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Developing Geometry Software for Exploration — Geometry Player

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The purpose of this study is to create an interactive tool *Geometry Player* for geometric explorations. In designing this software, we referred to van Hiele's geometric learning theory of and Duval's cognitive comprehension theory of geometric figures. With *Geometry Player*, it is easy to construct and manipulate dynamic geometric figures. Teachers can easily present the dynamic process of geometric figures in class, and students can use it as a leaning tool to construct geometric concepts by themselves. It is hoped that *Geometry Player* can be a useful assistant for teachers and a nice partner for students. A brief introduction to *Geometry Player* and some application examples are included in this paper.

Keywords: geometric software, Flash Action Script, Geometry Player
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INTRODUCTION TO GEOMETRY PLAYER

As school teachers, difficulties often occur when we integrate technology into the teaching of mathematics. A lack of powerful tools and instructional models are possible

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reasons. We strongly believe that electronic digital materials for teaching and leaning should help teacher's present mathematical concepts more clearly, and that they can be used to provide interactive learning environments so that students have a chance to explore mathematical concepts and improve their conceptual understanding. As we all know, Flash is a powerful tool for creating animations for use over the Web. With Flash we can design innovative and valuable animations in the internet. With its strong functions, Flash Action Script could be used to create highly interactive software. However, Flash is designed for creating interactive web pages and is inconvenient to use as an instructional tool to develop digital materials for geometric explorations. The purpose of this study is to create an interactive tool for geometric explorations — Geometry Player.

Rationale to Develop Geometry Player

van Hiele (1986) proposed a theory for learning geometry and emphasized that manipulations and explorations are important. However, being restricted to certain instructional tools, teachers often have difficulties in presenting the dynamic process of geometric figures in class. Studen ts also lack well-designed tools to help them make constructions, manipulations, experiments, observations, inductions and reasoning. Duval (1995) claimed that there are four kinds of cognitive comprehension about geometric figures. They are perceptual comprehension, sequential comprehension, discursive comprehension and operative comprehension. In Duval's point of view, it is based on object operations and transformations of geometric figures and getting these comprehensions so that people can solve difficult geometry problems. Some scholars (Bishop, 1989; Lean & Clements, 1981) also pointed out that learners need to mentally construct and manipulate the relative images when they are solving a geometry problem. Such mental ability plays an important role in students' leaning and understanding of mathematics.

In solving a geometry problem, an individual first interacts with the external-physical representation by visual perception. Initi ally, geometric figures are often used as an important external-physical representation. It is not until the external-physical representation is mentally transformed into an internal-physical representation that further mental operations on images are possible. How ever, the construction, transformation and connection of these representations is a complex cognitive process. It is especially not easy for students to construct relationships between the representations with static figures. Many students have difficulties in exploring a static drawing on paper or the blackboard. With the development of technology, students can manipulate the external-physical representations dynamically and have a chance to mentally construct the internal-physical representations.

Scher (2005) claimed that today's software should provide users the tools to design their own animations and students must learn how to build constructions. The Geometer's Sketchpad (GSP) is known as a powerful tool for constructing dynamic learning environments for exploring geometry (Albrecht *et al.*, 2001). However, using GSP to construct the expected figures is not easy work without having the concept of ruler-compass construction. For example, think about constructing a triangle which can be modified by dragging a vertex without changing its area. Using GSP to construct this triangle, you need to figure out how the area of a triangle can be fixed. One easy way to do this is to draw a line parallel to the base and pass through the vertex, and let the vertex move on the parallel line. Students need to know the fact that all triangles with the same base and height have the same area, and two parallel lines are always the same distance apart. If students do not understand this concept, the exact figure can not be constructed and they will not have a chance to do further explorations. In Geometry Player, this function button is provided. Stud ents only need to construct a triangle and select the vertex, then press the right mouse button and choose the "area unchanged" function. The "unchanged" function is one of the special features of Geometry Player.

Many geometric explorations involve activities such as figure translation, rotation, dilation, combination and decomposition. Co nducting these kinds of manipulations in a dynamic geometry program is called Operation of Dynamic Simulation (ODS). H owever, if one is not an expert user of GSP, these learning environments are not easily constructed. For example, think about using GSP to construct an exploratory activity "Cutting the three interior angles of an arbitrary triangle and combining them into a flat angle." It will take a lot of time to construct the three angles as three objects to be manipulated. Also the display of combining angles is not easy and is time consuming. To so lve this problem, what we called a "figure absorb" function is created in Geometry Player to help users more easily construct dynamic objects for manipulation. A rotate button is also provided to make object rotation easy. For example, we use the triangle tool to make an arbitrary triangle, and give A, B, and C as the names of the three vertices. A user makes and selects an arbitrary angle, then presses the right mouse button and chooses the "figure absorb" function to designate angle BAC as the absorbed angle. The same process is followed for the other two angles. Then the three absorbed angles are constructed for students to manipulate. The user selects one of the absorbed angles and presses the right mouse button and then selects a rotate function. A rotate sign will appear and the students can use it to easily rotate the angle.

Although GSP has been considered to be a powerful tool for exploring geometric concepts, if students do not have the knowledge to construct figures, it is not very useful for leaning. Bas ically, Geometry Player is a collection of major tools that elementary and junior high school students can use to learn geometry. We designed the basic functions students need in exploring a geometry problem. Stude nts at these levels may not have the ability and concepts to construct the desired figures for exploring. Just like talking to the software, Geometry Player was designed to provide students a useful tool to construct the desired figures.

Exploration geometry problems with Geometry Player

In this section, we will consider two geometry problems. Each problem will be discussed and solved with *Geometry Player.*

Problem 1. For the given isosceles triangles, which one will have the largest area? Figure 1 shows three examples of isosceles triangles. Do they have the same area? If not, which one has the largest area?

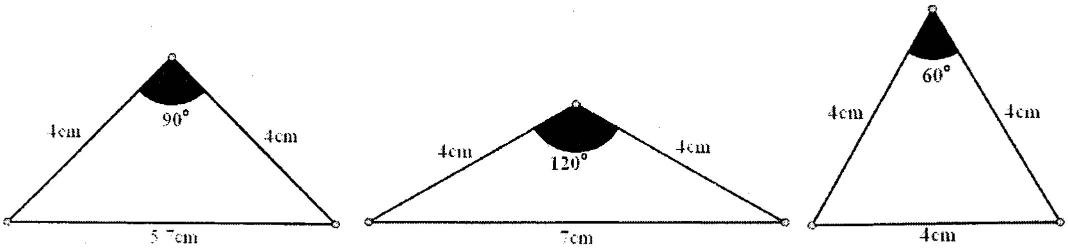

Figure 1. Three examples of isosceles triangles

At first glance, many students guessed that they have the same area because of a misperception. It seems that the triangle with the longest base has the least small height. One easy way to solve the problem is to measure the three areas.

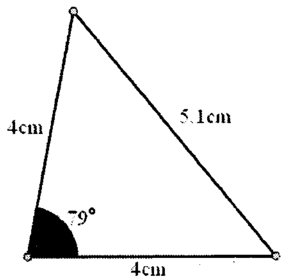

Figure 2. An isosceles triangle created with Geometry Player

We think this is a good beginning for exploring this problem. So far, Ge ometry Player does not have a function to measure the area of a figure. This function may need to be developed in the future, if it is possible. However, sho wing students the areas of the three triangles directly does not help their conceptual understanding. With Geometry Player, students can easily make isosceles triangles (Figure 2).

After students have constructed this triangle, they can drag point A in a way that does not change the length of AB. In manipulating point A, students have a chance to observe that the base BC is the same but the height changes if point A moves. Because the area of a triangle is half the product of a base and its height and to maximize the area of triangle ABC, we must maximize the length of its height from vertex A to base BC. Constructing the height AD from vertex A to base BC and showing the length of AD can help students figure out the answer (Figure 3).

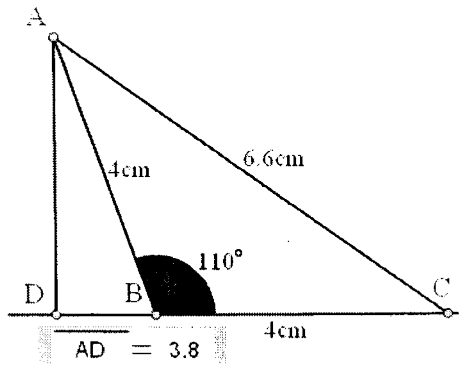

Figure 3. A triangle with a base BC and its corresponding height

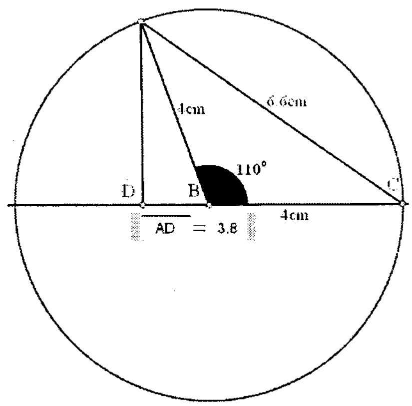

Figure 4. An isosceles triangle with one vertex on center and two vertices on the circle

Another way to construct the geometric figure for exploration is shown in Figure 4. In this case, A needs to be moved in a way that the length of AB is fixed. Since the hypotenuse is the longest side of a right triangle. It is clear that when height AD is coincident with AB, triangle ABC has a maximum area. After this exploration, even though students have not yet learned trigonometry, they can understand that the left triangle in Figure 1 has a maximum area.

Problem 2. A farmer has a rectangular shaped field. It is known that the rectangle has a length of 90 meters and a width of 60 meters. The farmer is thinking of creating two cross roads on the field (Figure 5). What will be the leftover area of the field?

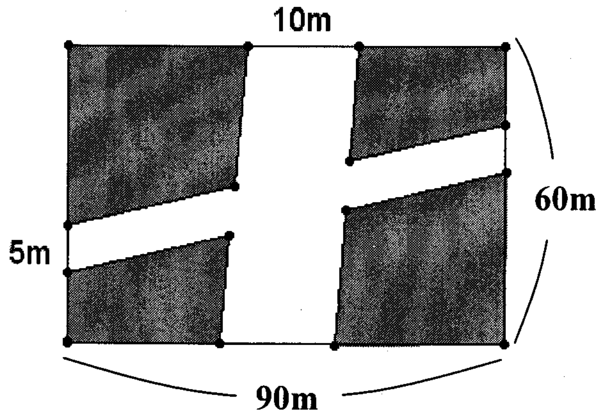

Figure 5. A farmer's rectangular field

Many students gave 4400 m^2 as the answer to this question. The students used one of two possible strategies. One is to put the four quadrilaterals together. It looks like it will be a new rectangle with a length 80m and a width of 55m. So 4400 m^2 (80×55) will be the answer. The other way is to subtract the road area from the whole field. It is easy to find the area of the road by calculating the area of two roads and subtracting the interaction area. The interaction area is a parallelogram, so it can be found as 10×5 m^2. We can then find the road area to be 1000 m^2 (90×5+10×60–10×5).

Subtracting 1000 from 5400 is also 4400, so 4400 is the answer. However, are these reasoning processes correct? Is there any doubt about this answer? Can we put these four quadrilaterals together to form a rectangle? After explorations with Geometry Player, we found that this answer is not correct. This answer is correct only in some specific situations.

Because we need a dynamic environment to manipulate the four quadrilaterals, the "unchanged" and "absorb" functions are used to create the figures. First we create a rectangle ABCD, a parallelogram EFGH and a parallelogram IJKL (Figure 6). Then we use the "area unchanged" to fix the area of two roads so that no matter what the parallelogram looks like, it will still have the same area. Figure 7 and Figure 8 show two examples of this construction.

We also need to create four quadrilaterals as the four fields so that they can be manipulated and combined. To do this, students can create a quadrilateral and let it be absorbed by an existing field. Af ter finishing creating the four fields, they can be moved and manipulated (Figure 9).

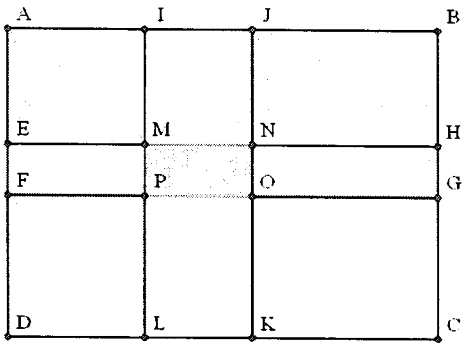

Figure 6.

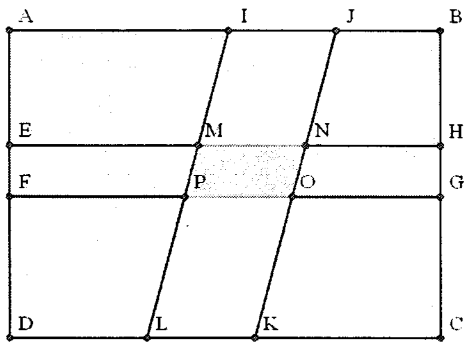

Figure 7.

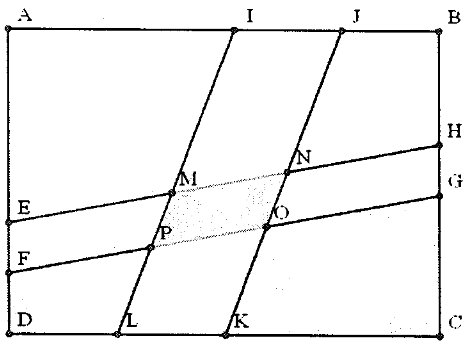

Figure 8.

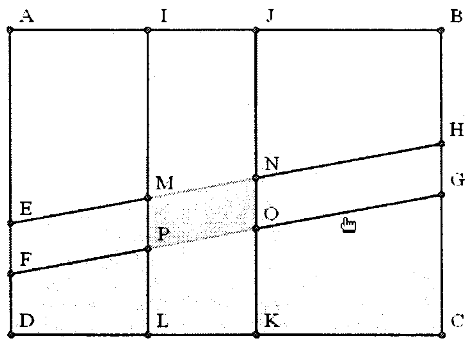

Figure 9.

Now, students can change the shapes of the two roads and manipulate the four fields to explore this problem. The following figures (Figures 10 to 14) show the explorations of this problem.

It is found that in Figures 13 and 14 the four quadrilaterals can not be combined to form a rectangle. Wh en neither of the roads is parallel to any side of rectangle ABCD, there is an overlap or gap of these combinations. On the other hand, the interaction of the two roads is really a parallelogram, but its area can not be calculated as 10 times 5. However, if any road is parallel to one side of rectangle ABCD, this will be true. Have you found this? In solving this problem, we proposed a way to use Geometry Player as a tool to create and manipulate objects for explorations.

A similar exploration used in class is to ask students to draw objects on paper, cut them out with scissors, and manipulate the cutouts. Compared to other interactive software packages, Geometry Player provides young students with an easy way to construct an exploring environment which is similar to their explorations in class. What is more, it is dynamic.

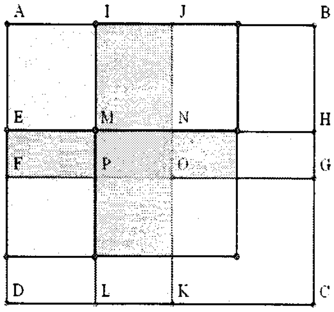

Figure 10.

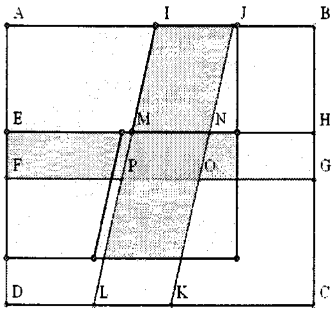

Figure 11.

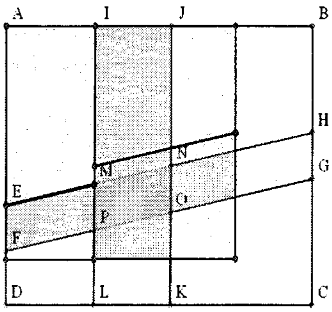

Figure 12.

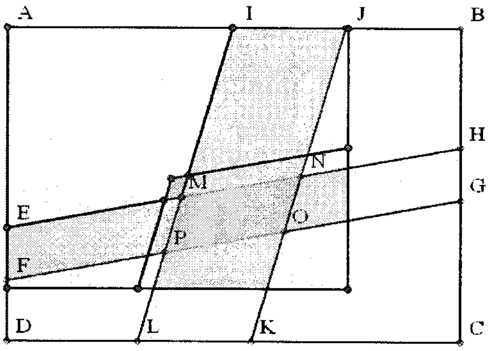

Figure 13.

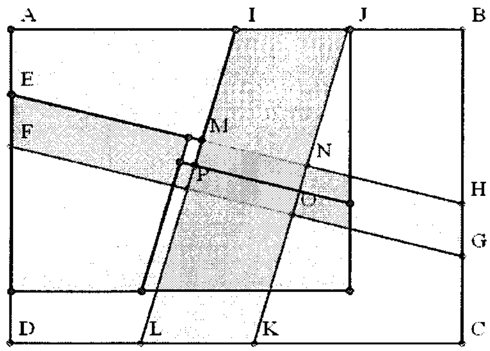

Figure 14.

This problem had been explored in a 9th grade class and Geometry Player was used to support cognitive conflicts. It was found Geometry Player can engage students in exploration more easily and help students prove the conjecture naturally (Lee & Chen, 2007).

CONCLUSION

The theory for learning geometry proposed by van Hiele emphasizes that manipulation and exploration are very important. In recent years computer programs which encourage exploration in geometry have been developed and are beginning to be widely used. GSP is one that provides opportunities for exploring and discovering geometric relationships.

But it is not easy for elementary and junior high school students to create their own geometric constructions by GSP. It is hoped that a powerful tool for these students will be developed. We found that Flash can be used to create highly interactive software and used it to develop a geometry program – Geometry Player. In designing this soft ware, we referred to van Hiele's theory, and Duval's cognitive comprehension theory of geometric figures. W ith Geometry Player it is easy to construct and manipulate dynamic geometric figures.

Teachers can easily present the dynamic process of geometric figures in class. And students can use it by themselves as a leaning tool to construct geometric concepts. It is hoped that Geometry Player can be a good assistant for teachers and a nice partner for students. In the future, similar research (Whang & Cha, 2002) should be conducted to further study the effectiveness of Geometry Player in exploring geometry problems.

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