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Visualisation of the Mathematical Process: Boolean Algebra and Graph Theory with TI-83/89

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Nowadays there are practically no mathematical courses in which Computer Algebra Systems (CAS) programs, such as MATHEMATICA, Maple, and TI-89/92, are not used to some extent. However, generally the usage of these programs is reduced to illustration of computing processes: calculation of integrals, differentiation, solution of various equations, etc. This is obtained by usage of standard command of type: Solve [...] in MATHEMATICA. At the same time the main difficulties arise at teaching non-conventional mathematical courses such as coding theory, discrete mathematics, cryptography, Scientific computing, which are gaining the increasing popularity now. Now it is impossible to imagine a modern engineer not having basic knowledge in discrete mathematics, Cryptography, coding theory. Digit al processing of signals (digital sound, digital TV) has been introduced in our lives.

Keywords: Computer Algebra Systems (CAS), algorithms of discrete mathematics
ZDM Classification: H55 , U75
MSC2000 Classification: 9 7U70

INTRODUTION

The first positive experience of using the opportunities of Computer Algebra Systems (CAS), in particular MATHEMATICA, was acquired by the author during the development of a course "Coding Theory in MATHEMATICA" (Gachkov, 1999; 2002).

During preceding years courses in Coding Theory have been given only for students on the postgraduate level. This is due to the complexity of the mathematical methods used in most of codes, such as results from abstract algebra including linear spaces over Galois Fields. With the introduction of computers and computer algebra the methods can be fairly well illustrated. The author (in cooperation with Kenneth Hulth) has developed a course, 'Coding Theory in MATHEMATICA', using the wide range of capabilities of

MATHEMATICA. An attempt to introduce Computer Algebra in the course program was made some years ago, as the course Discrete Mathematics with Computer Algebra (with TI-83/89) was given for the first time.

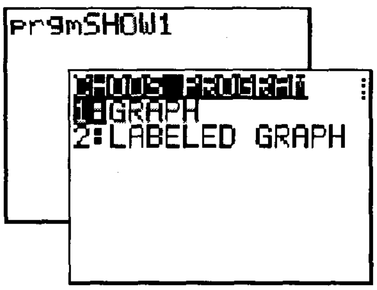
In developing the course modules the author used the MATHEMATICA package "Discrete Mathematics" and the package in TI-83/89 which was developed at the Karlstad University, Sweden and a set of laboratory sessions on an individual basis was worked out. The response from the students was very encouraging and indicated that a conceptually new course structure with hands-on sessions, using the powerful tools of MATHEMATICA and TI-83/89. The presentation "The algorithms of discrete mathematics and graph theory with MATHEMATICA and TI-83/89" is permanent developing of the course Discrete Mathematics with Computer Algebra.

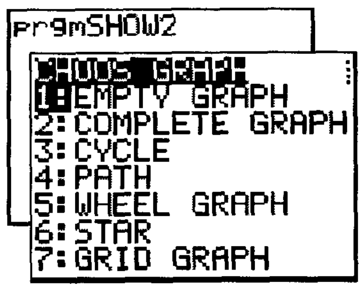
THE PACKAGES "BOOLEAN ALGEBRA" AND "GRAPH THEORY"

The packages "Boolean algebra" and "Graph theory" are a program packages, which was created especially for TI-83/89 and is used for teaching in course of Discrete Mathematics based on a traditional textbook "Discrete and Combinatorial Mathematics an Applied Introduction" (Grimaldi, 1999). Actually this package is a natural development and a further edition of the package "Boolean.m" in MATHEMATICA, which has been used by the author during a long time for teaching in this course (Gachkov, 1999; 2003). The packages "Boolean Algebra" and "Graph Theory" for TI-83/89 contains programs which allow to display different steps with illustrative explanations calculation in the Boolean algebra and Graph theory.

Calculators allow to change the teaching process by replacing of MATHEMATICA with TI-83 due to their safety, low price and because they are easy to use and are possible to develop and provide. Of course MATHEMATICA has more powerful calculating possibilities, but calculators are very flexible, and can therefore be used during lectures in big rooms without technical facilities.

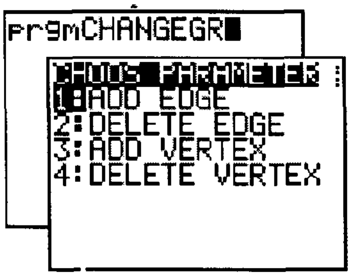
For example:

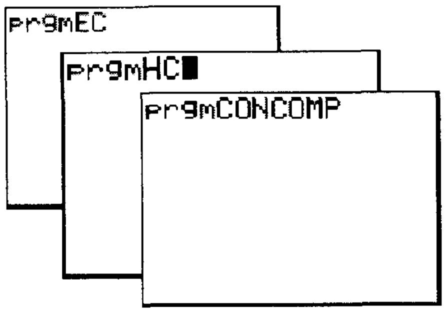




Programs from the other part can calculate different value parameters (for example number of vertices, edges, degree sequences) and also change the structure of the graphs with further display of final results.

For example:





Calculators allow to change the teaching process by replacing of MATHEMATICA with TI-83/89 due to their safety, low price and because they are easy to use and are possible to develop and provide. Of course MATHEMATICA has more powerful calculating possibilities, but calculators ar e very flexible, and can therefore be used during lectures in big rooms without technical facilities.

THE COURSE MODULE DISCRETE MATHEMATICS

The course in Discrete Mathematics in University of Karlstad consists of three main parts:

a) Mathematical Logic and Boolean Algebra.

b) Sets, Combinatorics, Generating Functions and Difference Equations.

c) Graphs, including Optimisation Problems.

Now we can compare the calculating possibilities of MATHEMATICA and TI-83/89 on simple examples from part a) and c).

A common case from Boolean algebra is the following example.

Example 1. Le t f: $B^4 \to B$ be defined by

$$f(w,x,y,z) = (w \wedge z \vee x \wedge y \wedge z) \wedge (x \vee \bar{x} \wedge \bar{y} \wedge z).$$

a) Find truth table for function f.

b) Find the *disjunctive normal form* (DNF) and the *conjunctive normal form* (CNF) for function f (Triantaphyllou, Evangelos & Soyster, 1995).

In[1]:= f=(w&&z||x&&y&&z)&&
(x||!x&&!y&&z)

Out[1]=

$(w\wedge z\vee x\wedge y\wedge z)\wedge(x\vee\ \ x\wedge\ \ y\wedge z)$

In[2]:= TruthTable[f]

Out[2]=

w	*x*	*y*	*z*	F(...)
T	T	T	T	T
T	T	T	F	F
T	T	F	T	T
T	T	F	F	F
T	F	T	T	F
T	F	T	F	F
T	F	F	T	T
T	F	F	F	F
F	T	T	T	T
F	T	T	F	F
F	T	F	T	F
F	T	F	F	F
F	F	T	T	F
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

In[3]:= DNF[f]

Out[3]=

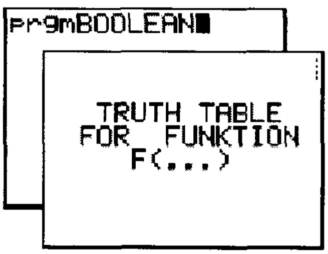
$w\wedge x\wedge y\wedge z\vee$
$w\wedge x\wedge\ \ y\wedge z\vee$
$w\wedge\ \ x\wedge\ \ y\wedge z\vee$
$w\wedge x\wedge\ y\wedge z$

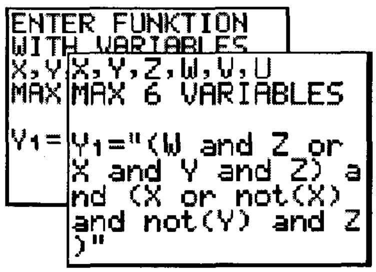
In[4]:= CNF[f]

Out[4]=

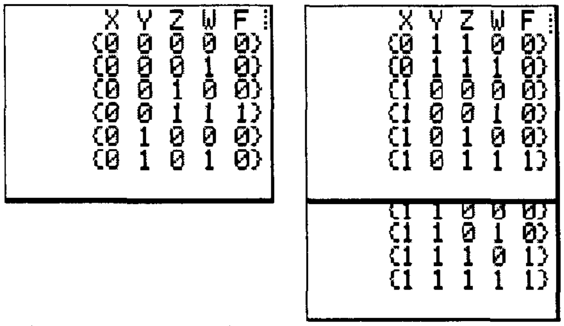
$(\ \ w\vee\ \ x\vee\ \ y\vee z)\wedge$
$(\ \ w\vee\ \ x\vee\ \ y\vee z)\wedge$
$(\ \ w\vee x\vee\ \ y\vee\ \ z)\wedge$
$(\ \ w\vee x\vee\ \ y\vee z)\wedge$
$(\ \ w\vee x\vee\ \ y\vee z)\wedge$
$(w\vee\ \ x\vee\ \ y\vee z)\wedge$
$(w\vee\ \ x\vee y\vee\ \ z)\wedge$
$(w\vee\ \ x\vee y\vee z)\wedge$
$(w\vee x\vee\ \ y\vee\ \ z)\wedge$
$(w\vee x\vee\ \ y\vee z)\wedge$
$(w\vee x\vee y\vee\ \ z)\wedge$
$(w\vee x\vee y\vee z)$

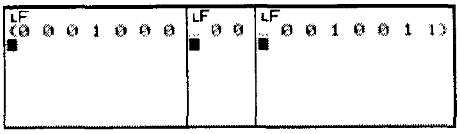
And the following program sequence allows displaying truth table for function *f* (*x*, *y*, *z*, *w*) with TI-83.



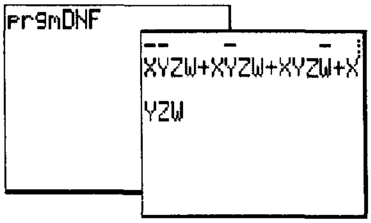


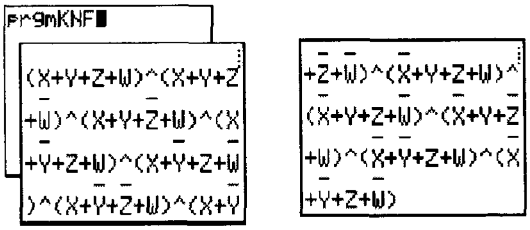
Truth table is and we can display value F



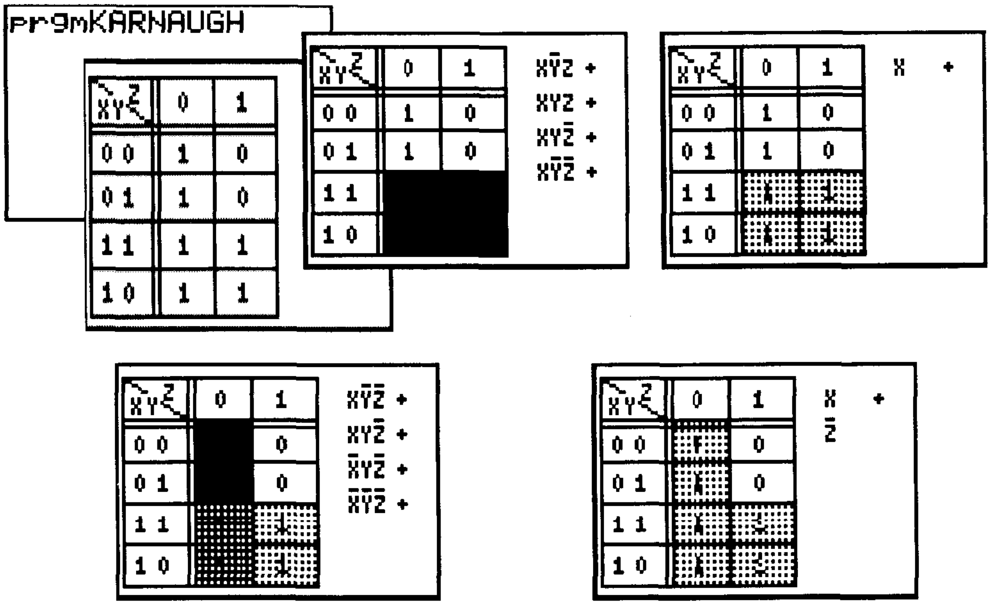


And DNF can get with program and CNF





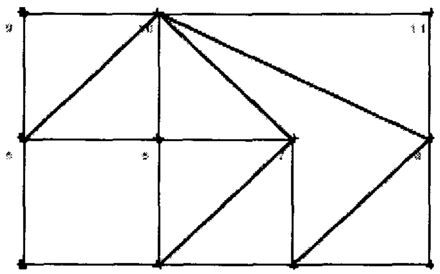
Now we will show how to simplify the function $f(x, y, z)$ by "Karnaugh maps".



and $f(x,y,z)=(x \vee \neg z)$

The following example shows how the package works with common examples from graph theory.

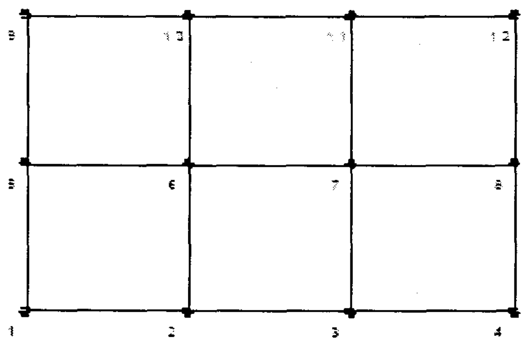
Example 2. F ind an Euler circuit (Copes, Sloyer, Stark & Sacco, 1989) for the graph *G*.



The following programs sequence allows displaying a graph with all the parameters. (With MATHEMATICA)

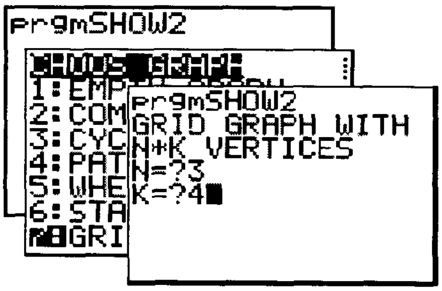
In[5]:= ShowLabeledGraph[G=GridGraph[4,3]]

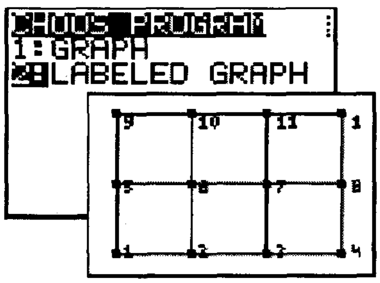
Out[5]=



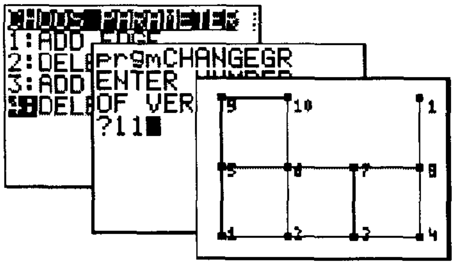
In[6]:= ShowLabeledGraph[G1=DeleteVertex[AddEdge[AddEdge[AddEdge[
AddEdge[AddEdge[AddEdge[DeleteEdge[DeleteEdge[G,{7,11}],
{7,8}],{5,10}],{7,10}],{2,7}],{3,8}],{10,8}],{10,12}],11]]

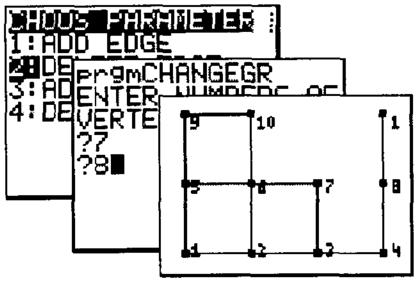
Out[6]=



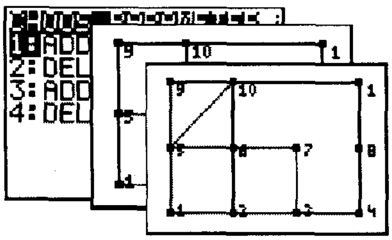


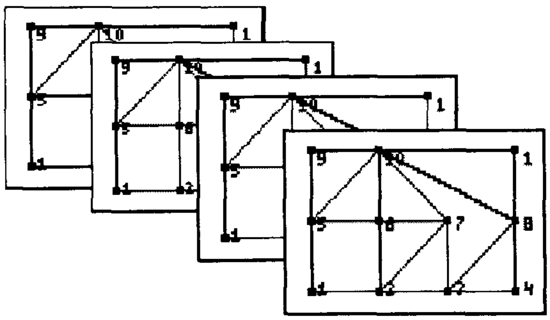
Second, we delete 11 vertexes and the edge between 7 and 8.



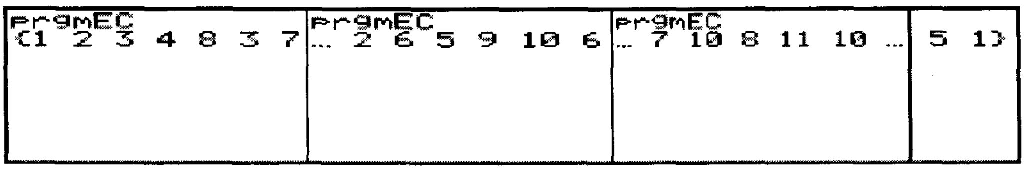


Third, with help of the program ADD EDGE (using this program 5 times), we add 5 edges.

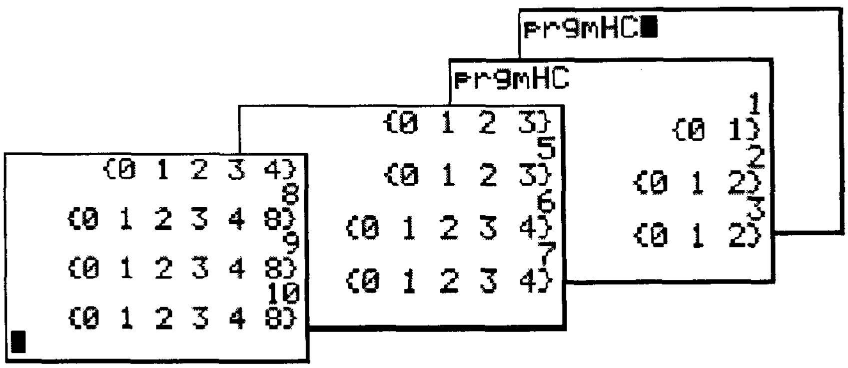




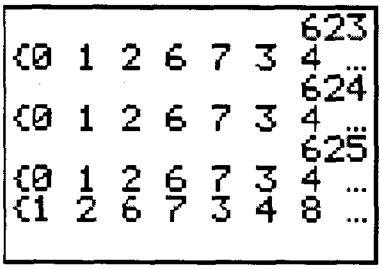
Finally, we find the Euler circuit for the graph *G*

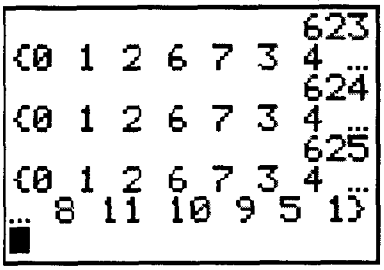


and Hamilton cycle (DeLeon, 2000) for the graph *G*.



There Hamilton cycle is {1,2,6,7,3,4,8,11,10,9,5,1}.





CONCLUSION

As conclusion we can point that there is an opportunity to complete the package described here with other programs if this package can be a basis. This gives a possibility to make the quality of teaching process higher.

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