드래그 감소를 위한 유체의 최적 엑티브 제어 및 최적화 알고리즘의 개발(2)
-개발된 기법의 검증 및 드래그 감소를 위한 유체의 최적 액티브 제어

Optimal Active-Control & Development of Optimization Algorithm for Reduction of Drag in Flow Problems(2)
-Verification of Developed Methodologies and Optimal Active-Control of Flow for Drag Reduction

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요 지

본 연구의 최종 목적은 유체가 빠른 속도로 가해지는 물체의 경계면에서 흡입(suction) 혹은 방출(injection)을 통해 유체를 제어함으로 드래그(drag)를 감소하고자 하는 것이다. 그러나 유체는 대용량, 비선형성을 가지고 있어서 직접적인 해석은 물론, 최적화를 적용한다는 것은 매우 어려운 일이다. 이를 위해 우리는 새로운 알고리즘과 기법들을 개발하였다. 본 연구에서는 이 기법들에 대한 검증을 하고, 나아가 최적화 기법을 사용하여 드래그를 감소하기 위해 흡입량과 방출량을 구하였다. 그리고 이 흡입과 방출을 가할 수 있는 구멍의 수와 위치에 따른 변화를 알아보았다. 본 연구에서 개발된 알고리즘과 기법들을 사용하였을 경우, 기존에는 해결 할 수도 없었던 문제를 가능하게 만들었으며, 기존에 저자가 1차로 개발한 바 있는 방법에 비해서도 더욱 효과적이라는 것을 입증하였다. 그리고 드래그 감소라는 차원에서 본다면 흡입과 방출을 가할 수 있는 구멍의 숫자가 많을수록 효과가 높으나 그다지 많은 수를 필요로 하지 않는다는 것을 알게 되었으며, 구멍의 위치는 유체의 경계층이 분리되는 약간 아래가 가장 최적의 위치라는 것을 알게 되었다.

핵심용어 : 최적제어, Navier-Stokes 유체, 흡입, 방출, 드래그

Abstract

The objective of this work is to reduce drag on a bluff body within a viscous flow by applying suction or injection of fluid along the surface of the body. In addition to minimizing drag, the optimal solution tends to reduce boundary layer separation and flow recirculation.

When discretized by finite elements, the optimal control problem can be posed as a large-scale nonlinearly-constrained optimization problem. The constraints correspond to the discretized form of the Navier-Stokes equations. Unfortunately, solving such large-scale problems directly is essentially intractable. We developed several Sequential Quadratic Programming methods that are tailored to the structure of the control problem. Example problems of laminar flow around an infinite cylinder in two dimensions are solved to demonstrate the methodology. We use these optimal control techniques to study the influence of number of suction/injection holes and location of holes on the resulting optimized flow. We compare the proposed SQP methods against one another, as well as against available methods from the literature, from the point of view of efficiency and robustness. The most efficient of the proposed methods is two orders of magnitude faster than existing methods.

Keywords : *optimal control, Navier-Stokes flow, suction, injection, Drag*

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1. Introduction

This work has two objectives. The first objective of this work is to reduce drag on a bluff body within a viscous flow by applying suction or injection of fluid along the surface of the body. Since for an incompressible external flow, drag is proportional to the rate of dissipation of viscous energy, the problem is formulated as an optimal control problem with dissipation as its objective and boundary velocities as its optimization variables. In addition to minimizing drag, the optimal solution tends to reduce boundary layer separation and flow recirculation.

When discretized by finite elements, the optimal control problem can be posed as a large-scale nonlinearly-constrained optimization problem. The constraints correspond to the discretized form of the Navier-Stokes equations. Unfortunately, solving such large-scale problems directly is essentially intractable.

Here, we developed several Sequential Quadratic Programming(SQP) methods, that are tailored to the structure of the control problem(Bark, 2007).

Therefore, the second objective of this work is to verify the effectiveness and validity of these methods.

Example problems of laminar flow around an infinite cylinder in two dimensions are solved to demonstrate the methodology. We use these optimal control techniques to study the influence of number of suction/injection holes and location of holes on the resulting optimized flow.

As an prototype example, we used a two-dimensional infinite cylinder. However, the technique of flow-control and the algorithm/techniques of optimization introduced in this study can be adopted on any kinds of objects subject to wind or water flow such as highrise buildings, cars, airplanes, ships, and etc.

2. Problem definition

We consider flow around a bluff body immersed in a stream of fluid. At low Reynolds number, the flow divides and reunites smoothly but with increasing Reynolds number the flow separates and recirculates on the downstream side, and the wake behind the body becomes unstable(Figure 1). Our aim in this study is to inhibit this boundary layer separation and flow recirculation numerically by controlling the velocities on the surface of the body, using optimization methods(Figure 2). As an objective function, we will use the rate of dissipation of energy due to viscosity, which we will show is equivalent to the drag force on the body, in the case of an external incompressible flow. An outline of the our approach to flow control is as follows:

1) Initially, the velocities on the surface of the bluff body are assumed to be zero, i.e. there is a no-slip boundary condition on the surface(see Figure 1).
2) Define number of n disjoint holes on the surface of the body. Since flow separation and recirculation occur on the back side of the body, we confine the holes that region. Suction or injection can be applied at each hole by controlling the velocity.
3) Find the optimal velocity vector at each hole, representing the optimal injection or suction, that minimizes the objective function, subject to the flow equations(see Figure 2).

Therefore, our optimization problem to be solved

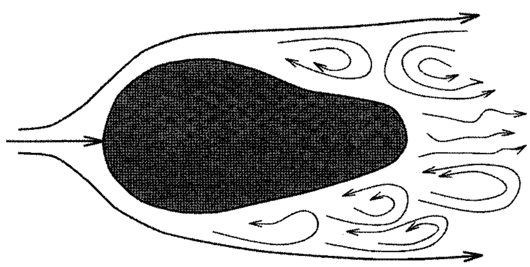


Figure 1 initial flow situation

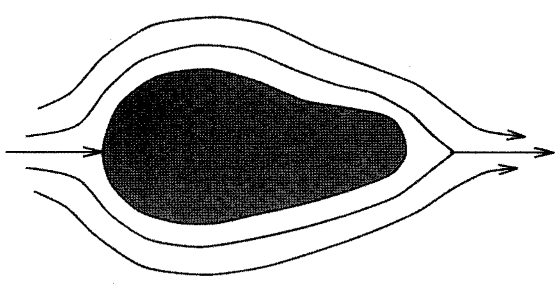


Figure 2 flow situation with suction/injection can be expressed as:

$$\text{minimize} \qquad 2\mu\int_{\Omega}[\mathbf{D}:\mathbf{D}]\,d\Omega \tag{1}$$

subject to

$$-\mu\Delta\mathbf{u}+\rho(\mathbf{u}\cdot\nabla)\mathbf{u}-\frac{1}{\epsilon}\nabla(\nabla\cdot\mathbf{u})=0 \tag{2}$$

where $\mathbf{D}=\mathbf{D}(\mathbf{u})=(\nabla\mathbf{u}+\nabla\mathbf{u}^T)/2$, and the symbol : represents the scalar product of two tensors.

3. Numerical Examples for optimal active control

The algorithm and techniques developed previous paper(Bark, 2007) are tested on a problem of flow around an infinite cylinder in two dimensions. However, our methodology and code can be applied to problems of arbitrary geometry, provided that an appropriate mesh is supplied. We study the influence of the number of holes and the hole location on the optimal flow. These tests include comparisons of the algorithm and techniques developed in quasi-Newton form, against one another as well as the steepest descent method from the literature.

3.1 Flow around an infinite cylinder

Without boundary control, separation for a cylinder is evident already for small Reynolds numbers(<10); the flowfield exhibits two symmetric standing eddies up to around Re=50; and beyond this range, the flow becomes increasingly unstable and vortices begin to be spun off asymmetrically. However, based on the experimental results of Prandtl, we expect that application of suction/injection would be capable of keeping the flow more-or-less attached for Reynolds number as high as at least 500. Thus, the steady Navier-Stokes equations are used to model the flow, with the knowledge that they may not be consistent with sub-optimal solutions at this value of Re, but at the optimum we expect steady flow. In addition, we make use of symmetry about the midplane of the cylinder to reduce problem size.

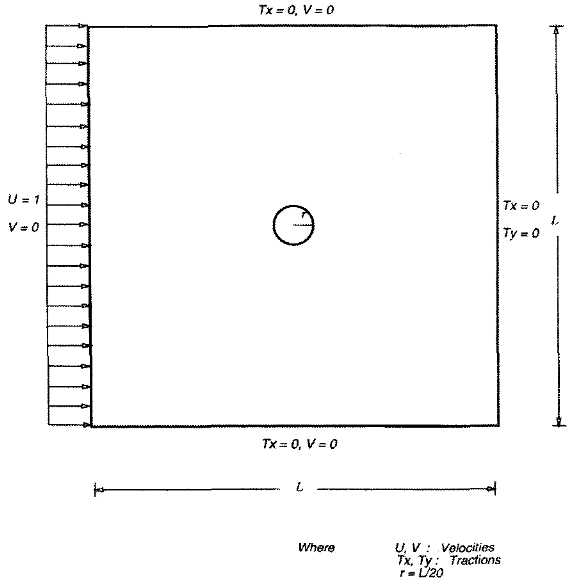


Figure 3 Problem description, flow around cylinder

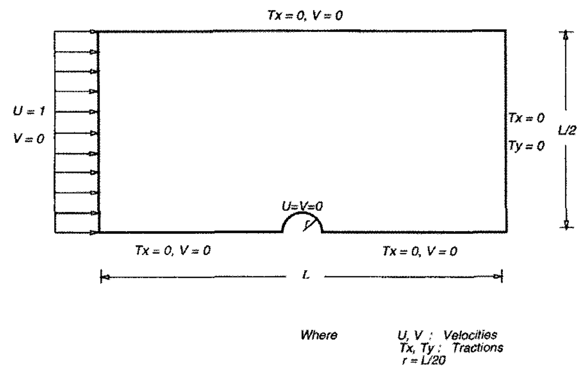


Figure 4 Computational domain

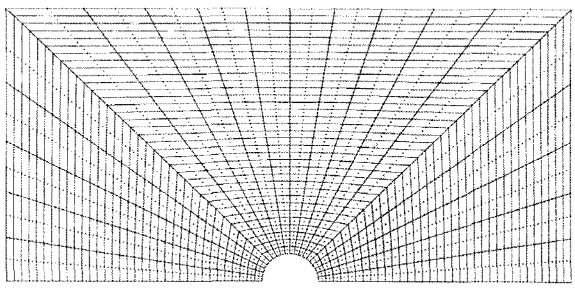


Figure 5 Mesh

A problem description is shown in Figure 3. Using symmetry about the midplane, we obtain the computational domain and boundary conditions shown in Figure 4. The corresponding mesh is given in Figure 5.

It has 2829 nodes and 680 isoparametric biquadratic 9-nodes rectangular elements. The Reynolds number is determined by

$$Re = \frac{\rho U d}{\mu} \tag{3}$$

where d is the diameter of a cylinder. Our target Reynolds number is 500 with step size Re=50.

Using symmetry and solving the steady Navier-Stokes equations, the flowfield shown in Figures 6 is obtained. Figure 6 shows the streamlines around the cylinder without any control, i.e. with the no-slip condition enforced on the surface of the cylinder. The flow separation and recirculation behind the cylinder are evident from the figures.

To demonstrate optimal control, we choose five points on the back side of the cylinder. Fluid is injected into the flow or sucked away from it at these chosen five points with the objective of minimizing the rate of viscous energy dissipation. Each point has two control variables which are the independent components of velocity. Thus this example has a total ten optimization variables.

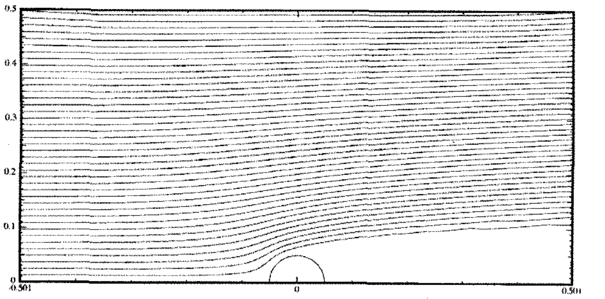


Figure 6 Streamline, without control

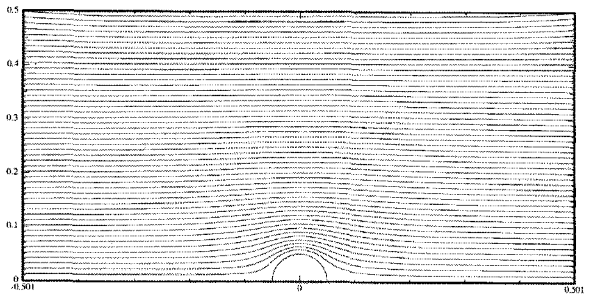


Figure 7 Optimal streamline

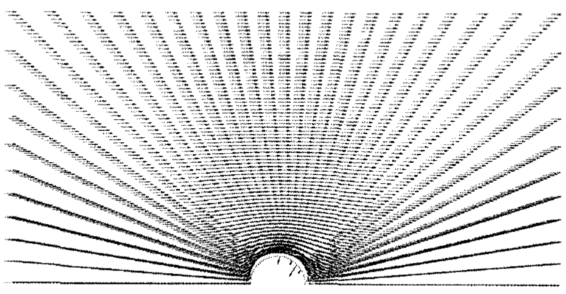


Figure 8 Optimal velocity vector

Figures 7~8 show the flow that results from optimal suction/injection. The streamlines corresponding to the optimal solution are shown in Figure 7. The flow pattern appears to be very similar to that of a potential flow. The velocity field corresponding to the optimum is depicted in Figure 8. The direction and magnitude of suction/injection is apparent from the figure. Suction works to keep the streamlines from separating on the back side of the cylinder, while injection at the base keeps the flow from *stagnating and recirculating.*

Table 1, 2 shows the dependence of the optimal solution. The table includes both initial and final objective function values as well as the optimal values of controls.

Table 1 Objective function

Objective	
No control	Optimal
11.83	4.26

Table 2 Optimal control velocities

Control Variables									
b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_1	b_{10}
.97	.27	-.16	-.20	-.73	-1.09	-.12	-.25	-.15	-.55

3.2 Comparison of Optimization methodologies

We have developed several methodologies for solving optimal flow control problems(Bark, 2007) that incorporates optimization-continuation schemes(*NA-SQP*), sensitivity-based initial guesses(*T1-NA-SQP*), approximate solution of flow equations(*T2-NA-SQP*), and approximate solution of optimality conditions (*T3-NA-SQP*).

In this section we compare these methods with respect to computational effort.

Tables 3 shows the number of *LU* factorizations and CPU time. We solved these problems with step size Re=50. *LU* factorization account for a sub-

Table 3 Number of *LU* factorizations and Total CPU time

	Number of LU factorizations	Total CPU time (minutes)
Steepest descent	failed	failed
OA-SQP	failed	failed
NA-SQP	401	84.7
T1-NA-SQP	318	67.2
T2-NA-SQP	261	55.0
T3-NA-SQP	111	23.4

stantial portion of the total CPU time, since it dominates all other computations.

The steepest descent method converges very slowly or not at all. It takes several days even for low Reynolds number problems. It failed for *Re*=200, 300, 400 and 500 problems. It failed for *Re*=400 and 500 problems even with step size *Re*=25. This is expected, since it does not attempt to use or approximate any curvature information of the control space. Therefore this method is not suitable for optimization of Navier-Stokes flows. Even *OA-SQP*, the straightforward SQP method, is much better than steepest descent method. It reduced by a factor of up to 50 the time consumed by steepest descent method.

By using *NA-SQP*, the continuation technique between optimization problems, the efficiency and robustness were improved further. Simply by using this method, a factor of four improvement was realized over *OA-SQP*. Furthermore, we were able to obtain converged solutions for *Re*=400, 500 which we were not able to do with *OA-SQP*

T1-NA-SQP improved the CPU time over *NA-SQP* by about 30%. *T1-NA-SQP* does not reduce the number of control iterations, but does reduce the number of *LU* factorizations resulting from better initial guesses for the analysis problem within each optimization iteration, obtained from sensitivity information.

Using *T2-NA-SQP*, one may observe an increase in the number of optimization iterations, since we are only approximately solving the optimality condition, though the tolerance reduces the closer we get to the optimum. However, *T2-NA-SQP* does reduce the number of *LU* factorizations, since the effort associated with incomplete convergences of the analysis and optimality conditions results in many fewer *LU* factorizations per optimal control iteration. For the problem studied here, the improvement was about 30%.

T3-NA-SQP uses the truncated idea, i.e. we solve each sequence of optimization problems approximately until we reach the final step. Here it reduced both the number of *LU* factorizations as well as optimization iterations significantly. This method is a factor of more than 2 faster than *T2-NA-SQP*, and a factor of about 15 faster than *OA-SQP*, and a factor of about 200 than the steepest descent method.

Taken together, the incorporation of approximate curvature information, continuation in the optimization problem, better initial guesses on the analysis problem through first-order sensitivity, and approximate solution early on of flow equations and optimality conditions yields a method that is at least two orders of magnitude faster than existing methods on the model problem studied, as well as more robust in the sense of converging for the higher Reynolds numbers.

3.3 Effect of the number of control holes

In this section, we study the effect of number of control holes on the optimal solution. We solved the infinite cylinder problem using one hole, three holes, and five holes. The central point of the five holes is chosen for the one hole problem, the central and two edge points are chosen for the three hole problem. Table 4 shows the optimal values of objective function and control variables. As expected, the objective function decrease with increasing number of holes. Apparently there is not much improvement over the no-control flowfield using central hole, but there is a big improvement using three and five holes. There is not a large difference in the optimal objective function between three hole and five hole problem. However the solution

Table 4 Optimal objective/control variables for three different number of holes

	1 hole	3 holes	5 holes
Objective	10.93	4.37	4.26
b_1		0.9959	0.9711
b_2		0.2641	0.2690
b_3			-0.1564
b_4			-0.1999
b_5	1.0837	-1.0445	-0.7250
b_6	0.9404	-1.3714	-1.0886
b_7			-0.1224
b_8			-0.2529
b_9		-0.1230	-0.1509
b_{10}		-0.7147	-0.5539

for three holes needs more power at each hole than the solution for five holes does. Figures 9~14 show the velocity vectors and streamlines in each case; the improvement between on hole and three holes is quite

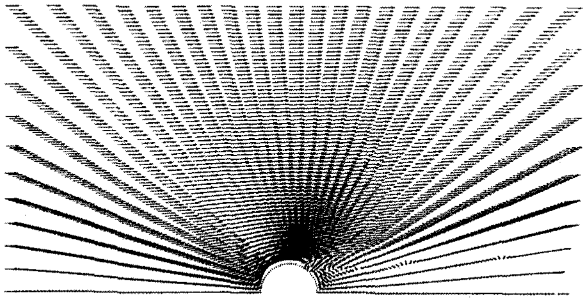


Figure 9 Velocity vectors with 1 control hole

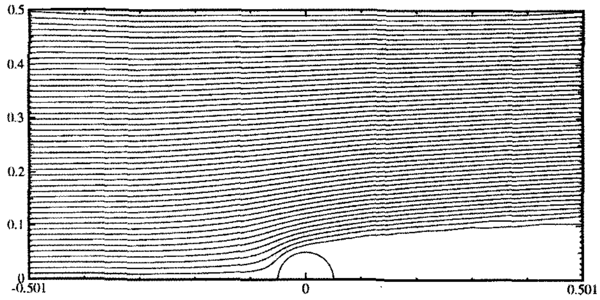


Figure 10 Streamline with 1 control hole

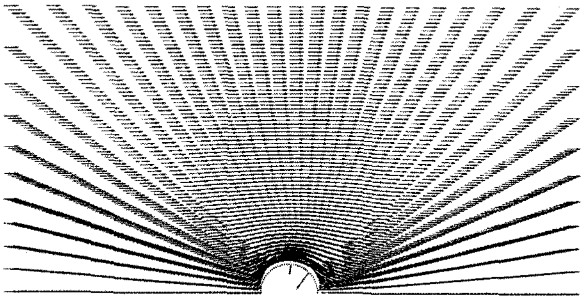


Figure 11 Velocity vectors with 3 control holes

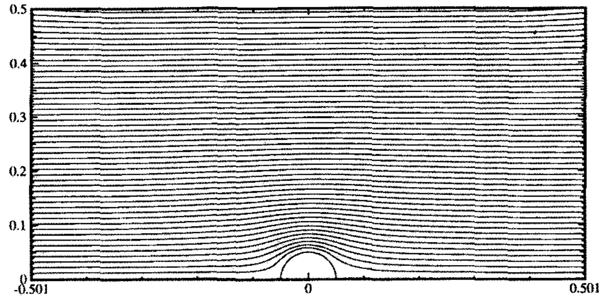


Figure 12 Streamline with 3 control holes

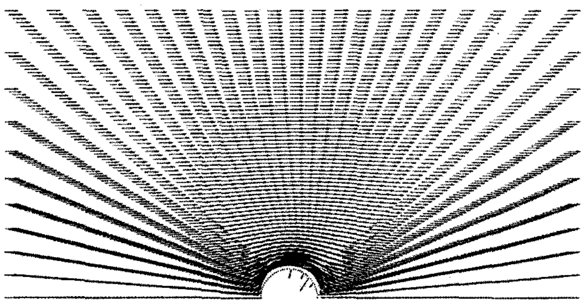


Figure 13 Velocity vectors with 5 control holes

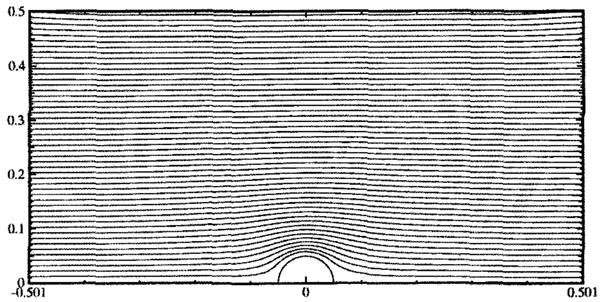


Figure 14 Streamline with 5 control holes

evident. Of course, it must be noted that the objective function does not reflect the cost of each suction/injection hole, nor does it include the cost of the controls: That latter may be included in the objective function easily, and the methodology remains unchanged.

3.4 Effect of hole location

When we use many holes, they may share the suction/injection and allow good distribution of control effort. However with a few holes, the effect of hole location becomes much more pronounced. To determine this, we studied the effect of location a hole at five different points. The location of the individual hole is chosen to be any of the locations of the five control holes of the previous section. Case 1 is the lower

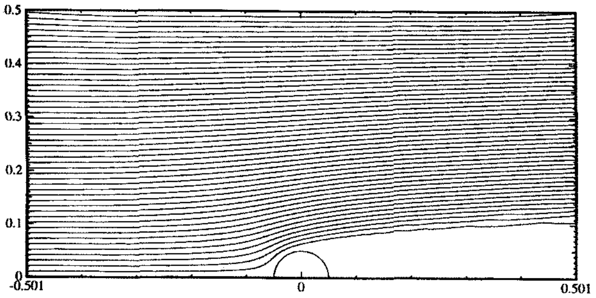


Figure 15 Streamlines-Case 1

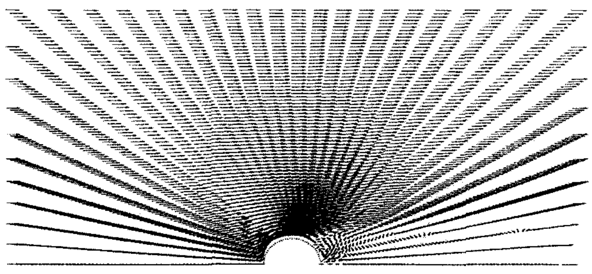


Figure 16 Velocity vector-Case 1

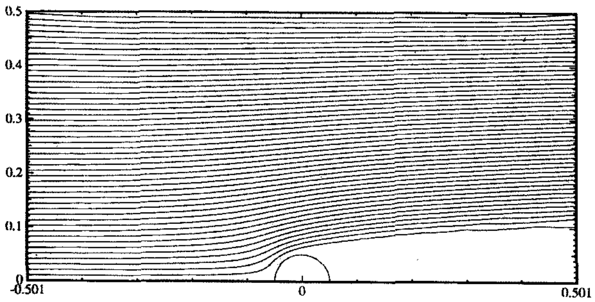


Figure 17 Streamlines-Case 2

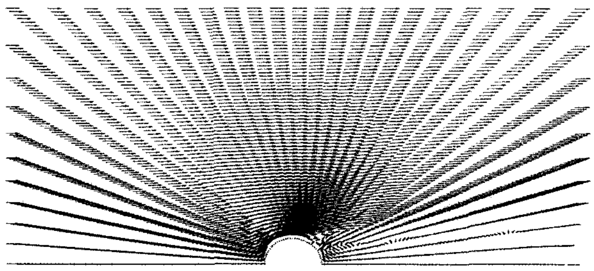


Figure 18 Velocity vector-Case 2

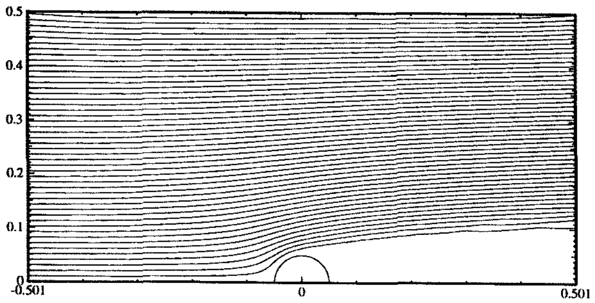


Figure 19 Streamlines-Case 3

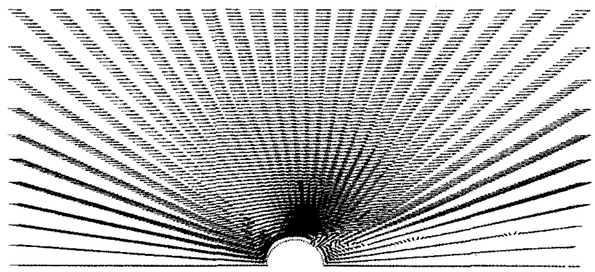


Figure 20 Velocity vector-Case 3

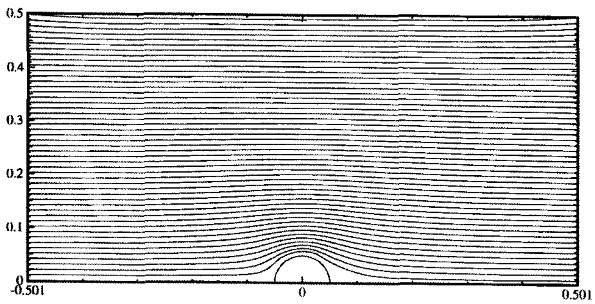


Figure 21 Streamlines-Case 4

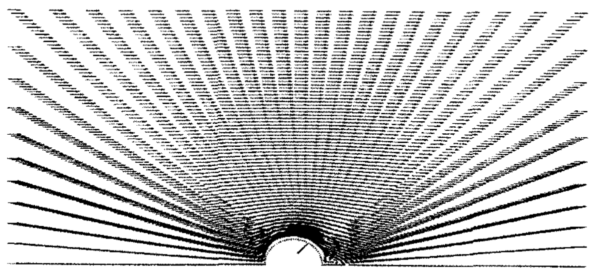


Figure 22 Velocity vector-Case 4

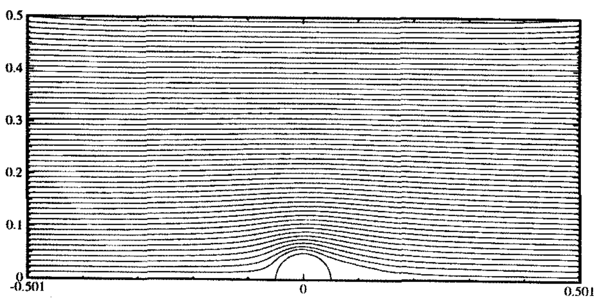


Figure 23 Streamlines-Case 5

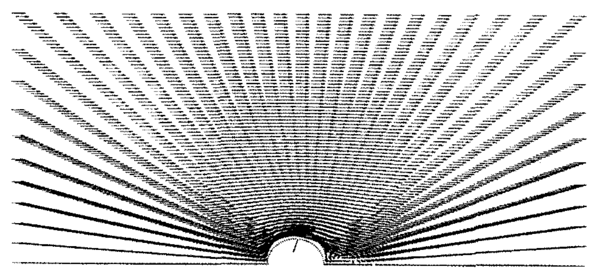


Figure 24 Velocity vector-Case 5

Table 5 Optimal objective/control variables for five different hole location

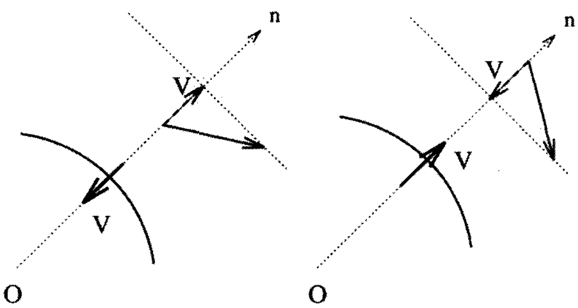
Case	Objective	b_1	b_2
1	11.6607	0.4508	0.3655
2	11.0251	0.6828	0.5817
3	10.9280	1.0837	0.9404
4	5.5621	-0.8950	-0.9349
5	7.2572	-0.3373	-1.1442

hole, and the other four cases correspond to a hole location moving count-clockwise. Table 5 shows the optimal objective and control variables for each case. Figures 15-24 show the velocity vectors and streamlines at each case. One may expect that in all cases suction would be applied. However in Case 1 through Case 3, injection is optimal and the optimum is relatively unchanged from the baseline no-control situation. Suction results in Case 4 and Case 5. At the best location, the optimal objective was 5.5621, which is not far from the optimum obtained by five points(4.2602). Case 5 corresponds to a location where boundary layer separation begins(in the absence of control). Case 4 is the point within the separated boundary layer, at some distance from the separation point. From the table and figures, we can see that the best location of the hole is within the region of separation but slightly downstream of the separation point, for the case of flow around an infinite cylinder

3.5 Comparison with cancellation scheme

In this section, we will compare the optimal control solution with that suggested by the heuristic cancellation scheme described in the previous paper(Bark, 2002). To apply this scheme, we establish sensor locations at several locations in the domain. In the context of the discrete problem, these correspond to different finite element mesh layers. Figure 25 shows the cancellation scheme for this problem.

When ejection/sweep is detected at the sensor locations, an equal amount of opposing normal velocity is applied at the suction/injection hole corresponding to each sensor.



(a) suction (b) injection

Figure 25 Cancellation scheme

Table 6 Objective of cancellation scheme

Y_d(mesh layer)	objective
1st	10.99513
2nd	11.62220
3rd	6.09299
4th	5.28132
5th	5.58749
6th	5.57967
7th	5.60926

We tested this scheme for Reynolds number 500. Without any control, the value of the objective function was 11.82614. The optimal objective value found by any of the methodologies in this study was 4.26017. Table 6 shows the values of objective function determined by the cancellation scheme at several layers corresponding to different sensor locations. The nearest layer from the cylinder is the first layer. Sensor locations near the cylinder(i.e. first and second layer) offer little improvement. By the fourth layer, the improvement is more substantial, but it still 24% from optimal.

The difficulty of deploying velocity sensors within the flow has been mentioned in the previous paper(Bark, 2002). It is best to view the cancellation scheme as a model-based scheme in which simulation is performed to obtain the velocity field. Even then, there is no guarantee of optimality, and the effectiveness depends strongly on sensor location. On the positive side, this technique is easy to implement, as it requires no optimization and only flow solution.

4. Conclusion

The objective of this study is to verify efficient numerical methods that we developed to enable solution of optimal control of fluids, and to apply these techniques to the problem of viscous drag minimization on a bluff body.

The flow is modeled by the incompressible Navier-Stokes equations. In a viscous fluid, boundary layer separation and flow recirculation result in viscous energy dissipation as well as drag. We manipulate an external boundary layer to reduce boundary layer separation and eliminate flow recirculation by controlling velocities on the surface of the body. That is, fluid is injected into the flow or sucked away from it at a small number of points on the surface of the body, with the objective of minimizing drag on the body, or equivalently, the rate of viscous energy dissipation.

Unfortunately, the problem of optimizing Navier-Stokes flows has been considered being out of the reach. However, by using the methods we developed, it can be solved even on a PC.

First, we introduce a continuation technique for both analysis and optimization problems. Second, we can approximate the state variables at each control iteration by using information obtained from sensitivity analysis. This gives us better initial guesses for solution of the Navier-Stokes equations. These two ideas help substantially reduce the number of iterations when a Newton-like method is used to solve the flow equations. Third, we introduce truncated analysis and optimization ideas, leading to a further significant reduction in number of analysis iterations, as well as a reduction in control iterations. Together, the methods proposed in this study lead to at least two orders of magnitude reduction in the cost of solving optimal flow control problems, relative to methods in proposed in the literature.

We verified our methods proposed are very powerful and effective. Those make it possible for flow optimization problem to be solved.

Basically, the methodologies we developed are based on quasi-Newton Method and we verified our methodologies with two-dimensional problem. However the actual problem lies on three-dimension. To solve three-dimensional problem, we need to develop a formulation of true Newton's method for optimization. Formulation for true Newton's method and application to three-dimensional problem will be presented in the next series of papers.

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