions such as normal, Frechet, Gumbel, Weibull, Generalized Pareto (GP), Generalized Extreme Value (GEV), and Generalized Logistic (GLO) in modeling some extreme values on US, UK, and Japan stock markets. Using the Anderson Darling test, they selected the GEV and GLO models among the aforementioned distributions as available distributions. In particular, they advocated the GLO distribution, which provided apparently better fitness than the GEV distribution in most financial dynamics.

Motivated by Gettinby et al. [7], we investigate the applicability of two generalized extreme distributions ; the GEV and GLO to the weekly minima of the KOSPI 200 and its five industry-based business divisions. Considered period is ranged from January 1990 to December 2005. Due to significant increments of variances of the underlying dynamics since International Monetary Funds (IMF) aids to the Korean government in the middle of 1997, we analyzed two sub-periods separately ; before the aid and after that. In order to catch the detailed tail behavior in the extreme distributions, the quantile estimates are also provided at the end of the paper. According to Gettinby et al. [7], the GEV model is not valid in many cases and the GLO provided significantly better fits than the GEV on the whole. On the other hand, our empirical results indicated that both models appear to be comparably applicable to the KOSPI 200 over the considered periods. Some reason for the different results will be mentioned in the main text.

Our paper and the article by Gettinby et al. [7] seem to be similar in the sense that the probability weighted moment (PWM) technique and the Anderson Darling test are applied in the main analysis. However, our paper is mainly different from Gettinby et al. [7] in the following two aspects. First, we clearly explained our estimation procedures step by step. We transformed the original weekly minima into maxima via the sign switch. Then, we apply the GEV and GLO distributions to the sign transformed data in order to obtain the PWM based parameter estimators. Second, we directly compute the empirical p-values of the Anderson Darling test statistics via Monte Carlo simulation studies. Through this approach, we show that the applicability of the aforementioned extreme models to the considered data sets can be investigated without using a well-made test table. Meanwhile, Gettinby et al. [7] did not clearly detail their estimation procedures. As an undesirable result, some inappropriate parameter estimates were obtained in their paper. Furthermore, they applied the Anderson Darling test table made by Ahmad [1], which is not easy to access.

The rest of the paper is organized as follows. In Section 2, two block-based extreme distributions and their parameter estimation method are described. The Anderson Darling test and its general decision rule using the p-value is briefly explained in Section 3. The weekly minima of the KOSPI 200 and its industry-based business divisions are considered to fit two extreme distributions in Section 4, where their applicability is investigated using the empirical p-value of Anderson Darling test. Some quantile values are also computed and compared in various business divisions. Some concluding remarks are given in Section 5.

2. Block-based Extreme Distributions

Maximum or minimum values in time series can be defined by two approaches ; block and threshold. Examples for the block-based approach are a collection of daily maximum values, a set of weekly minimum values, etc. Threshold-based one includes sets of bigger (smaller) values above (below) some cut-off point. The GEV distribution has been popular for modeling the first type, while the GP distribution has been useful for modeling the second one (Hosking and Wallis [11], Gilli and Kellezi [8]). Later, the GLO was suggested as an alternative to the GEV in case of left fatter tailed processes by Robson and Reed [20]. Since we are interested in modeling the first type, the GEV and GLO distributions are explained in this section. Note that we are considering the weekly minima, where the week works as a block.

Jenkinson [14] introduced the GEV distribution for the maxima. Its cumulative distribution function (cdf) is

$$F(x)=\exp[-\{1-\alpha_3(x-\alpha_1)/\alpha_2\}^{1/\alpha_3}],\ \alpha_3\neq 0,$$

where x is bounded as $\alpha_1+\alpha_2/\alpha_3\leq x<\infty$ if $\alpha_3<0$ and as $-\infty<x\leq\alpha_1+\alpha_2/\alpha_3$ if $\alpha_3>0$. Parameters, α_1, α_2, and α_3 are referred to the location, scale, and shape, respectively. The GEV is a generalized version for three types of limiting distributions of maximum values ; Gumbel, Frechet, and Weibull. Three distributions are characterized by the shape parameter, $\alpha_3=0$, $\alpha_3<0$, and $\alpha_3>0$, respectively. According to Fisher and Tippett [6], asymptotic distributions of maximum values belong to one of three distributions above. Regarding the minimum values, the sign transformation is requested to fit the GEV model (Ferguson, [5], Ch. 14). On the other hand, the inverse distribution function of the GEV distribution is :

$$x(F)=\alpha_1+\alpha_2\{1-(-\log F)^{\alpha_3}\}/\alpha_3,\ \alpha_3\neq 0 \qquad (1)$$

Using this function, the qunatiles are computed. Note that the q-quantile is $x(q)$, where $P(X\leq x(q))=q$ and $0<q<1$.

The cdf of GLO is as follows :

$$F(x)=1/[1+\left\{1-\frac{\alpha_3}{\alpha_2}(x-\alpha_1)\right\}^{1/\alpha_3}],\ \alpha_3\neq 0$$

Again, x is bounded as $\alpha_1+\alpha_2/\alpha_3 \le x < \infty$ if $\alpha_3 < 0$ and as $-\infty < x \le \alpha_1+\alpha_2/\alpha_3$ if $\alpha_3 > 0$. Under $\alpha_3 = 0$, this distribution reduces to a logistic distribution. Parameters, α_1, α_2, and α_3 represent the location, scale, and shape, respectively. Note that the GLO model above is built on the maximum values. Regarding the minimum values, a negative sign transformation is necessary to fit the GLO model. The quantile of the GLO distribution is obtained from the following inverse distribution function :

$$x(F)=\alpha_1+\alpha_2[(1-\{1-F)/F\}^{\alpha_3}]/\alpha_3,\ \alpha_3 \ne 0 \quad (2)$$

The PWM method was introduced by Greenwood et al. [9] as a parameter estimation method for extreme distributions. The maximum likelihood estimator (MLE) approach is its counterpart, and was studied in Prescott and Walden [19]. Since the MLE is derived based on the large sample theory, its performance in small or moderate sample sizes were not satisfactory. Furthermore, it is reported that the MLE sometimes does not have the local maximum of log likelihood in practice (Hosking et al., [12]). On the other hand, the PWM provided superiority in small sample properties (Landwehr et al. [15] and Hosking et al. [12]). However, the week point of the PWM in GEV and GP parameter estimation is that it is available only when the shape parameter is located between -0.5 and 0.5 (Hosking and Wallis [13] and Hosking et al. [12]). Even though the shape parameter is located within this range, its estimation accuracy is reduced as the shape parameter approaches to the boundary values. In case that the shape parameter exists outside the range, the MLE can be considered as an alternative. Some details for the PWM are explained in the followings.

The rth PWM of a random variable X with cdf $F(X)$ is defined as

$$b_r = E[X\{F(X)\}^r],\ r=0,1,\cdots \quad (3)$$

Hosking et al. [12] estimated (3) by $\hat{b}_r = \frac{1}{n}\sum_{i=1}^{n} p_{i,n}^r x_{(i)}$, where $p_{i,n}$ is a plotting position and $x_{(i)}$ is an ordered sample. A plotting position is usually applied as a graphical distribution-free cdf estimator. According to some literature, its reasonable choice is known to depend on the underlying distribution. One choice is

$$p_{i,n} = \frac{i+\kappa}{n+\nu},\ \nu > \kappa > -1 \quad (4)$$

Hosking et al. [12] and Hosking and Wallis [13] suggested $p_{i,n}=(i-0.35)/n$ for the GEV and GP. Cunnane [2] and Harter [10] also suggested several choices for the plotting position. In particular, Gettinby et al. [7] estimated the GEV and GLO distributions using a plotting position with $\kappa=\{\sqrt{n(n-1)}-(n+1)\}/2$ and $\nu=1+2\kappa$ in (4). For the parallel comparison purpose with the results in Gettinby et al. [7], we applied their version to the Anderson-Darling test application in this paper.

The PWM estimators for various distributions including the GEV and GLO were studied in Hosking [11]. Hosking [11] showed some relationships between the L-moments and the PWM such as $\theta_1=b_0$, $\theta_2=2b_1-b_0$, $\theta_3=6b_2-6b_1+b_0$, where θ_1, θ_2, and θ_3 are first, second, and third L-moments, respectively. The corresponding L-moments estimators $\hat{\theta}_1$, $\hat{\theta}_2$, and $\hat{\theta}_3$ can be obtained by replacing b_r with $\hat{b}_r$. For the GEV distribution, parameters can be estimated from the followings :

$$z = 2/(3+\hat{\theta}_3/\hat{\theta}_2) - \log 2/\log 3$$
$$\hat{\alpha}_3 \approx 7.8590z + 2.9554z^2$$
$$\hat{\alpha}_2 = \hat{\theta}_2\hat{\alpha}_3/(1-2^{-\hat{\alpha}_3})\Gamma(1+\hat{\alpha}_3)$$
$$\hat{\alpha}_1 = \hat{\theta}_1 + \hat{\alpha}_2\{\Gamma(1+\hat{\alpha}_3)-1\}/\hat{\alpha}_3$$

Using the same approach, parameters for the GLO distribution can be sequentially estimated as follows :

$$\hat{\alpha}_3 = -\hat{\theta}_3/\hat{\theta}_2,$$
$$\hat{\alpha}_2 = \hat{\theta}_2/\Gamma(1+\hat{\alpha}_3)\Gamma(1-\hat{\alpha}_3)$$
$$\hat{\alpha}_1 = \hat{\theta}_1 + (\hat{\theta}_2 - \hat{\alpha}_2)/\hat{\alpha}_3$$

Note that the PWM methods here are built on the maximum series since the underlying extreme models are based on maxima. Regarding the minimum series, the sign transformation is requested to apply the methods above.

3. Anderson Darling Test

When a continuous cdf and its parameter(s) are completely specified as F_0, the hypothesis of the goodness of fit test is built as follows :

$$H_0 : F = F_0 \text{ vs. } H_1 : F \neq F_0$$

where F is the cdf for the population. Several empirical distribution function (edf) based tests ; Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling were considered for this goodness of fit test. Among these, the Anderson-Darling has been popular due to its sensitive detection of departures in the tail of F_0. Its stronger power over the other edf based tests was investigated in Stephens [21] and D'Agostino and Stephens [3]. In particular, Stephens [22] applied this test statistic to the extreme value distribution case.

In practice, the Anderson-Darling test statistic is computed as follows :

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\{\log(F_0(x_{(i)})) + \log(1-F_0(x_{(n+1-i)}))\}, \quad i = 1, 2, \cdots, n,$$

where $x_{(i)}$ is the ith order statistic and n is the sample size. If $A^2 \geq A_\alpha^2$, the null hypothesis is rejected at the significance level α, where $\alpha = P(A^2 \geq A_\alpha^2)$. A usually suggested criterion for the evaluation of fit using the p-value (p) is as follows :

- If p > 0.2, then it is an "excellent fit" of the H_0 distribution.
- If 0.1 ≤ p < 0.2, then it is a "good fit" of the H_0 distribution.
- If 0.05 ≤ p < 0.1, then it is a "moderately good fit" of the H_0 distribution.
- If 0.01 ≤ p < 0.05, then it is a "poor fit" of the H_0 distribution.
- If p < 0.01, then it is an "unacceptable fit" of the H_0 distribution.

In this paper, the empirical p-values for the observed Anderson-Darling test statistic are computed using Monte Carlo simulation studies. More specific procedures are explained in the next section.

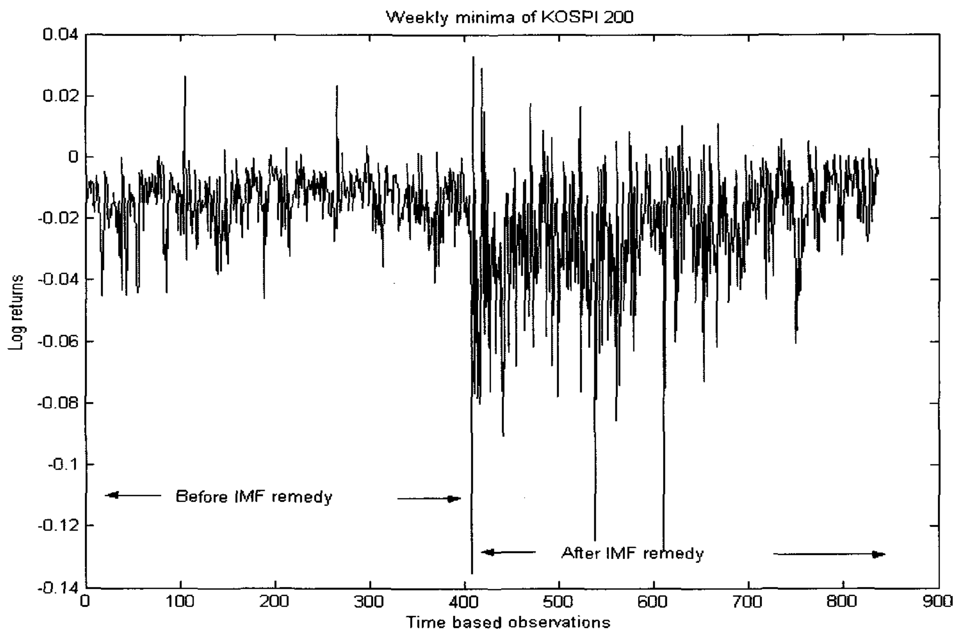
4. Empirical Results

In this section, we apply two extreme distributions to the weekly minima of daily log returns of KOSPI 200 and its five industry-based business divisions such as manufacturing, electronics and communications, construction, service, and banking. The daily log returns are obtained as follows :

$$x_i = \log(p_i/p_{i-1}),\ i=1, 2, \cdots, n,$$

where p_i is a daily released price index. Particularly, the daily ending prices of the underlying series are considered. Such data sets are released from the Korea Exchange. The KOSPI 200 is an index of a portfolio, which is composed of 200 companies. This is categorized into five industry-based business divisions mentioned above. Over the period ranged from January 1990 to December 2005, we obtained the minimum values for each week, i.e., weekly minima. After that, we considered separating the whole period into two sub-periods based on a cut-off point when the Korean government asked for the IMF remedy. From this point ; October 24, 1997, the abrupt increments of variances are documented. Nam and Kim [18] also commented that there have been significant increments in the variance magnitude since that. <Figure 1> verifies such changes in the weekly minima of daily log returns of KOSPI 200. These phenomena lead the whole process to non stationary series. Therefore, modeling the underlying process during the whole period does not seem to be appropriate. To avoid such situation, we basically analyze two sub periods separately. As is to be shown later, the extreme distribution modeling crossing two does not provide better fit than the modeling separating two periods.

First, two sub-periods (before the IMF remedy and after that) are considered separately. Total numbers of observations for the weekly minima before and after the bailout are 408 and 428, respectively. By multiplying a negative uni-



〈Figure 1〉 Weekly minima of daily log returns of KOSPI 200 ranged from January 1990 to December 2005. There have been significant increments in the variance since October 24, 1997 (starting point of the IMF remedy) ty, the minima were transformed into maxima. Two extreme distributions were applied to these sign transformed values. Using the PWM method, parameters of two distributions were estimated. The observed Anderson Darling test statistics were computed from the sign transformed data sets. Using these parameter estimates for each set of data, the Anderson Darling test statistics were generated 5,000 times. From the distribution built with these 5,000 test statistics, the empirical p-value can be computed using the observed test statistic. <Table 1> and <Table 2> provide the GEV and GLO model fits, respectively. In the tables, 'bf' and 'at' imply before and after the IMF remedy, respectively. As mentioned, for the validness of the PWM application, we need to check if the shape parameter is located between -0.5 and 0.5. All the documented shape parameter estimates were very within the range. According to the documented empirical p-values, both models seem to be appropriate in the weekly minima. Using the criterion for the evaluation of fit introduced in Section 3, we can conclude that most of them provide the excellent fits in both GLO and GEV models except for some cases. The weekly minima of electronic industry stock returns provided just good fits in the GLO fits of before the IMF aids and the GEV fits of after the IMF help. The weekly minima of bank industry provided only a moderately good fit in the GEV fits of before the IMF bailout. Overall, both GEV and GLO modeling seem to be applicable in the sense of providing appropriate p-values in the Anderson-Darling test.

These results are against the evidences in Gettinby et al. [7]. According to them, the significantly better fits were recorded in the GLO

〈Table 1〉 The PWM based parameter estimates and observed Anderson Darling test statistics (A2) for the GEV distribution with application to the sign transformed weekly minima of KOSPI 200 and its industry-based business divisions. Here 'Kospi', 'Man', 'Elect', 'Const', 'Serv', and 'Bank' imply the KOSPI 200, manufacturing, electronics and communications, construction, service, and banking divisions. The p-values were empirically computed through Monte Carlo simulation of 5,000 iterations. The whole period is divided into two intervals, before the IMF remedy (from January 1990 to October 1997) and after that (from October 1997 to December 2005). Here 'bf' and 'at' imply before and after, respectively

GEV	Type	location	Scale	shape	A2	p-value
week (bf)	Kospi	0.010	0.008	0.085	1.119	0.296
	Man	0.010	0.008	0.077	1.376	0.209
	Elect	0.013	0.010	0.112	0.900	0.405
	Const	0.013	0.011	0.126	1.029	0.335
	Serv	0.012	0.009	0.110	1.189	0.268
	Bank	0.015	0.009	0.067	2.187	0.073
week (at)	Kospi	0.015	0.017	0.041	0.605	0.643
	Man	0.015	0.017	0.010	0.366	0.882
	Elect	0.017	0.016	-0.056	1.666	0.138
	Const	0.020	0.022	0.023	1.368	0.218
	Serv	0.017	0.018	-0.038	0.541	0.696
	Bank	0.020	0.021	0.054	0.452	0.793

<Table 2> The PWM based parameter estimates and observed Anderson Darling test statistics (A2) for the GLO distribution with application to the sign transformed weekly minima of KOSPI 200 and its industry-based business divisions. Here 'Kospi', 'Man', 'Elect', 'Const', 'Serv', and 'Bank' imply the KOSPI 200, manufacturing, electronics and communications, construction, service, and banking divisions. The p-values were empirically computed through Monte Carlo simulation of 5,000 iterations. The whole period is divided into two intervals, before the IMF remedy (from January 1990 to October 1997) and after that (from October 1997 to December 2005). Here 'bf' and 'at' imply before and after, respectively

GLO	type	location	scale	shape	A2	p-value
week	Kospi	0.013	0.005	-0.117	0.544	0.707
(bf)	Man	0.013	0.005	-0.122	0.605	0.632
	Elect	0.017	0.006	-0.101	0.897	0.109
	Const	0.017	0.007	-0.092	1.018	0.346
	Serv	0.015	0.006	-0.102	0.700	0.555
	Bank	0.018	0.006	-0.128	1.298	0.556
week	Kospi	0.021	0.011	-0.144	1.039	0.343
(at)	Man	0.022	0.011	-0.164	0.994	0.351
	Elect	0.023	0.011	-0.206	1.015	0.355
	Const	0.029	0.014	-0.156	0.743	0.530
	Serv	0.024	0.012	-0.194	0.893	0.405
	Bank	0.028	0.014	-0.136	0.624	0.629

model than in the GEV on the whole. Furthermore, the GEV model was not valid by documenting poor fits and unacceptable fits in many cases. Regarding the different results, we carefully investigated some possible reasons, and found some parameter estimation problems in Gettinby et al. [7]. If we look into the shape parameter estimates for the GEV model in <Table 2> of Gettinby et al. [7], most of them are outside the range between -0.5 and 0.5 or are near the boundary values. Since Gettinby et al. [7] also used the PWM method, the shape parameter estimates should exist within the range in both theory and practice for the valid application. Searching for some possible reasons for these, we found that these results can appear when the raw minimum data are directly used to the GEV distribution. As mentioned, the minima are requested to have the sign transformation to fit the GEV or GLO models. As a pilot experiment, we applied the GEV model to our weekly minima without the sign transformation, and examined the outcomes. Interestingly, similar results from Gettinby et al. [7] were obtained. The shape parameter estimates were near the boundary values or outside the range between -0.5 and 0.5. If their estimation procedures were like above, their conclusion that the GLO model significantly outperforms the GEV model on the considered financial markets appears to be problematic.

Next, we considered modeling the weekly minima during the period crossing the cut-off point, the IMF bailout timing. Since we have significant changes in volatility of the dynamics in the middle of this period, an extreme distribution modeling with some regime switching may be recommended in this particular case.

However, that is beyond the scope of this paper. Some related studies will be considered in other places. In this paper, by simply modeling the dynamics without a regime switching factor for this kind of period, we will just show the reduced goodness of fit. In order to carry out parallel comparison with the previous two sub-periods analysis, we consider the similar number of observations like 400 from December 25, 1993 to August 18, 2001 ; 200 observations before the IMF remedy and another 200 observations after that. Note that the larger sample size can lead to the bigger power in the Anderson-Darling test, which may cause a distortion in the interpretation of the test results. Same approach used in two sub-periods analysis was applied here. Some results are in <Table 3>. According to <Table 3>, the weekly minima of KOSPI 200 returns in both GEV and GLO models still provided excellent fits documenting bigger than 0.2 p-values. The weekly minima of electronic business returns in the GLO model also showed excellent fits. However, the degrees of fitness in these dynamics were significantly reduced from the values in <Table 1> and <Table 2>. Similar results were observed in other stock returns. Most of all, the weekly minima of the banking industry provided poor fits in both GEV and GLO models. These results imply the decrements of goodness of fit under the existence of volatility change factors, and confirm the necessity of some model including regime switching factors.

Concluding this section, we investigated the 0.01 and 0.05 quantiles of the weekly minimum returns for the GEV and GLO models over two

〈Table 3〉 The PWM based parameter estimates and observed Anderson Darling test statistics (A2) for the GEV and GLO distributions with application to the sign transformed weekly minima of KOSPI 200 and its industry-based business divisions. Here 'Kospi', 'Man', 'Elect', 'Const', 'Serv', and 'Bank' imply the KOSPI 200, manufacturing, electronics and communications, construction, service, and banking divisions. The p-values were empirically computed through Monte Carlo simulation of 5,000 iterations. The implemented period is ranged from December 25, 1993 to August 18, 2001.

GEV	type	location	Scale	shape	A2	p-value
week	Kospi	0.013	0.014	-0.065	1.390	0.214
(cross)	Man	0.013	0.014	-0.086	1.706	0.132
	Elect	0.017	0.016	-0.066	1.538	0.165
	Const	0.018	0.019	-0.064	3.618	0.015
	Serv	0.016	0.015	-0.137	2.089	0.075
	Bank	0.019	0.018	-0.054	2.524	0.043
GLO	type	location	Scale	shape	A2	p-value
week	Kospi	0.019	0.009	-0.212	1.310	0.225
(cross)	Man	0.019	0.010	-0.226	1.562	0.159
	Elect	0.023	0.011	-0.213	1.161	0.274
	Const	0.025	0.013	-0.211	2.145	0.076
	Serv	0.022	0.011	-0.261	2.093	0.081
	Bank	0.026	0.012	-0.205	2.625	0.043

sub-periods. Hosking et al. [12] compared the quantile estimates based on the PWM and MLE through simulation studies. According to them, the MLE quantile estimators have smaller variance than the PWM estimators, but provide relatively large biases. Although the MLE is slightly more efficient than the PWM, the magnitude of biases of the MLE is not negligible in some cases. They also pointed out the importance of plotting position selection in the PWM quantile estimation, and suggested applying $p_{i,n}=(i-0.35)/n$. Following them, we obtained the PWM estimates using this plotting position. We applied the PWM estimates to the quantile equation (1) and equation (2). By plugging 0.99 and 0.95 into the equation (1) and equation (2) for the GEV and GLO, we obtained 0.99 and 0.95 quantiles for the sign transformed values. In order to obtain the quantiles for the original minima, we made the sign transformation again, which lead to the corresponding 0.01 and 0.05 quantiles of the weekly minima. <Table 4> summarizes the results. Throughout the studies, the 0.01 quantile values for the GLO model were smaller than those of the GEV model. Meanwhile, such systematic pattern did not appear in the 0.05 quantile values. From the obtained quantile values, we could compare the industry-based business divisions' weekly minima patterns in the tail areas. Overall, the construction industry showed the smallest 0.01 and 0.05 quantile values among the industry-based business divisions after the IMF remedy. This implies that the construction business is likely to provide more extreme weekly minima of returns than other industry-based business divisions after the IMF bailouts. The banking and service industries followed the construction business. The KOSPI 200, a portfolio index of these industry divisions indicated relatively bigger 0.01 and 0.05 quantile values leading to less extreme weekly minima of returns than other individual industries. Finally, to make a complete analysis, we also considered computing the PWM quantile estimators using the plotting position by Gettinby et al. [7], and compared the outputs. In most cases, the observed differences in two PWM quantile estimators were negligible. Due

〈Table 4〉 Estimates of 0.01 and 0.05 quantiles of weekly minimum returns for the GEV and GLO models over some time periods. Here 'Kospi', 'Man', 'Elect', 'Const', 'Serv', and 'Bank' imply the KOSPI 200, manufacturing, electronics and communications, construction, service, and banking divisions. The 'bf' and 'at' imply the before the IMF remedy and after the IMF remedy, respectively.

		Kospi	Man	Elect	Const	Serv	Bank
GEV	week (bf)	-4.16%	-4.17%	-4.89%	-5.14%	-4.54%	-5.34%
(0.01)	week (at)	-8.61%	-9.33%	-10.27%	-11.93%	-11.22%	-10.92%
GEV	week (bf)	-3.14%	-3.12%	-3.77%	-3.97%	-3.49%	-4.04%
(0.05)	week (at)	-6.15%	-6.51%	-6.86%	-8.43%	-7.54%	-7.90%
GLO	week (bf)	-4.48%	-4.47%	-5.24%	-5.55%	-4.86%	-5.74%
(0.01)	week (at)	-9.29%	-9.97%	-10.82%	-12.77%	-11.87%	-11.74%
GLO	week (bf)	-3.11%	-3.08%	-3.72%	-3.94%	-3.44%	-4.02%
(0.05)	week (at)	-6.09%	-6.40%	-6.72%	-8.30%	-7.39%	-7.80%

to the negligible differences in most places, we do not report the details here.

5. Conclusion

In this paper, we have investigated the applicability of two extreme distributions, the GEV and GLO to the weekly minima of the KOSPI 200 and its five industry-based business divisions. Due to the existence of a significant variance change in the middle of concerned period, we carried out the analyses after dividing the period into two sub-periods. Our empirical results indicated that both the GEV and GLO models seem to be comparably applicable to the underlying weekly minima series. These results are against the evidences in Gettinby et al. [7], who claimed the invalidity of GEV models in many cases. Regarding their results, we have pointed out some possible parameter estimation problems.

We also considered extreme distribution modeling over the period with significant volatility changes. Significantly reduced goodness of fits including poor fits in the Anderson-Darling test were documented in this analysis. These outcomes imply that some new models including regime switching factors need to be developed to increase the model fitness accuracy. Such studies will be interesting, and some works will be done in other places.

Furthermore, we investigated the tail behaviors of the weekly minima of returns in various industry-based business divisions using the 0.01 and 0.05 quantile values. Among the five industry-based business divisions, the construction industry provided the most extreme minima of returns in the 0.01 and 0.05 quantile values after the IMF aids. This implies that the construction industry is likely to provide more extreme minima than other industry business sectors after the IMF bailouts. The banking and service industries followed the construction field.

References

[1] Ahmad, M.I., Applications of statistics in flood frequency analysis, PhD Thesis, University of St. Andrews, 1988.

[2] Cunane, C., "Unbiased plotting position- a review," *Journal of Hydrology*, Vol.37(1978), pp.205-222.

[3] D'Agnostino, R.B. and M.A. Stephens, Goodness of fit techniques. Series Statistics : Textbooks and Monographs, Marcel Dekker, New York, Vol.68(1986).

[4] De Vries, G.C., J. Danielsson, Value at risk and extreme returns. Timbergen Institute, Erasmus University Discussion paper, 1997.

[5] Ferguson, T.S., A course in large sample theory, Texts in Statistical Science, Chapman & Hall, New York, 1996.

[6] Fisher, R.A., L.H.C. Tippett, "Limiting forms of the frequency distribution of the largest or smallest member of a sample," *Proceedings of the Cambridge Philosophical Society*, Vol.24(1928), pp.180-190.

[7] Gettinby, G.D., C.D. Sinclair, D.M. Power, and R.A. Brown, "An analysis of the distribution of extremes in indices of share returns in the US, UK and Japan from 1963 to 2000," *International Journal of Financial Economics*, Vol.11(2006), pp.97-113.

[8] Gilli, M. and E. Kellezi, "An application of extreme value theory for measuring finan-

cial risk," *Computational Economics*, Vol.27 (2006), pp.1-23.

[9] Greenwood, J.A., J.M. Landwehr, N.C. Matalas, and J.R. Wallis, "Probability weighted moments : definition and relation to parameters of several distributions expressable in inverse form," *Water Resource Research*, Vol.15(1979), pp.1049-1054.

[10] Harter, H.L., "Another look at plotting positions," *Communications in Statistics : Theory and Methods*, Vol.13(1984), pp.1613-1633.

[11] Hosking, J.R.M., "L-moments : analysis and estimation of distributions using combinations of order statistics," *Journal of the Royal Statistical Society Series B*, Vol.52(1990), pp.105-124.

[12] Hosking, J.R.M., J.R. Wallis, and E.F. Wood, "Estimation of the generalized extreme value distribution by the method of probability-weighted moments," *Technometrics*, Vol.27(1985), pp.251-261.

[13] Hosking, J.R.M. and J.R. Wallis, "Parameter and quantile estimation for the generalized pareto distribution," *Technometrics*, Vol.29(1987), pp.339-349.

[14] Jenkinson, A.F., "Frequency distribution of the annual maximum or minimum values of meteorological elements," *Quarterly Journal of the Royal Meteorological Society*, Vol.81(1955), pp.158-171.

[15] Landwehr, J.M., Matalas, N.C., Wallis, J.R., "Probability weighted moments compared with some traditional techniques in estimating Gumbel parameters and quantiles," *Water Resource Research*, Vol.15(1979), pp.1055-1064.

[16] Login, F.M., "The asymptotic distribution of extreme stock market returns," *Journal of Business*, Vol.69(1996), pp.383-408.

[17] Login, F.M., "From value at risk to stress testing : the extreme value approach," *Journal of Banking and Finance*, Vol.24(2000), 1097-1130.

[18] Nam, J.H. and S.D. Kim, "VAR estimation using the nonparametric approach," *Review of Financial Economics*, Vol.15(2001), pp. 87-113.

[19] Prescott, P. and A.T. Walden, "Maximum-likelihood estimation of the parameters of generalized extreme-value distribution," *Biometrika*, Vol.67(1980), pp.723-724.

[20] Robson, A. and D. Reed, Procedures for flood frequency estimation. Flood estimation handbook, Centre for ecology and hydrology. Wallingford, UK, Vol.3(1999).

[21] Stephens, M.A., "EDF statistics for goodness of fit and some comparisons," *Journal of the American Statistical Association*, Vol.69(1974), pp.730-737.

[22] Stephens, M.A., "Goodness of fit for the extreme value distribution," *Biometrika*, Vol.64(1977), pp.583-588.