Commun. Korean Math. Soc. **22** (2007), No. 1, pp. 111–119

GROWTH NORM ESTIMATES FOR $\bar{\partial}$ ON CONVEX DOMAINS

HONG RAE CHO AND ERN GUN KWON

ABSTRACT. We consider the growth norm of a measurable function f defined by

$$\|f\|_{-\sigma} = \text{ess sup}\{\delta_D(z)^\sigma |f(z)| : z \in D\},$$

where $\delta_D(z)$ denote the distance from z to ∂D. We prove some kind of optimal growth norm estimates for $\bar{\partial}$ on convex domains.

1. Introduction and statement of results

Let D be a bounded domain in $\mathbb{C}^n$ with C^2 boundary. For $z \in D$ let $\delta_D(z)$ denote the distance from z to ∂D. For $\alpha > 0$ we define a measure dV_α on D by $dV_\alpha(z) = \delta_D(z)^{\alpha-1}dV(z)$ where $dV(z)$ is the volume element. For $0 < p, \alpha < \infty$ let $\|f\|_{p,\alpha}$ be the L^p-norm with respect to the measure dV_α and we define $L^{p,\alpha}(D) = \{f : \|f\|_{p,\alpha} < \infty\}$. Let $A^{p,\alpha}(D) = L^{p,\alpha}(D) \cap \mathcal{O}(D)$, where $\mathcal{O}(D)$ is the space of holomorphic functions on D. We will denote the usual Hardy space $H^p(D)$ by $A^{p,0}(D)$, and the associated norm by $\|f\|_{p,0}$. We can identify $A^{p,0}(D)$ in the usual way with a subspace of $L^p(\partial D : d\sigma)$. For $\alpha \geq 0$ and $0 < p < \infty$ we have (see [6])

$$\sup\{\delta_D(z)^{(n+\alpha)/p}|f(z)| : z \in D\} \lesssim \|f\|_{p,\alpha} \quad \text{for} \quad f \in A^{p,\alpha}(D). \tag{1.1}$$

By using the estimate (1.1) we can prove embedding theorems among the weighted Bergman spaces (see [2], [3], [5], [7], and [6]). The estimate (1.1) motivated the author to consider the growth norm for general measurable functions.

Let $0 < \sigma < \infty$. For a measurable function f on D we define the growth norm

$$\|f\|_{-\sigma} = \text{ess sup}\{\delta_D(z)^\sigma |f(z)| : z \in D\}.$$

Let

$$L^{-\sigma}(D) = \{f : f \text{ measurable}, \|f\|_{-\sigma} < \infty\}.$$

Received November 9, 2006.

2000 *Mathematics Subject Classification.* 32W05, 32A26, 32A36.

Key words and phrases. growth norm estimates for $\bar{\partial}$, Lipschitz space, convex domains.

This work was supported for two years by Pusan National University Research Grant.

This work was supported by the Korea Research Foundation Grant founded by Korean Government(MOEHRD)(R05-2004-202-10990-0).

For $\sigma = 0$ we let $L^{-0}(D) = L^\infty(D)$. Then growth spaces $L^{-\sigma}(D)$ are Banach spaces, and we have the inclusion

$$L^{-\sigma}(D) \subset L^{-\sigma'}(D) \quad \text{for} \quad \sigma \le \sigma'.$$

Sobolev type growth spaces $L_k^{-\sigma}(D), k = 0, 1, 2, \ldots$ are defined by

$$L_k^{-\sigma}(D) = \{f \in L^{-\sigma}(D) : D^\alpha f \in L^{-\sigma}(D) \quad \text{for} \quad |\alpha| \le k\}.$$

The corresponding norm is given by

$$\|f\|_{-\sigma,k} = \sum_{|\alpha|\le k} \|D^\alpha f\|_{-\sigma}.$$

We denote by $\Lambda_\alpha(D)$ the Lipschitz space of order $0 < \alpha < 1$ and by $BMO(D)$ the BMO-space on D. By the classical Hardy-Littlewood lemma, we have

$$\begin{aligned} &L_1^{-\sigma}(D) \subset \Lambda_{1-\sigma}(D), \quad 0 < \sigma < 1, \\ &L_1^{-1}(D) \subset BMO(D). \end{aligned} \tag{1.2}$$

Let $L_{(0,1)}^{-\sigma}(D)$ be the Banach space of $(0,1)$-forms whose coefficients belong to the $L^{-\sigma}(D)$ space.

Theorem 1.1. *Let $D = \{\rho < 0\}$ be the bounded convex domain of C^2 class in $\mathbb{C}^2$. There is a bounded linear operator S such that $\bar\partial(Sf) = f$ for all $f \in L_{(0,1)}^{-\sigma}(D) \cap C_{(0,1)}^1(D) \cap L_{(0,1)}^1(D)$ with $\bar\partial f = 0$ and this operator satisfies the following estimates.*

(i) *For $\sigma = 0$,*

$$\|Sf\|_{BMO(D)} \lesssim \|f\|_{L^\infty(D)}.$$

(ii) *For $0 < \sigma < \infty$,*

$$\|Sf\|_{-\sigma} \lesssim \|f\|_{-\sigma}.$$

Remark 1.2. The same estimate (i) in Theorem 1.1 was proved by Range (see [10]). For $1 < p < \infty$, L^p estimates for $\bar\partial$ on convex domains in $\mathbb{C}^2$ was proved by Polking [8]. Moreover, Ahn-Cho [1] proved L^1 estimate and by using the estimate they characterized zero sets of holomorphic functions in the Nevanlinna type class on convex domains in $\mathbb{C}^2$.

2. Construction of the integral solution formula

One of the crucial points to prove Theorem 1.1 is to construct a certain weighted solution formula. We define

$$\tilde\phi(\zeta, z) = \sum_{j=1}^2 \frac{\partial\rho}{\partial\zeta_j}(\zeta)(\zeta_j - z_j) - \rho(\zeta).$$

The following is a well-known consequence of the convexity of D :

$$2\text{Re}\ \tilde\phi(\zeta, z) \ge -\rho(\zeta) - \rho(z) \quad \text{for all} \quad \zeta, z \in \bar D.$$

For any $r > 1$ we can define a kernel

$$K^r(\zeta, z) = c_{0,r} \frac{\rho(\zeta)^r}{|\zeta - z|^4 \tilde{\phi}(\zeta, z)^r} \partial_\zeta |\zeta - z|^2 \wedge \bar{\partial}_\zeta \partial_\zeta |\zeta - z|^2 + c_{1,r} \frac{\rho(\zeta)^{r+1}}{|\zeta - z|^2 \tilde{\phi}(\zeta, z)^{r+1}} \partial_\zeta |\zeta - z|^2 \wedge \partial\bar{\partial} \log \frac{1}{-\rho(\zeta)}$$

which induces a solution operator

$$Sf(z) = \int_{\zeta \in D} f(\zeta) \wedge K^r(\zeta, z), \quad z \in D$$

such that

$$f = \bar{\partial}(Sf),$$

for a continuous $(0,1)$-form f in $\bar{D}$ with $\bar{\partial} f = 0$ (see [4]). For a smooth form f, this formula holds for any $r > 0$. Note that

$$\partial\bar{\partial} \log \frac{1}{-\rho} = \frac{\partial\rho \wedge \bar{\partial}\rho}{\rho^2} - \frac{\partial\bar{\partial}\rho}{\rho}.$$

Thus

$$K^r(\zeta, z) = K_1^r(\zeta, z) + K_2^r(\zeta, z),$$

where

$$|K_1^r(\zeta, z)| \lesssim \frac{1}{|\zeta - z|^3} \frac{|\rho(\zeta)|^r}{|\tilde{\phi}(\zeta, z)|^r}$$

and

$$|K_2^r(\zeta, z)| \lesssim \frac{1}{|\zeta - z|} \frac{|\rho(\zeta)|^{r-1}}{|\tilde{\phi}(\zeta, z)|^{r+1}}.$$

Lemma 2.1 ([10]). *Let $(\zeta_0, z_0) \in \partial D \times \partial D$ such that $\tilde{\phi}(\zeta_0, z_0) = 0$. Then there exist neighborhoods V of ζ_0 and W of z_0 such that for each $z \in W$, there exists a C^1 local coordinate system $\zeta \mapsto t^{(z)}(\zeta) = (t_1, t_2, t_3, t_4)$ on V with the following properties:*

$$t_1(\zeta) = \rho(\zeta), \quad t_2(\zeta) = \operatorname{Im} \tilde{\phi}(\zeta, z), \quad t_3(z) = t_4(z) = 0;$$

$$|t^{(z)}(\zeta) - t^{(z)}(\zeta')| \sim |\zeta - \zeta'|$$

for all $\zeta, \zeta' \in V$ with the constants in (2.3) independent of $z \in W$.

For $j = 1, 2$ we define

$$I_j^r f(z) = \int_{\zeta \in D} f(\zeta) \wedge K_j^r(\zeta, z).$$

Then Theorem 1.1 can be induced by the integral estimates for $I_1^r f$ and $I_2^r f$ in Theorem 4.1 and Theorem 4.2, respectively.

3. Integral estimates for $I_1^r f(z)$

Theorem 3.1. *Let $f \in L^{-\sigma}_{(0,1)}(D)$. Let $r > 0$ be sufficiently large.*

(i) *For $0 < \sigma < 1$,*

$$\|I_1^r f\|_{\Lambda_{1-\sigma}(D)} \lesssim \|f\|_{-\sigma}.$$

(ii) *For $\sigma > 1$,*

$$\|I_1^r f\|_{-(\sigma-1)} \lesssim \|f\|_{-\sigma}.$$

Proof. (i) By (1.2), we prove that

$$\|I_1^r f\|_{-\sigma,1} \lesssim \|f\|_{-\sigma}.$$

Thus it is enough to prove that

$$\int_{\zeta\in D} |\rho(\zeta)|^{-\sigma} |\nabla_z K_1^r(z,\zeta)| dV(\zeta) \lesssim |\rho(z)|^{-\sigma} \quad \text{for all} \quad z \in D.$$

We have

$$|\nabla_z K_1^r(\zeta, z)| \lesssim \left|\frac{\rho(\zeta)}{\tilde{\phi}(\zeta,z)}\right|^{r+1} \frac{1}{|\rho(\zeta)||\zeta - z|^3} + \left|\frac{\rho(\zeta)}{\tilde{\phi}(\zeta,z)}\right|^{r} \frac{1}{|\zeta - z|^4}.$$

Since $|\tilde{\phi}(\zeta,z)| \lesssim |\zeta - z|$, it follows that

$$|\nabla_z K_1^r(\zeta, z)| \lesssim \left|\frac{\rho(\zeta)}{\tilde{\phi}(\zeta,z)}\right|^{r+1} \frac{1}{|\rho(\zeta)||\zeta - z|^3}. \tag{3.1}$$

Note that

$$|\tilde{\phi}(\zeta,z)| \lesssim \operatorname{Re} \tilde{\phi}(\zeta,z) - \rho(\zeta) - \rho(z) \quad \text{for all} \quad \zeta, z \in \bar{D}. \tag{3.2}$$

From (3.1) and (3.2) it follows that

$$\begin{aligned}
A_1(z) &= \int_{\zeta\in D\cap V} \frac{|\rho(\zeta)|^{-\sigma+r}}{|\tilde{\phi}(\zeta,z)|^{r+1}|\zeta - z|^3} dV(\zeta) \\
&\lesssim \int_{\zeta\in D\cap V} \frac{|\rho(\zeta)|^{-\sigma+r}}{(|\operatorname{Im} \tilde{\phi}(\zeta,z)| + |\rho(\zeta)| + |\rho(z)|)^{r+1}|\zeta - z|^3} dV(\zeta) \\
&\lesssim \int_{|(t_1,t_2,t')|<1} \frac{|t_1|^{r-\sigma} dt_1 dt_2 dt'}{(|t_1| + |t_2| + |\rho(z)|)^{r+1}|t|^3} \\
&\lesssim \int_{|(t_1,t_2)|<1} \frac{|t_1|^{r-\sigma} dt_1 dt_2}{(|t_1| + |t_2| + |\rho(z)|)^{r+1}(|t_1| + |t_2|)}.
\end{aligned}$$

If we make the change of variables $t_1 = |\rho|t_1'$ and $t_2 = |\rho|t_2'$, and omit the primes, this becomes

$$\begin{aligned}
A_1(z) &\lesssim |\rho(z)|^{-\sigma} \int_{(t_1,t_2)\in\mathbb{R}^2} \frac{|t_1|^{r-\sigma} dt_1 dt_2}{(|t_1| + |t_2| + 1)^{r+1}(|t_1| + |t_2|)} \\
&\lesssim |\rho(z)|^{-\sigma} \int_0^\infty \frac{t_1^{r-1-\sigma}}{(t_1+1)^r} dt_1 \lesssim |\rho(z)|^{-\sigma}.
\end{aligned}$$

Thus we get

$$|\nabla_z I_1^r f(z)| \lesssim \|f\|_{-\sigma}|\rho(z)|^{-\sigma} \quad \text{for all} \quad z \in D.$$

(ii) We have

$$\begin{aligned}|I_1^r f(z)| &\lesssim \int_{\zeta\in D} |f(\zeta)||K_1^r(\zeta,z)|dV(\zeta)\\ &\lesssim \|f\|_{-\sigma}\int_{\zeta\in D}|\rho(\zeta)|^{-\sigma}|K_1^r(\zeta,z)|dV(\zeta).\end{aligned}$$

From (3.2) it follows that

$$\begin{aligned}B_1(z) &= \int_{\zeta\in D\cap V}|\rho(\zeta)|^{-\sigma}|K_1^r(\zeta,z)|dV(\zeta)\\ &\lesssim \int_{\zeta\in D\cap V}\frac{|\rho(\zeta)|^{r-\sigma}}{|\tilde\phi(\zeta,z)|^r|\zeta-z|^3}dV(\zeta)\\ &\lesssim \int_{\zeta\in D\cap V}\frac{|\rho(\zeta)|^{r-\sigma}}{(|\mathrm{Im}\ \tilde\phi(\zeta,z)|+|\rho(\zeta)|+|\rho(z)|)^r|\zeta-z|^3}dV(\zeta)\\ &\lesssim \int_{|(t_1,t_2,t')|<1}\frac{|t_1|^{r-\sigma}dt_1dt_2dt'}{(|t_1|+|t_2|+|\rho(z)|)^r|t|^3}\\ &\lesssim \int_{|(t_1,t_2)|<1}\frac{|t_1|^{r-\sigma}dt_1dt_2}{(|t_1|+|t_2|+|\rho(z)|)^r(|t_1|+|t_2|)}.\end{aligned}$$

If we make the change of variables $t_1 = |\rho|t_1'$ and $t_2 = |\rho|t_2'$, and omit the primes, this becomes

$$\begin{aligned}B_1(z) &\lesssim |\rho(z)|^{-(\sigma-1)}\int_{(t_1,t_2)\in\mathbb{R}^2}\frac{|t_1|^{r-\sigma}dt_1dt_2}{(|t_1|+|t_2|+1)^r(|t_1|+|t_2|)}\\ &\lesssim |\rho(z)|^{-(\sigma-1)}\int_0^\infty\frac{t_1^{r-1-\sigma}}{(t_1+1)^{r-1}}dt_1 \lesssim |\rho(z)|^{-(\sigma-1)}.\end{aligned}$$

Thus we get the result (ii). □

4. Integral estimates for $I_2^r f(z)$

Theorem 4.1. *Let $f \in L^{-\sigma}_{(0,1)}(D)$. Let $r > 0$ be sufficiently large.*

(i) *For $\sigma = 0$,*

$$\|I_2^r f\|_{BMO(D)} \lesssim \|f\|_{L^\infty(D)}.$$

(ii) *For $0 < \sigma < \infty$,*

$$\|I_2^r f\|_{-\sigma} \lesssim \|f\|_{-\sigma}.$$

Proof. (i) By (1.2), we prove that

$$\|I_2^r f\|_{-1,1} \lesssim \|f\|_{L^\infty(D)}.$$

Thus it is enough to prove

$$|\nabla I_2^r f(z)| \lesssim \|f\|_{L^\infty(D)} \frac{1}{|\rho(z)|} \quad \text{for all} \quad z \in D.$$

We have that

$$\begin{aligned} A_2(z) &= \int_{\zeta \in D \cap V} |\nabla K_2^r(\zeta, z)| dV(\zeta) \\ &\lesssim \int_{\zeta \in D \cap V} \frac{|\rho(\zeta)|^{r-1}}{|\zeta - z|} |\zeta - z|^2 |\tilde{\phi}(\zeta, z)|^{r+1} dV(\zeta) \\ &\lesssim \int_{|w|<R} \int_{-T_2}^{T_2} \int_0^{T_1} \frac{t_1^{r-1} dw dt_1 dt_2}{(|w| + t_1)^2 (t_1 + |t_2| + |\rho(z)|)^{r+1}} \\ &\lesssim \int_{|w|<R} \int_{-T_2}^{T_2} \frac{dt_2 dw}{|w|(|t_2| + |\rho(z)|)^2} \\ &\lesssim \int_{T_2}^{T_2} \frac{dt_2}{(|t_2| + |\rho(z)|)^2}. \end{aligned}$$

If we change of variables $t_2 = |\rho| t_2'$, and omit the prime, this becomes

$$A_2(z) \lesssim \frac{1}{|\rho(z)|} \int_0^\infty \frac{dt_2}{(t_2 + 1)^2} \lesssim \frac{1}{|\rho(z)|}.$$

Thus we get the result.

(ii) Now for the case $\sigma > 0$ we have

$$|I_2^r f(z)| \lesssim \|f\|_{-\sigma} \int_{\zeta \in D} |\rho(\zeta)|^{-\sigma} |K_2^r(\zeta, z)| dV.$$

It follows that

$$\begin{aligned} B_2(z) &= \int_{\zeta \in D \cap V} |\rho(\zeta)|^{-\sigma} |K_2^r(\zeta, z)| dV(\zeta) \\ &\lesssim \int_{\zeta \in D \cap V} |\rho(\zeta)|^{-\sigma} \frac{|\rho(\zeta)|^{r-1}}{|\zeta - z||\tilde{\phi}(\zeta, z)|^{r+1}} dV(\zeta) \\ &\lesssim \int_{|w|<R} \int_{-T_2}^{T_2} \int_0^{T_1} \frac{t_1^{r-1-\sigma} dw dt_1 dt_2}{|w|(t_1 + |t_2| + |\rho(z)|)^{r+1}} \\ &\lesssim \int_{-T_2}^{T_2} \int_0^{T_1} \frac{t_1^{r-1-\sigma} dt_1 dt_2}{(t_1 + |t_2| + |\rho(z)|)^{r+1}} \\ &\lesssim |\rho(z)|^{-\sigma} \int_{-\infty}^{\infty} \int_0^\infty \frac{t_1^{r-1-\sigma} dt_1 dt_2}{t_1 + |t_2| + 1)^{r+1}} \\ &\lesssim |\rho(z)|^{-\sigma} \int_0^\infty \frac{t_1^{r-1-\sigma} dt_1}{(t_1 + 1)^r}. \end{aligned}$$

Note that

$$\int_0^\infty \frac{t_1^{r-1-\sigma}dt_1}{(t_1+1)^r} \lesssim \int_0^1 \frac{dt_1}{(t_1+1)^r} + \int_1^\infty \frac{dt_1}{t_1^{1+\sigma}} \lesssim 1.$$

Thus we get the result. □

5. Sharpness of the estimates

In this section we give an example to show that the estimates in Theorem 1.1 are sharp in some sense.

Let $D = \{(z_1, z_2) \in \mathbb{C}^2; \rho(z) = |z_1|^2 + ce^{-1/|z_2|^2} - 1 < 0\}$ where c is a constant such that the domain D is convex.

- *Sharpness of the case* $\sigma = 0$.

Define $v : D \to \mathbb{C}$ by $v(z) = \bar{z}_2/\log(1-z_1)$, where we use the principal branch $+2\pi i$ for the logarithm. It follows that the (0,1)-form

$$f = \bar{\partial}v = d\bar{z}_2/\log(1-z_1)$$

is $\bar{\partial}$-closed and bounded on D.

We have

$$|\nabla v| \lesssim \frac{|z_2|}{|1-z_1|} + |\log(1-z_1)| \lesssim \frac{1}{|1-z_1|} \lesssim \frac{1}{|\rho(z)|}.$$

By Hardy-Littlewood's lemma, $v \in BMO(D)$ (see [10]).

Proposition 5.1. *Suppose u satisfies $\bar{\partial}u = f$ on D. Then $u \notin \Lambda_\epsilon(D)$ for any $\epsilon > 0$.*

Proof. For any small $d > 0$, we consider the integral

$$I(d) = \int_{|z_2| = 1/\sqrt{|\log(d)|}} [u(1-d, z_2) - u(1-2d, z_2)]dz_2. \tag{5.1}$$

If $u \in \Lambda_\epsilon(D)$ for some $\epsilon > 0$, we see that

$$|I(d)| \lesssim d^\epsilon \cdot \frac{1}{\sqrt{|\log(d)|}} \tag{5.2}$$

by direct estimation. On the other hand, $\bar{\partial}(u - v) = 0$, so $u = v + h$, with $h \in \mathcal{O}(D)$. By Cauchy's theorem we can replace u by v in the integral (5.1). Therefore

$$\begin{aligned} I(d) &= \left[\frac{1}{\log(d)} - \frac{1}{\log(2d)}\right] \int_{|z_2| = 1/\sqrt{|\log(d)|}} \bar{z}_2 dz_2 \\ &= \left[\frac{1}{\log(d)} - \frac{1}{\log(2d)}\right] 2\pi i \cdot \frac{1}{|\log(d)|}. \end{aligned} \tag{5.3}$$

If $\epsilon > 0$, (5.2) and (5.3) lead to a contradiction as $d \to 0$. □

- *Sharpness of the case* $0 < \sigma < \infty$.

Let $\sigma > 0$. Define $v : D \to \mathbb{C}$ by $v(z) = \bar{z}_2/(1-z_1)^\sigma$, where we use the principal branch $+2\pi i$ for the $(1-z_1)^\sigma$. It follows that the (0,1)-form

$$f = \bar{\partial} v = d\bar{z}_2/(1-z_1)^\sigma$$

is $\bar{\partial}$-closed on D. We have

$$|\rho(z)|^\sigma |f(z)| \lesssim (1-|z_1|^2 - ce^{-1/|z_2|^2})^\sigma \frac{1}{|1-z_1|^\sigma} \lesssim 1.$$

Thus we have $f \in L^{-\sigma}_{(0,1)}(D)$.

Proposition 5.2. *Suppose $u \in L^{-\alpha}(D)$ satisfies $\bar{\partial} u = f$ on D. Then $\alpha \geq \sigma$.*

Proof. For any small $d > 0$, we consider the integral

$$J(d) = \int_{|z_2|=1/\sqrt{|\log(d)|}} u(1-d, z_2)dz_2.$$

If $u \in L^{-\alpha}(D)$, then

$$\begin{aligned}|J(d)| &\lesssim \int_{|z_2|=1/\sqrt{|\log(d)|}} |u(1-d,z_2)||dz_2| \\ &\lesssim \int_{|z_2|=1/\sqrt{|\log(d)|}} \frac{1}{|\rho(1-d,z_2)|^\alpha}|dz_2|.\end{aligned}$$

We have

$$\begin{aligned}|\rho(1-d,z_2)| &= 1-(1-d)^2 - ce^{-1/|z_2|^2} \\ &\geq 1-(1-d)^2 \geq d.\end{aligned}$$

Thus we have

$$|J(d)| \lesssim \frac{1}{d^\alpha} \cdot \frac{1}{\sqrt{|\log(d)|}}. \tag{5.4}$$

On the other hand, $\bar{\partial}(u-v) = 0$, so $u = v + h$, with $h \in \mathcal{O}(D)$. By Cauchy's theorem we can replace u by v in the integral (5.1). Therefore

$$J(d) = \frac{1}{d^\sigma} \cdot 2\pi i \frac{1}{|\log(d)|}. \tag{5.5}$$

If $\alpha < \sigma$, (5.4) and (5.5) lead to a contradiction as $d \to 0$. □

References

[1] H. Ahn and H. R. Cho, *Zero sets of holomorphic functions in the Nevanlinna type class on convex domains in $\mathbb{C}^2$*, Japan. J. Math. (N.S.) **28** (2002), no. 2, 245–260.

[2] F. Beatrous, *Estimates for derivatives of holomorphic functions in pseudoconvex domains*, Math. Z. **191** (1986), 91–116.

[3] F. Beatrous and J. Burbea, *Holomorphic Sobolev spaces on the ball*, Dissertationes Math. **256** (1989), 1–57.

[4] B. Berndtsson and M. Andersson, *Henkin-Ramirez formulas with weight factors*, Ann. Inst. Fourier **32** (1982), 91–110.

[5] H. R. Cho, *Estimates on the mean growth of H^p functions in convex domains of finite type*, Proc. Amer. Math. Soc. **131** (2003), no. 8, 2393–2398.

[6] H. R. Cho and E. G. Kwon, *Embedding of Hardy spaces into weighted Bergman spaces in bounded domains with C^2 boundary*, Illinois J. Math. **48** (2004), 747–757.

[7] P. L. Duren, *Theory of H^p spaces*, Academic Press, New York, 1970.

[8] J. C. Polking, *The Cauchy-Riemann equations in convex domains*, Proc. Symp. Pure Math. **52** (1991), 309–322.

[9] M. Range, *Holomorphic functions and integral representations in several complex variables*, Springer Verlag, Berlin, 1986.

[10] ———, *On Hölder and BMO estimates for $\bar\partial$ on convex domains in $\mathbb{C}^2$*, Journal Geom. Anal. **2** (1992), no. 4, 575–584.

Department of Mathematics
Pusan National University
Pusan 609-735, Korea
E-mail address: chohr@pusan.ac.kr

Department of Mathematics Education
Andong National University
Andong 760-749, Korea
E-mail address: egkwon@andong.ac.kr