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The Development of Subject-matter Knowledge and Pedagogical Content Knowledge in Function Instruction[1)]

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This study investigates preservice teachers' development of subject-matter knowledge and pedagogical content knowledge in teaching function concept. This development takes place in the pedagogical mathematics courses in which the theory of constructivism and cooperative learning theory are aligned.

Pre and post courses test were administered to examine the development and the follow-up interviews were conducted to gain more details. Analysis of the written questionnaire results and interview transcripts reveal that their limited concept image can be extended and developed in depth through pedagogical mathematics courses that apply reformed teaching methods.

INTRODUCTION

In the past decade, there has been increasing attention that prospective secondary mathematics teacher-educations need to prepare future teachers to support reform effort. This reform effort leads to create learning environment for their students that foster the development of students' understanding of mathematical concept (NCTM, 1991).

It has been evidenced that "direct instruction" may not provide an adequate implementation for students' development and for student use of cognitive activities. "Direct instruction"has been examined as the form of instruction with relatively familiar sequence of events: an introductory review, a development portion, a controlled transition to seatwork and a period of individual seatwork (Good & Grows, 1978; Peterson, Swing, Stark and Waas, 1984; Rosenshine, 1976).

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Confrey (1990) suggested that three key assumptions about mathematics instruction underlie direct instruction and are subject to challenge from a constructivist perspective:

1. Relatively short products are expected from students, rather than process-oriented answers to questions; homework assignment and test items are accepted as providing adequate assessment of the success of instruction.

2. Teachers, for the most part, can simply execute their plans and routines, checking frequently to see if the students' responses are within desirable bounds, and only revising instruction when those bounds are exceeded (Peterson & Clark, 1978 ; Snow, 1972).

3. The responsibility for determining if an adequate level of understanding has been reached lies primarily with the teacher.

Doyle, Sanford and Emmer (1983) examined students' views on the "academic work" in traditional classrooms and found that, as students convince teachers to be more direct and to lower the ambiguity and risk in classroom tasks, the teachers may inadvertently mediate against the development of higher cognitive skills. Other challenges to direct instruction come from the research on misconceptions (Confrey, 1987) wherein researchers have documented severe student misconceptions across topics and achievement levels. These misconceptions appear to be resistant to traditional forms of instruction (Clement, 1982; Erlwanger, 1975; Vinner, 1983). These studies point to a need to develop alternative forms of instruction. All of the research shares a commitment to the importance of an active view of the learner-constructive view (Confrey and von Glasersfeld).

Mathematics educators today are concerned with the way mathematics is taught. They call for making a change in the way teachers teach to emphasize teaching for understanding and meaningful learning (Davis, 1986; Educational Technology center, 1988; Lampert, 1988; Lappan and Schram, 1989; NCTM, 1989a; Peterson, 1988; Resnick, 1987; Romberg, 1983; Schoenfeld, 1987). The teacher's role is to help the learner achieve understanding of mathematical content. In order to do so the teachers need to have solid knowledge of the subject-matter knowledge emphasizing for understanding the concept and the ways in which they are organized. Reform efforts (Carnegie Task force, 1986; Holmes Group, 1986; NCTM, 1989b) are designed to improve professional teacher education related to the above context.

Teacher's subject-matter knowledge is represented as knowledge and understanding of facts, concepts and principles and the way in which they are organized, as well as knowledge about the discipline; that is, ways to establish truth (Ball, 1988, 1991; Even, 1990; Kennedy, 1990; Leihardt & Smith, 1985; Shulman, 1986; Tamir, 1987; Wilson et al., 1987).

Another category of subject-matter-specific knowledge of teachers that is also needed to be a well prepared teacher is pedagogical content knowledge. This knowledge is described as knowing the ways of representing and formulating the subject matter that make it comprehensible to others as well as understanding what makes the learning of specific topics easy or difficult (Ball, 1988; Even & Markovits, in press; Lampert, 1986; Shulman, 1986, 1987; Tamir, 1987; Wilson et al., 1987)
There were some researches to support and illustrate the relationships between subject-matter knowledge and pedagogical content knowledge (Ball, 1991 ; Buchmann, 1984 ; Kennedy, 1990 ; Shulman, 1986, 1987 ; Wilson et al., 1987).

Even(1990) and Ball(1988,1990) investigated that prospective secondary teachers' knowledge of functions tends to be weak and fragile, and they did not have a comprehensive and well-articulated knowledge of the mathematics they have to teach. Even(1990) employed a theoretical framework of subject-matter knowledge for teaching mathematical concepts in general and the function concept in particular. It consists of the following seven aspects:

(a) essential features—What is a function?; (b) different representations of functions; (c) alternative ways of approaching functions; (d) the strength of the concept—the inverse function and the composition of functions; (e) basic repertoire — functions of the high-school curriculum; (f) different kinds of knowledge and understanding of the function concept and (g) knowledge about mathematics.

Efforts to improve teaching style as well as special courses must be made in teaching the mathematics teachers have to teach.

One of the goals of the current attempts to reform teaching is to strengthen the subject-matter and pedagogical content preparation of teachers.

This study focuses on how preservice teachers develop their subject-matter knowledge and pedagogical content knowledge through reform-based mathematical courses for teaching function. Reform-based mathematical courses preparing prospective secondary teacher are consistof the psychology of mathematic education and the theory of instruction based on student-centered method.

Function concept play a central and unifying role in mathematics, yet few college teachers know how students come to understand functions. In a typical class, students are given the Bourbaki(ordered pair) definition or some version of the Dirichlet(rule) definition of function.

Function concept is described as a move from a dynamic-dependency notion to a static-set-theoretic one (Freudenthal, 1983), or from an operational notion as a process to a structural notion as an object (Sfard, 1991) and Tall(1991) has called the term, procept, to capture this dual process-object nature of thinking. Freudenthal points out that two essential features of the modern concept of function have evolved arbitrariness and univalence. Even(1993) investigates prospective secondary teachers' understanding of the arbitrary and univalence nature of the functions.

The arbitrary nature of functions refers to both the relationship between the two set on which the function is defined and the sets themselves. This relationship means that functions do not have to exhibit some regularity, be described by any specific expression or particular shaped graph. In this case two sets means that functions do not have to be defined on any specific sets of objects; in particular, the sets do not have to be sets of numbers. The univalence means that for each element in the domain there be only one element (image) in the range.

This study addresses the question, what aspects of teaching influence preservice teachers' subject-matter knowledge and pedagogical content knowledge? And do mathematics education courses and different teaching style develop a special facility for preparing subject-matter knowledge and pedagogical content knowledge?

METHOD

Subjects

The subjects were 23 preservice secondary school mathematics teachers. They were enrolled in a mathematics pedagogical courses at a natural sciences college in Fall 2006; all had recently completed substantial coursework in mathematics, as an undergraduate major and the theory of mathematics education. They had completed their mathematical coursework in a typical way (traditional way). All subjects had taken an advanced calculus course. Four participants of them had studied mathematics as second major (computer sciences is their first major), whereas the researcher has employed various teaching styles for mathematics pedagogical courses. Teaching method has comprised group study, cooperative learning

(Vygotsky's view) and discussion study. The theory of constructivism (von Glasersfeld's view) and posing problem has been applied as a cognitive developing process.

All the participants were seniors and partitioned into six groups. Each group comprises 4 preservice teachers except group 6.

Questionnaires and Procedures

The questionnaire consisted of sixproblems, each of which is shown in Figure 1 and Figure 4, addressing the different aspect of preservice teachers' subject-matter knowledge about function, and responses and analysis on their "Students'" mistaken solution and misunderstanding.

Given the nature of this study, the questionnaires including six items were presented to all subjects in twice pre and post mathematics pedagogical courses to see how they develop their subject-matter knowledge and pedagogical content knowledge about function instruction through mathematics education courses.

The pre-test was administered at the beginning of fall 2006. It took 75 minutes class time to complete the questionnaire. Each subject worked on each item individually and they were asked to give facilities to their "students" when mistaken solutions and misunderstanding were taken place in attempting to solve the same questionnaires.

Prior to the post-test in which six items were given in questionnaires and conducted at the end of Spring 2007, the subjects of this study were asked to participate in teaching practice in which they first experienced as "Learner."

Group study was employed in preparing the teaching practice of function instruction. The way of group study includes that individual's construction of function concept interact with other subjects' construction in a group, and then one of each group member presented their result in front of whole class.

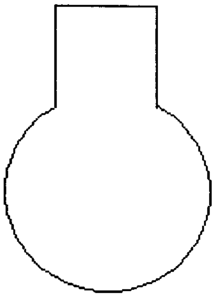
The follow-up interviews were administered right after the test aimed at clarifying their answers and adding more details to the written questionnaire. The interview items consisted of two parts based on the written questionnaire. In the first part, the subjects were presented with items related to the subject-matter knowledge that required more detailed responses than those of the questionnaire (Figure 2). The interview items of the second part were related to pedagogical content knowledge needed to be explained and probed on their answers to the questionnaires (Figure 3).

Guideline for Tests (Even. 1993, Carlson. 1995)

1. Solve the following problems
2. Let your student to solve the problems of same questionnaire. If your students' responses were one of the following cases, give an adequate statements or comprehensible guidance that might help the students' understanding.
   a) No response:
      i. The question was left blank.
      ii. The written information made no attempt to respond to the problem.
      iii. The written information was insufficient to allow any judgment.
   b) An inadequate response:
      i. Shows very limited understanding of the problem.
      ii. Contains words, examples or graph that does not reflect the problem.
      iii. Attempts, but fails to answer.
   c) Not enough response:
      i. Is not totally complete in responding to all aspects of the problem.
      ii. Exhibits a moderate amount of reasoning but reasoning is incomplete.
      iii. Shows some deficiencies in understanding aspects of the problem.

We introduce questionnaire items of pre-test as the following :

1. What is a function?
2. Describe the different ways a function can be represented and explain the meaning.
3. In following questions, give an example to confirm the existence of such a function. If one does not exist, explain why.
   a) Does there exist a function all of whose values are equal to each other?
   b) Does there exist a function whose values for integer numbers are non-integer and whose value for non-integer numbers are integer?
4. Express the diameter of a circle as a function of its area and sketch its graph.
5. Imagine this bottle filling with water. Sketch a graph of the height as a function of the amount of water that's in the bottle.



6. A man sees a ladder against a wall (in an almost vertical position). He pulls the base of the ladder away from the wall by a certain amount and then again by the same amount and then again by the same amount, and so forth. Each time he does this he records the distance by which the top of the ladder drops down.

Draw a graph which represents the relationship between the horizontal and vertical positions of a ladder as it slides down a wall, starting at a vertical position and finally resting on the ground. Explain.

Figure 1. Questionnaire items of pre-test.

We include here one example of interview item (Figure 2) based on the problem 4 in Figure 1 that can provide more detail responses.

1. Constructs graph from your formula.
2. Do you recognize that the range of your response is correct?
   a) If "No," is there any correct answer?
   b) What is your rationale to change your decision?
   c) How would you teach the reason to your student?
   d) If "Yes," why do you think the student give this answer?
3. Do you know the vertical line test for graph of function?
   a) Explain which aspect of function can be characterized by the vertical line test.
   b) If you know the vertical line test of function concept, can even functions expressed via polar coordinates be tested by this way?
   c) Do you find any other reason that your formula is a function?
   d) Can you apply this test to your graph? If "Yes," explain the reason.

Figure 2. Example of interview items

In the second part of the interview, the subjects were asked to reflect on their misconception and common properties that appeared in many of the written answers. One example of interview questions appeared in Figure 3.

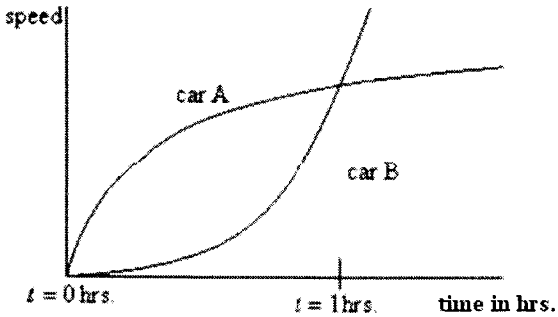
In case of that your student made no attempt or misconceptions to respond about the feature of function concept:

1. What did you think that students' mistaken solutions came from?
2. Pose the alternate questions to know how your students reveal their understanding of the features of function concept satisfying the following requirement:
   a) The requirement of having only one image for each element in the domain of function.
   b) The requirement of arbitrary nature of function concept.
   c) The requirement of "Static" nature of function concept. (The domain is a pointwise set)
   d) The requirement of covariant function property. (A change in the independent variables is seen as causing a change in the dependent variables) In view of this consequence, can constant function be considered as a function?
   e) The requirement of "Dynamic" nature of function concept. (The independent variables change continuously, as the result of this views the rate of change can be considered)
   f) The requirement of the relation between the function formula and everyday experience, other mathematics and physics.
3. State example or counter example of function concept in a view of 2, so that students can recognize their misconception by themselves.
4. State the alternative approach in defining function definition.

Figure 3. Interview questions

1. What is a function?
2. How are functions and equations related to each other?
3. If possible, describe the following situations using a function. If not, explain why.
   a) $y^4 = x^2$
   b) $\det(A) = a_{11}a_{22} - a_{12}a_{21}$ for any matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $a_{ij} \in \mathbb{R}\ (i, j = 1, 2)$
4. Find the equation of the line(s) through the point (a, a^2) that intersects the graph of $y = x^2$ exactly once. And explain your solution.

5. The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are traveling in the same direction.)



a) State the relationship between the position of car A and car B at $t = 1$ hr.: Explain.

b) State the relationship between the speed of car A and car B at $t = 1$ hr.: Explain.

6. Suppose this is the graph of height as a function of volume when a bottle is filling with water. . Sketch the shape of bottle.

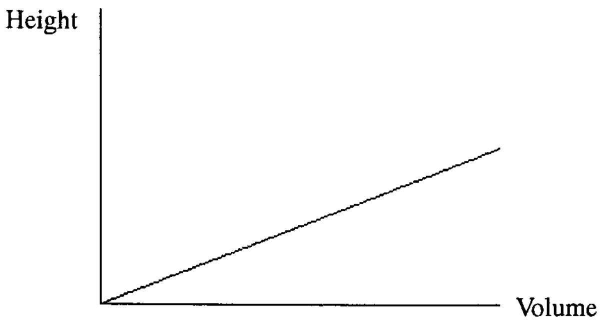


Figure 4. Questionnaire items of post-test

Data Analysis

The analysis of data needs to the creation of framework of responses for each question. The role of this framework was considered important according to what was known about students' difficulties and limited conceptions of function were added.

As the result, each of the question and sub questions of the questionnaire had a list of modified categories related both to the answers and the procedures used to obtain those answers and to themes that emerged from the different aspects of subject-matter knowledge for teaching (Even, 1990).

This study employs a theoretical framework of subject-matter knowledge for teaching function concept. This framework is based on that of Even (1990). It consists of the following aspects: (a) arbitrariness (b) univalence (c) static view (d) dynamic view (e) covariant view and (f) function concept in different context.

In many cases, an answer was categorized several times and several responses can be analyzed for a one aspect. The framework for pedagogical content knowledge was categorized as follows: (a) visualization (b) example-based or counterexample - based statement (c) alternate statement and (d) posing problem. Posing problem mean that students learn the function concept by posing problem based on their cognitive experiences and enable them to recognize their misconception by themselves.

The interview transcripts were analyzed and each subject's answer to an interview question was summarized and significant comments were recorded.

RESULTS

The results consist of two parts, the first part is the result of pre-courses test and the second part is the result of post-courses test.

In the aspect of arbitrariness, the subjects' definition of a function was categorized as "Modern" if there seemed to be some reference to the arbitrary nature of function. Subjects' definitions were categorized as "Old" if some regularity of the function behavior was included. (Even, 1993)

The subject's responses from the all items (Figure 1 and Figure 4) revealed that the difficulties of understanding the concept of function and subjects' activities that might help a student with difficulties. Table 1 includes the features of function definition that were given by subjects, both with and without reference to a student.

The method of presenting data of table 1 is due to the method of Even (1993.)

Table 1 Distribution of function definition and function definition for students.

Concept	Modern		Old		Static		Dynamic		Covariant		DC		N/R	
	PRE	POST	PRE	POST	PRE	POST	PRE	POST	PRE	POST	PRE	POST	PRE	POST
Function definition	4	21	17	2	18	3	3	20	3	12	7	15	2	0
Function definition for students	0	17	17	16	18	12	3	16	0	6	7	12	2	0

※ Note. N/R = No Response, DC = Function Concept in Different Context.

1. The result of pre-courses test

Table 1 shows that the subjects had more "old" and "static" views than "modern" and "dynamic" views in understanding the function definition. They tended to emphasize these views (Old and Static view) in helping their students in pre-test. The result of Item 1 and 2 (Figure 1) revealed that functions are seen as rules with regularities; this is the "function as formula" idea, and functions are identified with just one representation - either the symbolic, or the graphical (17 from 23).

The relatively large number of 17 who had "old" view in the "function definition for students" category is the result of subjects who emphasize function definition as equation, algebraic expression or formula.

Table 1 and interviews indicated that many subjects (18 from 23) who had static view of function concept gave their students a function definition with variety of examples. Some of them defined a function as a corresponding from the pointwise set to some set, input and output processing and a mapping from the set of integers. For example one subject said;

> "Here, I have 12 bottles of yellow color water. When I put 12 different colors, green, red, and etc. in each bottle, I get yellow green, scarlet and etc. 12 different colors. I think that it is a good example of function."

As for the univalence nature of function definition, most subjects wrote easily "A mapping from the set X to a set Y is a function if the image is unique for all element x in X." But the interview based on Item 4 in the questionnaire (Figure 1), show that they had many difficulties in helping their students' understanding of unvalence aspect.

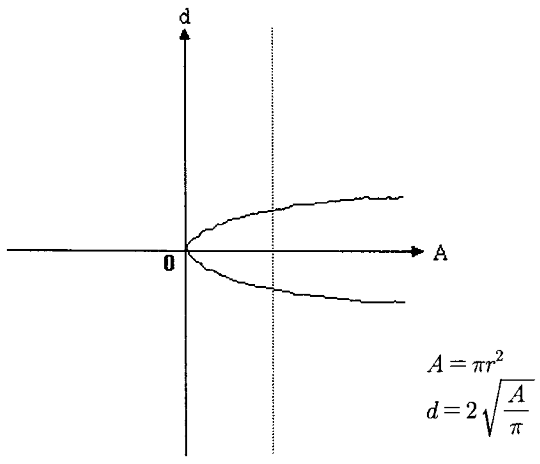


Figure 5

For example,

Interviewer: Do you think that your algebraic representation obtained from item 4 is a function?

Interviewee: Yes, I think so.

Interviewer: Give me a graph of your algebraic expression.

Interviewee: I find that there are two images for each elements in the domain of the graph.

Interviewer: Can you modify this algebraic expression to be a function?

Interviewee: I can do that on the graph by "vertical line test." Since the graph of the algebraic expression and the vertical line intersected at two points, I will take the positive part of the range.

Interviewer: Do you have any other way of approaching to help your students?

Interviewee: I can only show it by the "vertical line test" by visualization.

Interviewee was not capable of helping their student by algebraic approach.

Table 2 summarizes the result of follow-up interview about the pedagogical content knowledge from the item 4 of the questionnaire (Figure 1).

Table 2 Distribution of responses of subjects from Item 4

	Correct Equation	Incorrect Equation
Algebraic Approach for Students	3	0
Vertical Line Test for Students	13	7

Even though small members of subjects (4 of 23) whose definition of a function (Item 1, 2 and 3, Figure 1) were categorized as the "modern" did not use the modern nature of function in teaching situation.

On Item 6 (Figure 1), relatively small number of subjects (3 from 23) who had a dynamic aspect of function concept used their function concept for their students.

The aspects of the "dynamic" and the "covariant" had not been appeared in their concept image of function and in their teaching (Figure 1),and all those who had static view and the view of function concept in different context (other mathematics, physics and real world context) used various example-based function definitions in teaching situation. (Item 3, 5 and 6. Figure 1)

The interview results of Item 5(Figure 1) revealed that two subjects demonstrated no ability to attend to the covariant aspect of situation but they recognize the dynamic nature of this relation of two variables — height vs. volume—in the middle of the interviews. It seems that no subjects' concept image with ambiguity provide their students with accurate explanations.

There is only one subject who says "I had come to discover that I couldn't understand the rate of change if I did not understand the dynamic and the covariant aspect of function."

Some of them did not even know what the pedagogical content knowledge is. In fact, in response to problem situations of their pre-test, most students did not use all aspect of function definition, rather they used their own incomplete concept images — "the total cognitive structure that is associated with the concept including all the mental pictures and associated properties and process." (Tall and Vinner, 1981).

2. The result of post-course test

The written questionnaire (Figure 4) was designed to gain subjects' aspects of subject-matter knowledge and pedagogical content knowledge in function instruction. The results of the pre-courses test was the product of traditional learning method of mathematic courses in which subjects did not have sufficient opportunity to develop their cognitive activities.

The post-courses test was administered for same subjects after that they have learned various learning theory in pedagogical mathematics courses.

Prior to solve the questions (Figure 4), all subjects had experienced to learn function concept in comprehensive and meaningful way under the group study in which cooperative learning (Vygotsky's view) and the theory of constructivism (Von Glasersfeld, 1990) are aligned, i.e., knowledge is derived from experience not passively received but rather actively built up. Therefore, subjects had more opportunity for cognitive activities.

Table 1 indicates that the relatively large number of subjects changed their conceptual view of function from "old" to "modern" (17 subjects). This phenomenon is due to subjects who solve the problem of Item 3.b) (Figure 4). They have come to have an idea viewing determinant as a function whose domain is $M_2(R)$ (the set of 2x2 matrices with real entries) and the range is R.

One of them used a "posing question" method to help their student, she said "Do you think that a mapping from each R-R function to its image is a function? If yes, explain why." This showed "posing question" is the dominant tool in developing and extending their concept image of function.

About half of the subjects claim that the univalence requirement of function definition is a criterion for checking whether given mathematical forms were functions or not.

After struggling with Item 3.a) (Figure 4), some of subject (12 of 23) mentioned that this formula is not a function with justification of interpreting graphical information. They concluded that given algebraic expression can not be explained to have the univalence nature using the vertical line test.

Whereas, four subjects who tried to solve the question with algebraic manipulation did not give a correct solution.

Among them, as the interview continued, two subjects demonstrated the inability, in translating from complicated algebraic to graphic representations, somewhat inconsistent with the way in which they used aspects of function concept. It suggests that understanding and assimilating the aspects of function requires a great deal of "Sense making" on the part of the student. And they have little confidence in their abilities to solve unfamiliar problems.

Static and dynamic aspects

The following Item was borrowed from Monk (1992)

Item 5. (Figure 4.) The given graph represents speed vs. time for two cars.

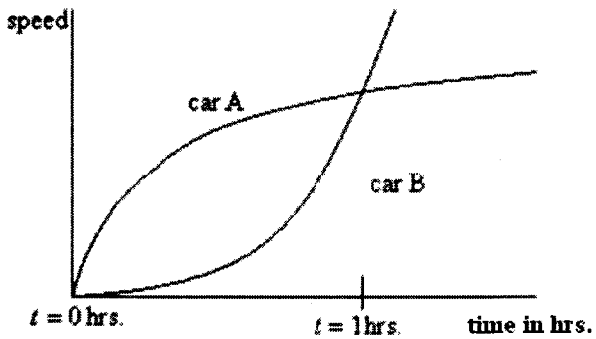


Figure 6.

State the relationship between the position of car A and car B at t = 1 hr. (assumes the two cars start from the same position and are traveling in the same direction)

On item 5 a) and 5 b) (Figure 4), most of the entire subject (17 from 20) extended their view of function concept from static to dynamic in their responses of written questionnaire. Twenty subjects interpreted the graphs as functional relation whose domain consisted of time changed continuously. However, only nine students provided the correct response with justifying their answer by comparing the relative areas under the curves. Even though eleven subjects had read the dynamic relation from the graph, they interpreted functions' image as the paths of the cars, making no connection with solutions.

Two interview subjects state that their interpretation includes static and dynamic view at the same time considering static view as t=0, 1, 2...

Four subjects, who interpret this graph as dynamic relation of function, indicated that two cars are colliding at t=1 hr, since two curves intersects.

Some of the subjects, who had a dynamic view of function claimed that dynamic view of function was always defined as a function of time.

Covariant aspect

From the results of Item 4 and Item 6 (figure 4) in the written questionnaire, twelve subjects showed the line of equation passing through (a, a^2) as the tangent line, but 3 subjects indicated the vertical line passing through (a, a^2) on Item4. This show that the subjects who take the tangent line in approaching Item 4 had the view of rate of change (covariant aspect), but the others who concluded the vertical line as only one line passing through (a, a^2) did not have covariant view of function.

In Item 6 (Figure 4), given the graph of height as a function of volume when a bottle is filling with water, the subjects were asked to sketch the shape of bottle. Prior to this test, they are already used to sketch the graph of height as a function of volume conversely. Hence they are confronted with the inverse case which is not familiar.

One of three subjects, who did not used covariant term in Item 4, illustrated the development of covariant aspect in her response of interview of Item 6 (figure 4):

"I imagine the situation — filling a bottle with water. When I fill a bottle with water, the volume increase and the height of bottle also changes related to change of the volume. Since the graph of function showed that the rate of change is constant, I think that the bottle is a cylindrical shaped form."

Most subjects did not provide teaching strategy for students about covariant aspects in the written questionnaire. But during the interview, they revealed that "the thinking of inverse case" (Item 6, Figure 4) was very useful to help student in understanding the covariant aspect.

Aspect of Function Concept in Different Context

All the interview subject indicated that it take long time to capture the aspect of function concept in different context — other field of mathematics, physics and "real world." They had a belief that they have more facilities to help students in teaching situation when they have broad and deep aspects of function concept in different context.

Pedagogical Content Knowledge

On responses to Item 5.a) (Figure 4), only seven subjects of the 23 used the area under the curves in their solutions and used it as an explanation for students with explaining how the positions of cars can be represented as the areas under the curves. They also revealed in the interview that their students have the difficulty to relate between the area and the distance. The following excerpt illustrates this.

"I'm sure that they knew that the distance is determined by velocity and time. But they did not recognize the relation between the distance and the area under the curves. When they were asked to construct the sum of rectangular that intersect with curve as time pass, they can see right away why the area works. They concluded that the positions of cars are the limit of the sum of these rectangulars."

Furthermore, these subjects insist on that their knowledge of physics and mathematics (aspects of function in other context) made them to analyze their knowledge in teaching situation.

Table 3 summarizes the development of subjects' pedagogical content knowledge by analyzing the result of their responses.

Table 3. Distribution of the aspects of pedagogical content knowledge by the number of using

	Visualization	Example or Counter Example	Alternate Statement	Posing Problem
Pre-test	21	38	2	0
Post-test	53	82	12	11

Table 3 shows that preservice teachers used more pedagogical strategies in post-test comparable with pre-test in each aspect, especially example based aspect. The interview results revealed that successful subject felt free from choosing their pedagogical strategies and especially they tend to satisfy with their ability to posing problem activities.

Subjective-matter knowledge and pedagogical content knowledge appear to have an impact on what subjects know and what they can do in learning process.

Conclusion and Implications

The development of preservice teachers' subject-matter knowledge appears to be facilitated by the instructional method from which their cognitive activities are promoted.

In this study, we have reported that instructional method drawn from the group study can contribute more to the reform of function instruction than "right instruction." The group study in which cooperative learning (Vygotsky's view) and the theory of constructivism (von Glasersfeld, 1990) are aligned invites preservice teachers to explore and to discuss the specific subject-matter knowledge of function concept. These experiences can provide them with concrete ways to engage actively in constructing the concept image of function and in presenting their thinking to whole class. Exploration interacting with peers in a group encourages preservice teachers to continually revise their interpretations, consider alternative perspectives and pursue new questions.

For this reason, this instructional strategy can do more than help preservice teachers to understand the subject-matter knowledge of function concept.

The analysis of our data suggested that better understanding of subject-matter knowledge enables subjects to develop their pedagogical content knowledge.

Analysis of the written questionnaire indicated that the development of abilities in employing the graphs enables them to use easily the "the vertical line test" in determining whether a given expression is a function.

Through their abilities to interpret dynamic and covariant graphical information, the subjects can effectively use "the tangent line" based on the "rate of change."

They showed also that their efforts give students more aid in solving problem related to "rate of change" when they were able to represent graphically covariant and dynamic aspect of real world situations.

Furthermore, the interview result shows that the development of pedagogical content knowledge also depends on the function concept in different context of subject-matter knowledge, especially the more subjects prepare their function concept in different context, the more they can pose questions, not only for problem situation but also for cognitive process. But it appears that the learning of knowledge about the function concept in different context develops slowly, thus it demanded more time to learn.

Therefore, a good subject-matter preparation based on deep and broad understanding would enable preservice teachers to teach in the spirit of the Professional Standards for Teaching Mathematics (NCTM, 1991). Prospective secondary teachers with this preparation would be powerful in helping their students' understanding of the function concept.

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함수개념의 교수 · 학습과정에서 나타난 subject-matter knowledge와 pedagogical content knowledge 능력의 발전에 관한 연구2)

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본 논문은 예비교사들이 함수교육과 관련된 subjective-matter knowledge와 pedagogical content knowledge를 어떻게 효율적으로 의미있게 학습하고 발전시키는 가에 대하여 조사하였다.

함수의 기본개념과 원칙, 그리고 그들이 어떻게 조직되었는지를 이해하는 능력과 의미있는 함수학습이 가능하도록 그들을 표현하고 구성하는 능력을 증진시키기 위하여 본 연구에서는 구성주의와 협동학습에 기반 한 학습방법을 채택하였다.

사전, 사후테스트와 인터뷰를 통하여 평가한 결과 소그룹 구성원들과의 상호작용 결과를 전체 구성원과의 토론을 통하여 학습하는 과정에서 보다 깊이 있고 확장된 subject-matter knowledge와 다양한 pedagogical content knowledge를 획득하게 되는 결과를 얻게 되었다.

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