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Mathematics Teachers' Understanding of Students' Mathematical Comprehension through CGI and DMI

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This paper compares and analyzes mathematics teachers' understanding of students' mathematical comprehension after experiences with the Cognitively Guided Instruction (CGI) or the Development of Mathematical Ideas (DMI) teaching strategies. This report sheds light on current issues confronted by the educational system in the context of mathematics teaching and learning. In particular, the declining rate of mathematical literacy among adolescents is discussed. Moreover, examples of CGI and DMI teaching strategies are presented to focus on the impact of these teaching styles on student-centered instruction, teachers' belief, and students' mathematical achievement, conceptual understanding and word problem solving skills. Hence, with a gradual enhancement of reformed ways of teaching mathematics in schools and the reported increase in student achievement as a result of professional development with new teaching strategies, teacher professional development programs that emphasize teachers' understanding of students' mathematical comprehension is needed rather than the currently dominant traditional pedagogy of direct instruction with a focus on teaching problem solving strategies.

Keywords: Cogniti vely Guided Instruction (CGI), Development of Mathematical Ideas (DMI)
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INTRODUCTION

In every decision an individual makes, comprehension and analysis of information play a vital role on what and how certain actions and attitudes are conceptualized and performed. Whe n the complexity, source and limit of a particular knowledge is exposed from individual perspectives, conflicting points of view follow the beliefs and values a

person has that typically determines the person's behavior given that the person is aware of the possible principles and truths involved.

In any professional discipline, the relevance of development and progress is more worth noting than only the knowledge that governs the field; the application and benefits of the implementation as well as sharing of a particular body of thought are important. In particular, the teaching profession has been identified for its implementation of professional development programs since the future of all other professions is dependent on the efficiency and effectiveness of the people in the academic arena with the responsibility of imparting knowledge. Successful professional development addresses the concerns of teachers in resolving the discrepancies of current teaching programs and the strategies employed in the academic community compared to the ideal educational environment where the achievement of a series of goals and objectives are desired for the improvement of the teaching profession. S uch an outcome not only is an advantage to result-oriented teaching fellows but also to the students from whom much is expected.

Currently, the teaching profession is confronted with developmental issues calling for immediate attention and deliberation of academicians. T raditional techniques of teaching are being challenged in terms of their efficiency and effectiveness for the improvement of both the teachers and the students. The passive process of the one-way teaching-learning approach in schools, as evidenced in the roles of teachers as individuals of authority in the classroom where students are considered as mere performers of school activities, must be examined and integrated with more interactive teaching and learning approaches.

Assessment and evaluation of modern teaching processes, not just in the perspective of faculty but also of the parents and students, helps to clarify the developmental issues that beset the teaching profession, and likewise enumerate the benefits and advantages of implementing current teaching guidelines in the academic community. If the desire is to improve individual learner's achievement of specific educational objectives, validity and reliability measures of new teaching strategies must be carefully examined.

Since teachers are professionals who are considered to have the most knowledge of prevalent teaching and learning conditions in schools, their ability to discern the most appropriate methods of teaching is invaluable in the development and improvement of the goals of the profession. B y the same token, teachers' participation and contribution to studies that are conducted for the sole purpose of investigating the learning environment in schools most likely reflects the actual scenario and reality of teaching. Therefore, teachers' experiences inside the classroom and in the academic community in general must be considered as an insightful account of real-life situations to be considered in a research study.

Attention to instructional problems and concerns of teachers, as well as successful teacher decision-making incidents, could provide information to serve as a constant

evaluation of the teaching and learning interplay in the academe. Conti nuous and regular assessment of the failures and achievements that nest every teacher's career should provide a base of knowledge to contribute significantly to the development and improvement of the profession.

This paper provides a review and analysis of mathematics teachers' understanding of students' mathematical comprehension when using the Cognitively Guided Instruction (CGI)[1] and the Development of Mathematical Idea (DMI) teaching strategies. The report sheds light on current issues confronted by the educational system in the context of mathematics teaching and learning. In particular, the declining rate of mathematical literacy among adolescents is discussed. Examples of CGI and DMI teaching strategies are presented to focus the impact of these teaching styles on student-centered instruction, teachers' beliefs, and students' mathematical achievement on conceptual understanding and word problem solving skills.

TEACHING STRATEGIES

Cognitively-Guided Instruction (CGI) is a problem-solving mathematics program for students in kindergarten through third grade following a non-textbook approach better suited to school children with diverse cultural and social backgrounds (*cf.* Villasenor & Kepner, 1993). This program was originally developed and facilitated in Wisconsin by educational researchers (Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Fennema, Carpenter & Peterson, 1991) and is now a popular teaching strategy throughout the United States. Acc ording to Waxman, Padrón & Knight (1991), CGI highlights learning strategies that enhanced students' "metacognitive" development. This development is achieved by focusing on direct teaching and modeling of cognitive learning strategies to help students enhance their problem solving and reasoning skills through a regimen of practice during which they are engaged in effective methods for monitoring their own learning. This particular teaching program is also an area for professional development with the goal of enabling teachers to predispose their students to making use of their intuitive knowledge in dealing with mathematical problems and learning processes successfully. In doing so, the program embraces each student's unique ways of applying reasoning and problem solving strategies and attempts to integrate their unique approaches with formal mathematical definitions and operations they encounter during lectures and class discussions.

The CGI teaching model is based on two assumptions. Th e first assumption states that

[1] Cogni tively Guided Instruction (CGI). *Promising Practices Network.* Retrie ved October 11, 2005, from http://www.promisingpractices.net

children possess intuitive knowledge about mathematical definitions and operations which they develop during their attendance in school. The CGI model proposes that this growing body of intuitive knowledge in a child's mind needs to be recognized by the educational system as basis for introducing the young learner to formal mathematics instruction in the elementary grade levels. Fu rthermore, the employment of specific cognitive strategies to help establish a link between a student's mathematical intuitions and formal mathematical concepts is determined through a series of assessments. The second assumption claims that teaching mathematics should be grounded on emphasizing the importance of problem solving schemas against the repetitive approach of number facts and desk work (as in worksheets). In this way, problem solving strategies are developed and enhanced through innate schemas among students instead of the traditional familiarization with formal concepts and solution strategies, thus, emphasizing qualitative learning. This particular teaching program for mathematics teachers has been introduced as a cyclical professional development program that is part of on an integrated program of research focusing on the development of students' mathematical thinking, instruction that influences such development, teachers' knowledge and beliefs that influence their instructional practices, and the impact of teachers' knowledge and beliefs that influence their understanding of students' mathematical thinking (Fennema, 1996; Carpenter, Fennema, Franke, Levi & Empson, 2000).

In addition, the program is designed to present a formal way of how students understand mathematical knowledge for the teachers to integrate such understanding and CGI theories into a curriculum that focuses on problem solving. As an alternative teaching approach in mathematics, CGI highlights the advantage of identifying the inherent skills and knowledge of children in dealing with mathematics problems, by building on these abilities that likewise enlighten teachers regarding the level of difficulty confronting students with specific topics. The process also enables teachers to determine the common mistakes that students commit when faced with mathematical challenges. The errors made by students serve as a beginning point for teachers to discern concepts and processes that students had not been able to comprehend through the identification of valid concepts in their answers and solutions. Mathematics teachers make use of the CGI framework in developing their own mathematics curriculum. Thi s personalized version of a mathematics curriculum offers a new perspective of how teachers perceive their students' thinking, thereby allowing them to restructure their teaching techniques about what constitutes student-centered mathematical learning processes. CGI teachers go through intensive training and need enough time to develop their CGI-based mathematics curriculum through workshops and seminars, and technical support and mentoring led by the program developers and experienced CGI teachers.

CGI calls for teachers to understand a set of principles that is most useful in

identifying specific types of mathematics problems, and examining different approaches that students may use in comprehending and applying concepts of addition, subtraction, multiplication and division. In this teaching program, students' different solution strategies are identified and the intellectual processes that they undergo while progressing in their application of these solution strategies is evaluated. The program can also be perceived as a pre-teaching evaluation program among children, wherein teachers allow students to explore their own solutions in solving problems along with providing them ample opportunities to examine the relevant experiences and knowledge they already have. In addition, a CGI -based curriculum can be used to draft and conceptualize specific instructional framework for the particular needs of individual students in a class, thereby enhancing the students' mathematical comprehension and reasoning skills. Various tools are made available so that students are able to communicate their solution strategies with each other as well as with their instructor in order to help them develop their mathematical thinking and reasoning skills in a collaborative environment. Teachers who are CGI-trained apply methods of inquiry, problem-posing, and assessment in dealing with complexities of their students' thinking as a launching point for addressing further developments in learning mathematical concepts, symbols and procedures (Carpenter, 1996; 1999).

In the traditional way of teaching mathematics, the informative/instructive teaching approach, where students normally work alone and the curriculum is heavily dependent on textbooks and workbooks, is the dominant strategy. In a teaching environment that is built around this strategy, the topics that are discussed follow a particular schedule, and the time spent on each task is important and is geared toward meeting the goals outlined in the schedule. Fu rthermore, because meeting the schedule to cover a certain volume of material is given priority over student learning, the instructional approach assumes a specific-to-general trend in covering a sequence of topics; students are regarded as sponge-like passive absorbers of information without the characteristics of a pre-learned individual. Likewise, students of teachers of mathematics who employ traditional methods of instruction are evaluated primarily based on the tests and examinations facilitated to them after each lesson, wherein their performance is considered independent of the teaching strategy employed by their instructors. Th is method of assessing student understanding does not take into account individuality in the context of learning, and results in an inaccurate stratification of students in a classroom.

On the other hand, CGI pedagogy promotes the interactive roles of teachers and students where the teacher has the role of a medium or channel for the students to appreciate the concepts available to them through the group enhancement of skills. Students usually work in groups to reflect and discuss the concepts presented to them, a process that consumes much time. "Curricular activities rely heavily on primary sources

of data and manipulative materials" and most of the class time is spent solving complex mathematical problems (Carpenter, Fennema, Peterson, Chiang & Loef, 1989). In contrast to the traditional method of instruction, a CGI teaching strategy follows a general-to specific approach in exploring the content material, and in doing so, it ensures the relevance of the big ideas and general theories that display the content with the specific concepts and operations that students deal with in class. As mentioned before, CGI-trained teachers acknowledge learners' prior knowledge of ideas as an assessment integrated within the teaching approach and is facilitated through questioning and observation of students' work. The instructors also recognize the difference in the level of learning that each student is likely to exhibit (Hankes, 1998).

Padrón, Waxman & Rivera (2002) provide an example of CGI through the use of reciprocal teaching procedure where the students are instructed to perform four reading comprehension monitoring strategies including summarizing, self-questioning, clarifying, and predicting. T he intent of the exercise is described as an increase in students' reading achievement that is expected to help student's master school-based knowledge. In this process, the students serve as their own instructors by reflecting on their lessons and self-understanding of the concepts and operations they need to learn.

Development of Mathematical Ideas (DMI) is a framework designed to assist teachers perceive the major ideas of elementary mathematics and investigate the students' comprehension and development of those ideas. This particular teacher education framework aims to help participants learn more mathematical content, define and select mathematical objectives for their students, recognize key mathematical ideas which appears to be difficult for their students to understand, guide students' mathematical thinking, appreciate the power and complexity of students' perception and comprehension of mathematics, explore the depths of students' understanding of mathematical concepts, study the curriculum itself to extract information that will be helpful to students, enhance instructor's ability to apply mathematical concepts and be able to demonstrate this information to students, and continue learning about children and mathematics. The primary teacher training material in this teaching approach is comprised of cases of class observations illustrating children's learning styles as reported by their teachers. The DMI framework offers opportunities for teachers to develop mathematics lessons that are guided by a group of teachers as facilitators who discuss the work of their students, conduct interviews with their students, write case studies, adapt innovative and successful teaching measures as well as review administered research studies that are related to the goals and objectives of the profession (Schifter, 1994; 1997a; 1997b).

As a long-term professional development program for teachers, DMI supports deep and powerful exploratory experiences with teaching and learning mathematics, promotes

alternative assessment and evaluation of teaching practices, utilizes inquiry-based teaching approaches extensively, and fully incorporates a standards-based curriculum. Teaching under this particular framework includes questioning, listening to and evaluating students, and making strategic teaching decisions and reflection. Ref lection on the part of the instructors calls attention to the content of the lessons, alternative approach and perception of teaching, two-way learning, qualitative assessment, and curriculum coherence and growth of teaching practices and students' performances over time. In planning the curriculum for a particular set of students, analyzing the mathematical ideas presented in the lessons must be carefully examined. Dec iding on the units of instruction as well as exploring and understanding the learning processes of the students is helpful in evaluating the real comprehension and performance of mathematics and serves as a basis for constructing and developing instructional strategies. Collaboration with teaching fellows and educational experts likewise increases understanding of students' mathematical thinking and appreciation as well as fostering development in the discipline of teaching as a noble profession (Phillips, Lappan & Grant, 2000).

Therefore, educational seminars and workshops are usually facilitated to engage the mathematics teachers in alternative and more effective ways of teaching mathematics to increase and improve the performance of their students. DMI is particular in its goal for changing teachers' knowledge and beliefs. To this end, the knowledge, beliefs, and classroom practices of DMI-trained teachers are modified through seminars designed to promote deeper understanding of elementary mathematics concepts, which in turn, is meant to support the development of their students' mathematical knowledge and skills, and in this way enhance professional development (among teachers) and mathematics learning (among students), simultaneously.

Acknowledging the effectiveness and efficiency of alternative teaching techniques and approaches based on the goals of professional development is important for the future of teaching mathematics. The changing perspectives of teachers about teaching functions along with sharing experiences that have contributed to this change in perspectives impart a better understanding of how the learning process occurs among mathematics students to subsequently expose opportunities for further improvement in the educational system (Simon & Tzur, 1999). It is indeed a challenge for those in the pursuit of professional development to identify ways and measures that display teachers' shift from the conventional concentration on mathematical teaching sequence and scope, to a more individualized development of students' mathematical ideas and forms of reasoning (Stigler & Hiebert, 1999).

THE RESULTS

A number of other research studies recognize the important role that teachers have to play in achieving fundamental reforms in mathematics education. Cur rently, there is a call for teachers to learn new ways of teaching so as to enhance the development and effectiveness of their profession. Jaworski (2004) pointed out that educators have the responsibility for building on the findings and results revealed through research on this topic in order to implement the necessary changes that the academic society calls for in teaching mathematics. Davenpor t & Morse (2001) support the proposed teaching reforms through a dramatic transformation in the current educational practices. Their study investigates the impact of utilizing successful reform-based curricula on the new role of the mathematics teacher in learning the concepts of the subject in a more in-depth manner.

Pioneer efforts in the reform of teaching mathematics through a thorough investigation and attention to research studies conducted to determine the most ideal teaching conditions for teachers and the most effective learning experiences for students has resulted in remarkable improvements in overall educational environment. S tudent-centered instructional environments have resulted in positive changes in teachers' beliefs and impacted the change in their students and their classroom practices. An increase in in-depth and improved understanding of mathematical concepts and higher achievement in dealing with word problems as well as an overall improved school performance among students have been some of the salient outcomes observed in such educational settings.

Student-centered instruction. Th e call for necessary reforms in teaching mathematics in ways that benefit both teachers and students in promoting quality education has increased the attention and interest of educators in examining classroom conditions and experiences as well as the perspectives being brought in by students in regards to their concerns with learning mathematical concepts. Cogni tive processes are considered more intently in order to determine the thought processes students have been engaged themselves in the study mathematics. M oreover, in pursuing studies for the development of the teaching profession, a need for detailed descriptions of classroom practices focusing intently on the students' responses and behaviors is necessary for the invention and discovery of sound solutions and measures of improvement. Such attention is primarily recognized by teachers in making observations and narratives out of the classroom environment to shape their teaching approaches to the educational needs of students.

Relevant information and data gathered from studies focused on students' approaches to mathematical reasoning and problem solving contributes to the development, not just of the teaching profession, but also to improved performances of students in the

classroom, thereby resulting in an overall increase in the quality of education. M artino & Maher (1999) conducted a study on the changing approaches to mathematics teaching and professional development providing data that strengthens the claim that attention to students' mathematical reasoning implies the need for a more effective student-teacher interaction.

Teachers' beliefs change positively. It is widely recognized that developing a successful teaching practice grounded in the principles guiding the current reform effort in mathematics education requires a qualitatively different and significantly richer understanding of mathematics than most teachers possessed in the 1990s (Schifter, 1994; 1997a; 1997b). The standards-based elementary mathematics curricula serves as a new tool for mathematics education reform by offering opportunities for teachers' professional development, and at the same time presents serious challenges in utilizing the curricula as intended through a necessary shift in teachers' ways of thinking regarding the subject, mathematics learning and teaching. Mu ch responsibility is required of them as channels of quality education.

Davenport (2001) accumulated narratives on the use of the innovative elementary mathematics curriculum to illustrate the importance of teachers' understanding of the curriculum and the thinking of their students as a promising professional development experience. In line with this information, Goldsmith & Schifter (1994) recognized the need for teachers to understand thoroughly the ideals of the new mathematics teaching curricula for effective implementation in the classrooms. Thei r research paper described a set of components of developmental models that can be used to guide efforts in building models of the process of teachers' development in mathematical practice with a focus on three components of the change process:

(1) Qualitative reorganizations of understanding;
(2) Orderly progression of changes; and
(3) The contexts and mechanisms by which transitions are affected. Meanwhile, Russell, Schifter, Bastable, Yaffee, Lester & Cohen (1994) concern themselves in examining the cases of elementary grade teachers' mathematics learning in the context of their own teaching as they explore mathematics content they use with their students, consider student strategies and representations that are new to them, and try to understand how students are thinking about complex mathematical ideas. The study considered what the teachers must already understand and know in order to implement transformation in teaching mathematics.

Increase in student achievement. W ith an increase in teachers' understanding of the thought processes of their students in dealing with mathematical concepts and operations in the classroom leading to a more student-centered instruction, the students' level of

performance has reflected an increase in their mathematical achievement levels. Since teachers are inclined to know and understand the perspective and learning strategies employed by their students, details of each student's learning style are carefully investigated to support them with their learning and comprehension skills. The attention to students' reaction and performance when confronted with mathematical challenges in the classroom made it more plausible to recognize each student's performance independently from that of his or her classmates. This individualized recognition of student work marks a key phase in mathematics teaching reform, identifying it as an essential ingredient of quality instruction. The irrelevance of competition between and among students inside their mathematics class at the elementary level is deemed to allow more freedom for abilities and levels of understanding that each young student is charged with to surface. Thus, the standards-based mathematics teaching reform fosters a healthier learning environment for students. Since competition is unnecessary at the elementary school level, an assessment approach that considers students individually and independently from each other seems to create an environment where learning mathematical concepts and operations appear to be less stressful on students. This ideal learning environment is likely to maximize student achievement.

Students understand more conceptually. CGI and DMI along with other innovative standards-based elementary mathematics curricula are concerned with a shift in teaching mathematics in order to provide development within the teaching profession, and answer the call of the educational system for improving the quality of mathematics learning among students. Since these alternative curricula distinguish the importance of general-to-specific approach of teaching mathematics, learning the more general mathematical concepts in order to understand the operations and line of thought in solving specific problems in mathematics is highlighted as teachers consider general mathematical rules and ideas as launching points for students to contemplate before asking them to attempt to solve more pointed and complex problems. Along with this approach to teaching mathematics, learning the subject calls for more than performing complicated solutions, but rather a deeper understanding of mathematical principles and postulates within which mathematical applications are rooted. Such teaching practices inside the classroom emphasize the importance of comprehension rather than the volume of the material to be covered. With mathematics as a foundation for many other subject areas, a more meaningful learning of mathematics is likely to have a positive impact on learning competency of students in other areas of study.

Students' word problem solving skill is enhanced. Traditionally, the dominant pedagogy for handling word problems in mathematics has been one that requires students to memorize formulas and procedures that are necessary for manipulating the variables in

questions. Al though students are quite often able to memorize strategies for solving mathematical equations, the challenge in solving word problems lies in the comprehendsion of the concepts presented in the question. Thus, if students can understand the concepts in word problems, the likelihood of them being able to correctly answer these questions is increased. The pedagogical shift from a traditional way of teaching mathematics to one based on inquiry and exploration not only has been shown to enable students learn more than in the mere act of memorizing mathematical equations; but, it has also been shown to be more superior in applying basic mathematical concepts when dealing with the demands of word problem solving. T he reformed curricula's recognition of unique problem solving strategies and differing levels of mathematical skills of students has intensified and contributed to students' innate mathematical knowledge and skills and ultimately provided new opportunities for students to apply their individual tactics for solving word problems without having to experience the difficulty of adapting a dominant traditional approach—an approach that is likely very different from their own line of thought and reasoning in solving word problems. Thi s result demonstrates that reliance on the knowledge and skills that elementary students are already characterized with in dealing with mathematics problems is a more efficient way of teaching and more effective means of imparting and acquiring quality education.

DISCUSSION

The proposed reforms in teaching mathematics require more than the mere acquisition of new teaching techniques and strategies but more importantly addresses the need for a redefinition of the fundamental knowledge of teaching practices, learning, the nature of mathematics as a discipline, and the availability of constructing new and unexplored ways of learning in the formal education.

In CGI, trained instructors analyze students' mathematical understanding and cognitive approaches to problem solving through careful studies of observations that have been captured on videotapes. In DMI, teachers focus on the development of mathematical ideas that guide students in understanding more effectively through discussions between teachers and students. Through CGI and DMI mathematics teachers conduct student-centered instructions, facilitate students' mathematical communication, listen to students' mathematical problem solving strategies, and emphasize conceptual understanding. As the result, teachers' beliefs are changed positively, students' conceptual understanding is improved through use of various problem solving strategies, and student achievement is increased.

The roles and responsibilities of mathematics teachers in the successful implement-

ation of CGI-and DMI-based educational reform have been discussed extensively in many research studies. Through this discussion these frameworks, the power and influence the teachers' potential exercises on the development of trends in the academic community has been recognized. In this regard much is expected from teachers of mathematics for the realization of the goals of this educational movement, since only through their initiative can the promotion of change in the teaching discipline result in more effective and efficient teaching and learning mathematics.

The study conducted by Sassi (2002) supports the belief that teachers need to be intellectually curious, and need to cultivate a particular type of attention toward their teaching situations by pointing out the significance of teachers' sensitivity to their students' thought processes and perceptions. Moreover, Sassi points out the importance that teachers need to be cognizant about the subtle factors that may contribute to or hinder effective teaching and learning in general. In line with these outcomes, Geist & Remillard (2000) highlighted the importance of support to the teachers who are engaged in the implementation of an innovative elementary mathematics curriculum through investigation of teachers' learning, and via efforts to change their educational practice. Their research primarily recognizes the teachers' needs for dealing with unexpected discussion incidents and new perceptions that result from the reformed classroom teaching in the form of unanticipated questions, challenges, and misconceptions that call for on-the-spot resolution during classroom discussions.

Goldsmith & Davenport (1995) deepened the discussion by exploring the role of teachers' emotions in the process of developing a mathematical practice based on constructivist principles of learning and teaching, wherein emotions potentially influence the process of change that promotes growth in teaching. At the same time, Shifter (1994) has investigated the function of narratives in the enhancement of mathematics teaching and urged that narratives of observations made by DMI-trained teachers serve as a medium through which the teachers become more involved in broad conversations and discourse about mathematics and educational reform.

However, the task for setting up a regular professional development program among teachers and a format for quality education among students do not fall solely on the shoulders of teachers. In order to bring the aforementioned educational reforms to fruition, individuals who have the obligation to review and pass judgment about the new approach to teaching mathematics must also do their part in attaining the common goals of the educational system by committing themselves to the improvement of service conditions for teachers and adopting the reformed teachers' goal of producing more learned individuals.

Nelson (1995; 1997) argues that the claims of the reforms cannot be fully realized through successful conceptualization of new mathematics curricula and educational

policies alone, unless the reform-oriented transformation is implemented aggressively among a large number of schools. He believes that school and district administrators will need to become involved centrally, rather than peripherally, if the intellectual norms and values embedded in the mathematics education reform movements are to move beyond individual classrooms and significantly influence entire schools and districts. Moreover, he reasons that since administrators' ideas about the nature of mathematics, learning, teaching, and school culture affect teachers' interpretations of the nature and intent of the mathematics reform movement; their thoughts affect how they might support the change.

The academic community and the society in general needs to be informed of the implications of the innovative mathematics curricula. The common beliefs and expectations about children's cognitive processes call for a new line of study and exploration under the assumptions and claims of the alternative mathematics teaching approach.

Hence, with the gradual spread of reformed ways of teaching mathematics in schools and the reported increase in student achievement as a form of professional development, there is now a need to assert the necessity of teacher professional development programs to emphasize teachers' understanding of students' mathematical comprehension rather than emphasizing the currently dominant traditional pedagogy of direct instruction with a focus on problem solving strategies.

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