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A Literature Survey of the Internet Topology Generation Models

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There has been much effort to improve the accuracy of the Internet topology model and include its quantitative and/or qualitative effects on studies of a variety of network problems. Such improvement is the primary motivation of this paper in listing and classifying the body of literature addressing the Internet topology. The metrics, which characterize the fundamental properties of the Internet, are also divided into five categories and their importance and applications are discussed. Finally, we suggest several future research topics for the Internet topology models to be more realistic and applicable.

Keywords: Power-law, Internet Topology Model

1. Introduction

The explosive growth of the Internet has been accompanied by a wide range of internetworking problems related to routing, resource reservation, and administration. The study of algorithms and policies to address such problems often involves simulation or analysis using an abstraction or model of the actual Internet structure. Naturally, we have the questions like the following: "What does the Internet look like?", "Are there any topological properties that don't change in time?" and "How can I generate Internet-like graphs for my simulation?". However, modeling the Internet topology is an important open problem despite the attention it has attracted last few years. Paxson and Floyd (1997, 2001) discuss why simulating Internet is an immensely difficult undertaking.

An accurate topological model can have significant impact on network research. First, network topology models play the important role in assessing network algorithms (Magoni and Pansiot, 2001). That is, the effectiveness or performance of proposed algorithm is highly sensitive to the underlying Internet AS connectivity structure. Doar and Leslie (1993) find that the efficiency of their dynamic multicasting algorithms was reduced by as much as half when using random graph versus using hierarchical structured graph. And protocols that work seamless on prototypes fail to scale up, being inefficient on the larger real network (Yook *et al*., 2001). Second, we can design more efficient algorithm that takes advantage of its topological properties. In designing route-based distributed packet filtering, Park *et al*. (2001) show that power-law structure of Internet AS topology plays an important role in facilitating efficient proactive/reactive filtering. Third, network topology models can help to understand large-scale properties such as reliability and robustness to accidents, failures, and attacks on network components (Willinger, 2004). And fourth, as the Internet continues to expand exponentially, a topology generator is required which can yield insight into future behavior

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and suggest novel strategies for planning and long term network design as well.

There is a real desire to account for the difference between the artificial and actual Internet topology. Thus, there has been much effort to improve the accuracy of Internet topology model and include its quantitative/qualitative effects on studies of a variety of network problems. Such improvement is the primary motivation of this paper in listing and classifying the body of literature addressing Internet topology. In this study, we deal with the AS-level Internet topology. However, most of the contents can be applicable to the router-level Internet topology.

In Section 2, we describe the power-laws. After the introduction of the power-laws by Faloutsos *et al*. (1999, 2001, 2003) there has been a big change in modeling the Internet topology. They become to serve as a litmus test for the realistic topology generator. The early generator failed when tested against power-laws, so after that a number of new generators were proposed. In Section 3, we summarize the Internet topology metrics. The goal of the topology generator is not to produce exact replicas of the Internet, but instead to create network topologies that embody the fundamental properties of real network. Therefore, the importance of the Internet topology metric, which can well capture the topological properties of the Internet, cannot be overemphasized. In Section 4, several kinds of Internet topologies are reviewed. According to the basic approach methods and the topology metrics, they are compared. The physical interconnection structure at AS level is not enough to understand whole aspect of Internet structure. In Section 5, the logical relationships of AS are reviewed. Section 6 concludes the paper with future works.

2. Power-Laws

2.1 Power-laws in the Internet

Faloutsos *et al*. examined the inter-domain topology of the Internet from the end of 1997 until the end of 1998. The information was collected from BGP routing tables (University of Oregon RouteViews Project). RouteViews collects and archives both static snapshots of the BGP routing tables and dynamic BGP data in the form of BGP message dumps. They showed empirically that certain properties of the AS-level Internet topology are well described by power-laws of the form $y = x^{\alpha}$. This implies that those same distributions of interest in the Internet topologies are skewed. For example, for a particular snapshot of the Internet topology in 1998, 85% of the nodes had degree less than the average. They propose and measure graph properties, which demonstrate a regularity that is unlikely to be a coincidence. The exponents of the power-laws can be used to characterize graphs. They observe three power-laws of the Internet and one Approximation.

① **Power-law 1 (rank exponent)** *Given a graph, the degree, d_{ν}, of a node v is proportional to the rank of the node, r_{ν}, to the power of a constant, R*:

$$d_v \propto r_v^R$$

② **Power-law 2 (degree exponent)** *Given a graph, the CCDF, D_d, of a degree, d is proportional to the degree to the power of a constant, D*:

$$D_d \propto d^D$$

③ **Power-law 3 (eigen exponent)** *Given a graph, the eigenvalues, λ_i, are proportional to the order, i to the power of a constant, ε*:

$$\lambda_i \propto i^{\varepsilon}$$

④ **Approximation 1 (hop-plot exponent)** *The total number of pairs of nodes, P(h), within h hops, is proportional to the number of hops to the power of a constant, H. δ is the diameter of the graph*:

$$P(h) \propto h^H, \ h << \delta$$

Chou (2000) shows that the first two-laws are in fact equivalent. That is, as long as any one of them is true, the other can be derived from it, and vice versa. He argues that for nodes of not very large degree, the first Faloutsos' power law is superior to the second one in giving a better estimate of the exponent, while for nodes of very large degree the power-law relation may not be present, at least for the relation between the frequency of degree and node degree.

2.2 Observations of Power-Laws

There have been several studies over last few years, which investigate power-laws in the AS-level Internet. The results are summarized in <Table 1>. Most of them could observe the power-laws in the Internet. And the presence of power-laws in the Internet is now considered to be empirically well established. However, Chen *et al*. (2002) and Chang *et al*. (2004) find that the AS degree distributions constructed from the Oregon BGP (University of Oregon RouteViews Project) and extended data sources are slightly different from the strict power-law curves. The extended data sources include Looking Glass (Traceroute), RIPE (IRR: List of Routing Registries, Merit Network, Internet Routing Registry) and other publicly available full BGP routing tables, which capture 20 ~ 50% more physical links than Oregon BGP. In Mahadevan *et al*. (2005) and Mahadevan *et al*. (2006), they find that Traceroute (http://www.Traceroute.org) and BGP topology are similar to one another but differ substantially from the WHOIS (Internet Routing Registries) topology. Traceroute is a tool that captures a sequence of IP hops along the forward path from the source to a given destination. A tool, Skitter (Claffy *et al*., 1999), was developed to collect continuous traceroute-based Internet topology measurements. WHOIS is a collection of database containing a wide range of information useful to network operators.

2.3 Possible Causes for Power-laws

There have been several studies to identify possible causes for power-laws. Followings are three major findings.

(1) Barabasi-Albert (BA) model (Barabasi, 1999)

It shows that the scale-free power-law distribution is a consequence of two generic mechanisms: i) networks expand continuously by the addition of new nodes (incremental node growth) and ii) new nodes attach preferentially to nodes that are already well connected (preferential connectivity). They showed analytically that these two mechanisms suffice to produce networks that are governed by a power-law. Based on the BA model, several topology generators have been developed (Albert and Barabasi, 2000; Bar *et al*., 2004; Bar *et al*., 2005; Bu and Towsley, 2002; Median *et al*., 2001 Medina *et al*., 2000; Park and Lee, 2001; Zhou and Mondragon, 2004), which try to capture the actual processes that govern the creation of power- laws.

(2) Heuristically Optimized Tradeoffs

In Fabrikant *et al*. (2002), they argue that power-laws appear when one greedily optimizes a balanced tradeoff between two objective functions: last mile connection cost and transmission delay measured in hops.

(3) Highly Optimized Tolerance

In Carlson and Doyle (1999), they propose that power-laws are the result of an optimization, either through

Table 1. Power Law Observations

Researcher	Year	Data source	Results
Jin *et al*. (2000)	Apr.1999 ~ Nov.1999	BGP routing table	Demonstrates power-laws 1 and 2 with exponents -2.2 and -0.75, respectively.
Mihail *et al*.(2002)	Nov.1997 ~ May 2001	BGP routing table	Both indegree (number of customers and peering/siblings) and outdegree (number of providers and peering/siblings) follow power law.
Bu *et al*. (2002)	Nov. 2000	BGP routing table	Demonstrates power-law 1 with exponent -2.26.
Chen *et al*. (2002)	March 2001	BGP routing table	Agrees with the power-law curve
		Oregon BGP + extended sources of data	· Slight difference from the strict power-law curve · The distribution is certainly heavy-tailed.
Zhou *et al*. (2004)	April 2002	Traceroute	Good agreement with the power-law 1 with exponent -2.22.
Mahadevan *et al*. (2005)	March 2004	Traceroute	Exhibits the power -law.
		BGP routing table	Does not show the strict power-law curve.
		WHOIS	Does not follow power-law.

natural selection or engineering design, to provide robust performance despite uncertain environments. Regarding the Internet, they mention that the survivability built in the Internet and its protocol can be the cause of power-laws.

2.4 Open Questions

There are several open questions that still need to be answered.

(1) There may be more topological properties of the Internet topologies that are not captured by the power-laws.

The Internet topologies which possess the power-law properties resemble that of Internet in some topological metric, such as average degree and average path length (Median *et al*., 2001; Zhou and Mondragon, 2004). The power-law-based topology models also exhibit large-scale global properties of the Internet, such as expansion, resilience, distortion (Tangmunarunkit *et al*., 2002) and hierarchical properties (Jaiswal *et al*., 2004; Tangmunarunkit *et al*., 2002). On the other hand, there are some properties of Internet that are not captured by the power-laws. Bu *et al*. (2002) demonstrate that Internet is a small world graph, a topological properties suggested by Watts and Strogatz (1998). They show that BA model does not possess the properties of a small world graph. The same is true for the rich-club connectivity metric suggested by (Zhou and Mondragon, 2004; Zhou and Mondragon, 2004). Li *et al*. (2004) show that networks having the same power-law degree distributions can have vastly different features and appear deceivingly similar from a view that considers only graph theoretic properties.

(2) How complete is the AS-level topology captured by the Oregon BGP routing table?

It is demonstrated in Chen *et al*. (2002) that the Internet maintains much richer connectivity than can be observed by aggregating a handful of BGP routing tables. They compare the AS node degree distribution of the Oregon data sets to that constructed from sources beyond the Oregon data sets. The latter shows more ASes with node degrees ranging from 4 to 300, resulting in a curved line in the distribution, even though the distribution is certainly heavy-tailed. In Mahadevan *et al*. (2005), they find that Traceroute and BGP topology are similar to one another but differ substantially from the WHOIS topology.

(3) It needs to explore further the meaning and the values of the exponent.

Faloutsos *et al*. (2003) argue that such analysis could reveal interesting inter-plays and trade-offs between the forces that govern the creation of the topology. And, it might be necessary to investigate the effect of the exponent values on the network topology metrics.

3. Internet Topology Metrics

Tangmunarunkit *et al*. (2002) insist that the goal of the Internet topology generator is not to produce exact replicas of the current Internet, but instead to produce graph whose properties are similar to the Internet graph. The question is what properties are relevant to this comparison. There seems to be no single answer to this question, as the relevant properties may well depend on how the generated networks are used.

The metrics that have been reported in the previous studies on the Internet topology can be classified several ways. Govindan *et al*. (1997) propose three classes of metrics such as 1) domain degree distribution, 2) diameter and 3) connectivity. In Zegura *et al*. (1997), the metrics are classified into the following two categories : 1) topological metrics, which are independent of any potential application and 2) application-specific metrics, which depend on topology and application. On the other hand, Park *et al*. (2004) divide the metrics of the Internet topology into two categories: 1) static, which is constant over time and 2) dynamic, which traces the behaviors of the Internet topology over time. In this study, metrics are classified into 5 categories as follows.

- Degree-based metrics
- Node-connectivity metrics
- Large scale global metrics
- Hierarchical metrics
- Other metrics

3.1 Degree-Based Metrics

These are simple and basic metrics that characterize local connectivity in a network, which are summarized in <Table 2>. They represent basic statistics of node and link degrees such as average node degree, number of leaves, etc.

Table 2. Degree-Based Metrics

Measure	Definition	Importance/Applications	Internet characteristics
Average degree	Average number of edges incident to the node	Coarsest connectivity characteristics of the topology	4.68(Claffy, *et al*. 1999), 6.29 (University of Oregon RouteViews Project) and 15.22 (Internet Routing Registries)
Maximum degree	Maximum number of edges incident to the node		Internet has much higher value than BA model.
Number of Leaves	Number of nodes with degree 1	Pure BA model never produce leaves.	Around 26% of all nodes (Zhou and Mondragon, 2004)
Degree distribution	Degree distribution of a node	· Most frequently used characteristics · Contains more information about connectivity in a given graph than the average degree.	Follows power-laws.
Joint node degree distribution (JDD)	Probability that a randomly selected edge connects k1 and k_2 degree nodes.	· Summary statistics of JDD : average neighbor connectivity and assortative coefficient (Dorogovtsev; Newman, 2002) · Can be used to produce more realistic topology generator (Mahadevan, *et al*., 2005).	
Link-degree ratio distribution	L(i) and H(i) denote the degrees of lower degree and higher degree nodes of link i, respectively. The ratio is defined as L(i) / H(i).	It clearly differentiates the distinct topology (Park *et al*., 2004).	
Average-node-degree ratio distribution	A(i) denotes the average degree of the set of neighbor nodes of the node i. The ratio is defined as 'the degree of node i / A(i)'.		It is invariant both under the Oregon BGP and the extended topology (Park *et al*., 2004).

Mahadevan *et al*. (2005) list the degree-based metrics in the order of increasing amount information about local connectivity structure of the network, as shown in <Table 3>.

Table 3. Connectivity characteristics of a network with a maximum distance D (Mahadevan, *et al*., 2005)

Tag	Name	Degree correlations of node at distance
0K	Average degree	None
1K	Degree distribution	1
2K	Joint node degree distribution	2
3K	Joint edge degree distribution	3
….	….	….
(D+1)K	Full degree distribution	D

3.2 Node -Connectivity Metrics

These are more sophisticated metrics of node connectivity, which are summarized in <Table 4>. Rather than just specifying node and link degrees, these metrics show a measure of how close a node is to its neighbor. Those include clustering coefficient, assortativity coefficient, rich club coefficient and coreness.

3.3 Large Scale Global Metrics

<Table 5> shows metrics, which represent large scale global properties of the Internet, not purely local quantities. These are measures related to resilience, network performance, robustness to link failure, expansion and reachability. One of the global metrics, characteristics path length, can indicate overall routing efficiency.

Table 4. Node-Connectivity Metrics

Measure	Definition	Importance / Applications	Internet Characteristics
Clustering coefficient (Bu and Towsley, 2002)	A measure of how close a node's neighbors are to form a clique	· Basic connectivity metrics · Local robustness in the graph	Much larger clustering coefficient than a random graph with the same characteristic path length (Bu and Towsley, 2002)
Assortativity coefficient (Mahadevan *et al*., 2005)	A measure which shows a preference for high degree nodes to attach to other high degree nodes	· Disassortative networks; excess radial links connecting nodes of dissimilar degrees · Assortative networks; excess tangential links connecting nodes of similar degrees · related to likelihood metric (Li *et al*., 2004)	Internet exhibits disassortative mixing behavior (Mahadevan *et al*., 2005; Zhou and Mondragon, 2004).
Rich club Connectivity (Zhou. and Mondragon, 2004)	A measure of how well rich club members, a small number of high degree nodes, are connected to each other	· Rich club is a super traffic hub of the networks. · The rich club connectivity and the disassortative mixing properties together contribute to the routing efficiency of the networks. · Not captured by BA model	Shows strong rich club connectivity (Zhou and Mondragon, 2004).
Coreness (Mahadevan *et al*., 2005)	A measure how 'deep in the core' a node is; a more sophisticated measure of a node connectivity	· Shows how easily the node is disconnected by removing poorly connected neighbors. · Signature of topology dynamics under different types of anomalies (Gaertler and Patrignani, 2004)	

Table 5. Large Scale Global Metrics

Measure	Definition	Importance / Applications	Internet characteristics
Spectrum (Mahadevan *et al*., 2005)	Set of eigenvalues of the graph adjacency matrix	· Related to the resilience (Chung 1997) and network performance (Gkantsidis *et al*., 2003) · Can be used to discover clusters of highly interconnected nodes (Gkantsidis *et al*., 2003; Vukadinovic *et al*., 2001).	The eigenvalues follow power-law.
Resilience (Tangmunarunkit *et al*., 2002)	Size of a cut-set for a balanced bipartition	Measure of the robustness of the graph to link failure	· High resilience (Tangmunarunkit *et al*., 2002) · High degree nodes connect to each other (Jaiswal *et al*., 2004).
Distortion (Tangmunarunkit *et al*., 2002)	Minimum-communication-cost spanning tree	It reflects the manner in which spanning tree can be embedded into the topology.	Low distortion (Tangmunarunkit *et al*., 2002)
Expansion (Tangmunarunkit *et al*., 2002)	Measure of the ability of a node to reach other node within a given distance	Important for many application, most prominent being routing	High expansion (Tangmunarunkit *et al*., 2002)
Dense, or sparse core (Mihail *et al*., 2002; Sagie and Wool, 2003)	Qualitative coarse structure characteristics		Internet is a dense core network (Mihail *et al*., 2002; Sagie and Wool, 2003; Zegura, 2002)
Distance distribution (Mahadevan *et al*., 2005)	Probability for a random pair of nodes to be at a distance of x hops from each other	· Closely related to the metrics, expansion and reachability function (Phillips *et al*., 1999) · Performance of routing algorithm strongly depends on this measure. · Robustness of the network to viruses	A Gaussian-like shape (Mahadevan *et al*. 2005)

(continued)

(Table 5 continues)

Characteristic path length (Bu and Towsley, 2002)	Median of the means of the shortest path lengths connecting each node to all other nodes	Indicates overall routing efficiency.	Small compared to the random network (Bu and Towsley, 2002).
Small-world network (Watts and Strogatz, 1998)	It has a much larger clustering coefficient than a random graph with the same characteristic path length.	Topological properties that are not captured by BA model	AS-level Internet graph is a small-world graph (Bu and Towsley, 2002).
Skewness (Park *et al*., 2004)	It measures how preferential the network is.	· Value 0: extremely Preferential and · Value 1: Uniform	Internet has a skewness of about 0.4 (Park *et al*., 2004).

3.4 Hierarchical Measure

There seems to be little doubt that the Internet has a significant degree of hierarchy. We mean hierarchy in the sense that the nodes (aside from those in the topmost level) would depend on the nodes in the level above for paths to the rest of the graph. Tangmunarunkit *et al*. (2002) argue that there are two symptoms of hierarchical structure. The first is that some links are used more often than others. The backbone link is expected to be more highly used than peripheral links. Here usage is measured by the set of node pairs whose traffic traverses the link when using shortest path routing, called link's traversal set. The second symptom is that paths tend to first go up, and then down, the level of hierarchy. That is, a path between two nodes at the edge of the network works its way up the hierarchy until it reaches the backbone, and then works its way back down. On the other hand, Jaiswal *et al*. (2004) suggest the decomposition procedure of the graph, which allows us to examine whether the graph has hierarchical properties. The resulting decomposition shows the evidence of both hierarchical and non-hierarchical properties of the Internet. Hierarchical measures or methods to infer hierarchical properties are summarized in <Table 6>.

Table 6. Hierarchical Measures and methods to infer hierarchy

	Definition	Importance / Application	Internet characteristics
Link value (Tangmunarunkit *et al*., 2002)	Number of node pairs whose shortest paths traverse the link	Identify the degree of hierarchy in the Internet.	More loose hierarchy than that of the structural generators such as Tiers (Doar, 1996) and GT-ITM (Calvert *et al*., 1997)
Up/down analysis (Tangmunarunkit *et al*., 2002)	Looks at the series of link values along a path and asks what fraction of paths have an 'up-down' pattern.	Shows the hierarchy in the Internet.	
Betweenness (Mahadevan *et al*., 2005; Zhou and Mondragon, 2004)	Measure of the number of shortest paths passing through a node or link	· Estimate potential traffic load on nodes/links and can be used to infer hierarchy in the Internet. · Directly related to link value distribution and router utilization (Li *et al*., 2004)	The cumulative distribution of betweenness exhibits power-law behavior (Zhou and Mondragon, 2004).
Decomposition procedure (Jaiswal *et al*., 2004)	Computes the connected components of the graph obtained after removing the selected nodes, which belong to same level (or tier) of decomposition.	It can show the nature of the hierarchy - balanced or how deep.	AS-level Internet has similar hierarchical properties as power-law-based models (Jaiswal *et al*., 2004).

3.5 Other Measures

Li *et al*. (2004) claim that commonly-used metrics (graph-theoretic quantities and their statistical properties) are inadequate for capturing what matters for real network topologies for the following reasons.

- They lack a direct networking interpretation.
- They all rely largely on qualitative criteria, making their application somewhat subjective.

They also show that networks having the same (power-law) node degree distribution can have vastly different features, and appear deceivingly similar from a view that considers only graph theoretic properties. Instead, they suggest two types of metrics as follows.

- Performance related metrics; throughput, router utilization and user bandwidth distribution
- Likelihood metric; A measure of randomness differentiating between multiple graphs with the same degree distribution. It can be used to evaluate the amount of order (engineering design constraints) present in a given topology.

4. Internet Topology Generator

Various topology generators have been developed to generate realistic topologies. They fall into one of three classes, random generators, structural generators and degree power-law generators.

The first topology generator is based on the random method. The Waxman (1988) method is an instantiation of this method. A link is added between each pair of nodes with a certain probability. Focusing on the hierarchical structure of the network, structural generators such as GT-ITM (Calvert *et al*., 1997) and Tiers (Doar, 1996) follow. The introduction of power-laws brought a revision of the graph generation models in the networking community. A number of power-law-based topology generators have been proposed.

Table 7. Comparison between Network Topology Generators

	Random generator	Structural generator	Power-law-based generator
Generators	Erdos-Renyi (1959) and Waxman (1988)	GT-ITM (Calvert *et al*., 1997) and Tiers (Doar, 1996)	BA (Barabasi and Albert, 1999), AB (Albert and Barabasi, 2000), PLRG (Aiello *et al*., 2000), Brite (Median *et al*., 2001), Inet (Jin *et al*., 2000; Winick and Jamin, 2002), GLP (Bu and Towsley, 2002), PFP (Zhou and Mondragon, 2004), PLOD (Palmer and Steffan, 2000) and GDTANG (Bar *et al*., 2005)
Basic idea	Long-range links are expensive.	Focuses on the hierarchical structure.	Faithfully reproduces local properties, power-law degree distributions.
Degree-based Metric	Does not follow power-Laws.	Does not follow power-laws.	· Naturally good agreement with power-laws · BA model produces no leaf and has much smaller value of maximum degree. · Modified BA models show good agreement.
Connectivity Metric	Does not agree with the Internet.	Generally not good agreement with the Internet	Shows good matches with the Internet for the chosen metrics of the corresponding generator.
Global metric	Does not agree with the Internet.	Generally not good agreement with the Internet	Shows good matches with the Internet for the chosen metrics of the corresponding generator.
Hierarchical metric	Complete lack	The hierarchy is more strict than that of the Internet (Tangmunarunkit *et al*., 2002).	Well captures the hierarchy of Internet. Two main reasons are; · long tail distribution of degrees · backbone links are merely the links connecting two high degree nodes (Jaiswal *et al*., 2004; Tangmunarunkit *et al*., 2002).

4.1 Comparison between Network Topology Generators

In this Section we compare the Internet topology generators of three classes, and summarize in <Table 7>. The discovery of power-laws in the Internet has brought new constraints upon the generated topologies. Since power-laws are key elements, implicitly or explicitly, in the power-law-based models, the power-law-based generators are more realistic than other two, random and structural generators. In addition to reproducing power-law degree distributions, the power-law-based models also try to fit certain properties of measured Internet topology. So, they obtain a good fit on these chosen metrics and outperform other two classes of generators. However, this is not to say that the power-law-based models are sufficient to describe the Internet topology in all its complexity. We still find Internet topology properties, which the power-law-based models cannot exhibit.

4.2 Power-law-based Internet Topology Generators

The power-law-based Internet topology generators can be classified as,

- Degree-driven models: They take the power-laws as given, and a style of reverse engineering by first having an appropriate degree distribution. They do not attempt to emulate the process that leads to power-laws.
- Evolution-based models: They produce a topology incrementally, by adding one node at a time to an existing topology. They try to capture the actual process that governs the creation of power-laws.

4.2.1 Degree-Driven Models

There are several topology generators using this approach such as

- Inet (Jin *et al*., 2000; Winick and Jamin, 2002),
- Power- law random graph (Aiello *et al*., 2000),
- Power-law outdegree algorithm (Palmer and Steffan, 2000) and
- Markov-chain simulation method (Gkantsidis *et al*., 2003).

The general procedure to construct network topology is as follows.

① Number of nodes are given as N.
② Assign degrees to N nodes drawn from a power-law distribution with a given exponent.
③ Construct a topology meeting a degree sequence using a preferential connectivity, which is defined in Section 4.2.2, or some random method.

The evaluation metrics for degree-driven models are how well they produce power-law relationships of Siganos *et al*. (2003). So, the used metrics are rank-degree relationship, degree-frequency relationship, pair size within h hops and eigenvalue-rank relationship.

Mihail *et al*. (2002) show that degree-sequence is not sufficient for producing topologies that are a good match to real data, in the sense that one can produce topologies that differ significantly from one another, despite meeting the same degree sequence. Instead of just using preferential connectivity they suggest models, which allow flexibility in connecting nodes. That is, the algorithm can start by connecting the highest degree node(d_h) with d_h other high degree nodes and obtain a residual degree sequence by reducing the degrees of these vertices by one, which is called 'dense core' model. Alternatively, the algorithm can connect the lowest degree node (d_l) with the d_l highest degree vertices, which is called 'sparse core' model. So, for the same degree sequence, we can generate models of three types: sparse core, dense core and preferential.

4.2.2 Evolution-Based Models

Just after the introduction of power-laws, we have generators which try to capture the actual process that governs the creation of power-laws. These are

- BA model (see Section 2.3) and
- Brite (Median *et al*., 2001; Medina *et al*., 2000).

In Medina *et al*. (2000), they examine four factors for possible causes of power-law existence: preferential connectivity, incremental growth, node placement and connection locality. They find that preferential connectivity and incremental growth are the main causes for power-laws, which are defined as follows.

- Incremental growth: It places nodes gradually at a time as nodes join the network. In this case, a new node considers as candidate neighbors only those nodes that have already joined the network.

- Preferential connectivity: A newly considered node v connects to a candidate neighbor node i with the following probability

$$\frac{d_i}{\sum_{j \in C} d_j}$$

where d_i is the current degree of node i, and C is the set of candidate neighbor nodes.

Their finding is the same as that of BA model (Barabasi and Albert, 1999), which suggests two mechanisms as the origin of a scale-free power-law distribution. Yook *et al*. (2001) also try to uncover the mechanisms that shape the Internet's large-scale topology. They extract several parameters, whose values uniquely parameterize a family of Internet models, generating potentially different large-scale topologies.

The models mentioned above are based solely on the attachment of new nodes. However, the appearance of new internal links among already existing nodes has also been observed in the evolution of the Internet (Satorras *et al*., 2001; Vazquez *et al*., 2002). Albert *et al*. (2000) generalize their previous work, BA model, by incorporating the addition of new nodes, new links, and the rewiring of links. Links can be added from a newly created node to the existing network (external link addition). Or links can be added between already existing nodes in the network (internal link addition).

The network researchers soon find out that there are some topological properties of the Internet that are not well captured by the initial BA model and the degree-driven models. So, they try to develop methods that match these metrics. Some revisions are also required for topology generator to be more realistic.

① The data present in (Chen *et al*., 2002) suggests that rewiring rarely happens in the Internet.
② In the Internet, new ASes have a much stronger preference to connect to high degree ASes than predicted by the linear preferential model (Bar *et al*., 2005; Bu and Towsley 2002; Zhou and Mondragon 2004).
③ It needs to describe the AS-graph as a directed graph,

Table 8. Revised Topology Generators

Generator	Newly added topological properties or internet features	Modifications
Albert *et al*. (2000)	Depending on the frequency of processes, two fundamentally different topologies can be developed : 1) power-law and 2) exponential.	More realistic description of the local processes : 1) addition of new links, 2) rewiring of links and 3) addition of new nodes
Generalized linear Preference (Bu and Towsley, 2002)	Small-world graph	· Generalized linear preference model: It has a tunable parameter that indicates the preference for a new node connecting to more popular nodes. · Both external and internal link additions · Delete rewiring operation.
Positive feedback preference model (Zhou and Mondragon, 2004)	· Rich club connectivity · Maximum degree	· Nonlinear preferential attachment · Both external and internal link additions
Geographic directed preferential internet topology model (Bar *et al*., 2004, 2005)	· Directed graph · Number of leaves · Exact power-law exponent · Dense core	· Superlinear preferential attachment · Both external and internal link additions · To allow edge direction, both in-degree and out-degree are considered. · A probability of any new edge to be a peer to peer is given. · For geography, a regional distribution and locality parameter are introduced.
Park *et al*. (2004)	· Link degree ratio · Average-node-degree ratio · Skewness	· A new attachment policy : new nodes select host node only among candidate nodes which have higher degree than their own. · Dynamic probability of internal link addition

considering both customer-provider and peerto peer relationships, and taking geography into account. And there is a lack of dense core in the generated topology (Bar *et al*., 2005).

④ The BA model produces much lower value of maximum degree than the real Internet and does not produce any leaf (Zhou and Mondragon, 2004).

Following <Table 8> summarizes the revised topology generators.

4.3 Some Critics of the Power-law-based Models and Other Works

Chen *et al*. (2002) found that the node-degree distributions resulting from the extended data sources (see section 2.2) deviate significantly from a strict power-law. So, they argue that BA modeling approach (or power-law-based model), which faithfully reproduces power-law degree distributions, can not explain the structure of the node degrees of the real Internet. They suggest an alternative approach using the concept of "Highly Optimized Tolerance", called HOT (Carlson and Doyle, 1999).

Alderson *et al*. (2003, 2004) argue that the current Internet topology generator is developed so that it can well match the newly introduced metric or observed feature of interest, whose importance in judging and comparing different topologies is not clear. And a generator that does a good job in matching the chosen metric often does not fit other metrics well. So, they suggest an alternative approach, which tries to identify the causal forces at work in design and evaluation of real topologies. The basic idea from (Carlson and Doyle, 1999) is that when deploying their infrastructure, network owners and operators are, in fact, approximately solving optimization problems that express the ways in which they build up and evolve their networks. So, they propose formulating appropriate optimization problems to model the process by which Internet connectivity is established and evolves. This work is still at its infant stage, and further study is required to identify the causal relationships between the objectives and constraints of a network design problem and the resulting topology.

The most popular topology generators today (Inet, Brite, etc) incorporate the idea of reproducing the node degree distribution observed in the real network. Mahadevan *et al*. (2005) call these models 1K-random graph (see <Table 3>). They develop a topology generator that creates random graphs with a given form of the joint node degree distribution, called 2K-random graph generator. They argue that this generator reproduces all the important metrics such as distance, betweeness and spectrum. They try to build 3K generator to better match clustering.

Magoni *et al*. (2002) develop the Internet topology generator based on map sampling. Their work consists of two steps: i) it performs a randomized node sampling on a real Internet map to produce a tree and ii) adds redundant links to produce graphs that have appropriate topological properties. For comparison with the Internet, the measures of average path length and power-laws are used.

5. AS Relationships in the Internet

Previous works on the Internet topology have been focused on the physical interconnection structure at AS level. This is reasonable in the sense that connectivity is perhaps the most basic characteristic of a topology. On the other hand, since routing between ASes is controlled by BGP-a policy-based routing protocol, connectivity does not imply reachability. So, the internet topology alone does not provide enough information regarding routing problem. For example, suppose that AS B connects to two providers, AS A and AS C. An AS graph would show connectivity from A to B and from B to C: however, AS B's routing policies would not permit transit traffic between A and C. Therefore, knowing a global picture of AS relationships is an important aspect of the Internet structure.

AS relationships are classified into customer-provider, peering and sibling. A customer pays its provider for connectivity to the rest of the Internet. Therefore, a provider does transit traffic for its customers. However, a customer does not transit traffic between two of its providers. A pair of peers agree to exchange traffic between their respective customers free of charge. In a sibling relationship, a mutual-transit agreement allows a pair of administrative domains to provide connectivity to the rest of the Internet for each other.

Several studies have been conducted for inferring AS relationships in the recent years. Since data about these relationships is not easy to obtain directly, a natural idea is to infer AS relationships from the routing paths in the networks. These paths can be determined from BGP information. Pursuing this approach, the first study on inferring AS relationships is performed by Gao (2001). He proposes a heuristic algorithm for inferring AS relationships, which utilizes the valley-free AS path pattern of BGP routing table entry and AS degree. Formally, an AS path is valley-free if and only if the following conditions hold true.

- A provider-to-customer edge can be followed by only provider-to-customer or sibling-to-sibling edges.
- A peer-to-peer edge can be followed by only provider-to-customer or sibling-to-sibling edges.

Subramanian *et al*. (2002) also propose a heuristic algorithm based on the observation of the Internet from multiple vantage points of BGP routing table, which relies on the rank of the ASes. A rank is assigned to each vertex by applying the reverse pruning algorithm. Although both algorithms have good overall accuracy, the accuracy on the peer-to-peer relationships is found to be quite low [49% of Gao (2001) and 24.6% of Subramanian *et al*. (2002)].

To increase the accuracy of inference, Xia *et al*. (2005) propose a new algorithm, which consists of two major components. One is to filter non-valley-free path, and the other is to infer AS relationships from partial information, which defines three inference rules and one refreshing rule. They argue that their algorithm achieve 96.37% overall accuracy and 91.45% on peer- to-peer relationships.

There is an another approach for inferring AS relationships. Subramanian *et al*. (2002) propose a formulation of this inference problem as a combinatorial optimization problem, called the Type-of-Relationship (ToR) problem. That is,

ToR Problem: Given an undirected graph G with vertex set V and edge set E and a set of paths P, label the edges in E as either −1, 0 or +1 to maximize the number of valid paths in P.

Here, a path is valid if it starts with zero or more customer-provider edges; followed by at most one peer-to-peer edge; followed by zero or more provider-customer edges.

Both Erlbach *et al*. (2002) and Battista *et al*. (2003) prove that ToR problem is NP-hard. Erlbach *et al*. (2002) proposes an approximation algorithm under bounded path length, and (Battista *et al*., 2003) also suggests a heuristic for detecting the AS relationships with a small number of anomalies, respectively. While the former work puts more emphasis on the approximability of the problem, the latter focuses more on the engineering and the experimentation of an effective heuristic approach. While these techniques for inferring AS relationships have yielded extremely few invalid BGP paths, some relationships inferred are incorrect and unrealistic, e.g. well-known global providers appear as customers of small ASes. To get more realistic results, Dimitropoulos *et al*. (2005) generalize the ToR problem as multiobjective optimization problem with node-degree-based corrections (direct edges from adjacent nodes of lower degrees to nodes of higher degrees) to the original objective function of minimizing the number of invalid paths. In this model, tradeoff occurs between i) directing edges along the node degree gradient and ii) percentage of valid path.

6. Conclusions and Future Works

There have been much efforts to improve the accuracy of Internet topology model and include its quantitative/qualitative effects on studies of a variety of network problems. In this study, we listed and classified the body of literature addressing the Internet topology models. The metrics, which characterize the fundamental properties of the Internet, were also divided into five categories and their importance and applications are discussed.

Based on our survey, followings are believed to be the possible topics or directions for future works.

(1) We need to determine the Internet topology metric based on the intended use or application of the network topology model.

There seems to be no dominantly favored topology metric over all applications. What is “right” metric is apt to vary, depending on the intended use of the topology. In some application, as we have seen in Section 3, a certain metric is more important than others. For example, if the purpose is to test the efficiency of routing algorithm, we need the network topology model which resembles the Internet in metrics such as ‘distance distribution’ or ‘expansion’. For the problem of network robustness, a measure of ‘resilience’ is the most important one. Therefore, it might be necessary to determine which metric is good for some intended use or application of the network model, and develop application-specific metric.

(2) We need to develop the application-specific Internet topology model.

This is closely related to the future work 1) mentioned above. As we have seen in the Section 4, it seems to be difficult to develop the network topology model which resembles the internet in every metric. Some model can well capture a certain topological properties over others, and vice versa. Therefore, it is necessary to develop the topology model depending on the intended use or application. For example, if the purpose is to stress test the algorithm, the model should generate instances which are, in some sense, "difficult". Unfortunately, the development of the new network topology model has not been motivated by its intended use, but by newly observed or introduced metric, which is believed to represent the Internet topology property that currently available topology models do not exhibit. We can have several problems. First, this tends to yield a generated topology that matches observations on the chosen metric but looks very dissimilar on others. Second, since the new metrics can be introduced additionally at any time, the model development seems to be an endless task. Third, it is difficult for users to determine the topology models which fit best to their needs since the intended uses of models are not considered at the time of model development.

(3) It is shown that networks having the same power-law degree distributions can have vastly different features. That is, the power-laws alone may not be sufficient in describing the topology in all its complexity. So, several metrics are suggested to represent the global and hierarchical properties of the Internet. However, the chosen metrics are introduced without being tested whether they are "good" in the following sense:

- They should have a direct networking interpretation.
- They should be unique, not shared by distinct topologies.
- They should be "inclusive" in a sense that by reproducing this inclusive metric in a generator we can capture all other important properties of the Internet. Power-law is one of the inclusive metrics to some extent.
- They should be quantitative, making their application objective.

By applying above criteria for newly introduced metrics, we can develop more realistic and applicable Internet topology generator.

For this literature survey we have tried to be reasonably complete; those papers not included were either inadvertently overlooked or considered not to bear directly on the topic of this survey. We apologize to both the readers and the researchers if we have omitted any relevant papers.

References

Aiello, W., Chung, F. and Lu, L. (2000), A Random Graph Model for Massive Graph, *STOC'2000*, Oregon Portland.

Albert, R. and Barabasi, A. (2000), Topology of Complex Networks: Local Events and Universality, *Physical Review Letters*, **85**(24), 5234-5237.

Alderson, D., Doyle, J., Govindan, R. and Willinger, W. (2003), Toward an Optimization-Driven Framework for Designing and Generating Realistic Internet Topology, *ACM SIGCOMM Computer Cmmunication Review*, **33**(1).

Alderson, D. (2004), Optimization-based Approaches to Understanding and Modeling Internet Topology, Tutorial, *INFORMS Telecom Conference.*

Bar, S., Gonen, M. and Wool, A. (2004), Incremental Super-Linear Preferential Internet Topology Model, *Proc. 5th PAM, LNCS 3015*, 53-62.

Bar, S., Gonen, M. and Wool, A. (2005), A Geographic Directed Preferential Internet Topology Model**,** eprint arXiv.cs/0502061.

Barabasi, A. and Albert, R. (1999), Emergence of Scaling in Random Networks, *Science*, **286**, 509-512.

Battista, G. D., Patrignani, M. and Pizzonia, M. (2003), Computing the Types of the Relationships between AS, *IEEE INFOCOM.*

Bu, T. and Towsley, D. (2002), On Distinguishing between Internet Power Law Topology Generator, *IEEE INFOCOM.*

Calvert, K., Zegura, E. and Doar, M. (1997), Modeling Internet Topology, *IEEE Commun. Mag*., 160-163.

Carlson, J. M. and Doyle, J. (1999), Highly Optimized Tolerance : A Mechanism for Power Laws in Designed Systems, *Physics Review E*, **60**(2).

Chang, H., Govindan, R., Jamin, S. and Willinger, W.(2004), Towards Capturing Representative AS-level Internet Topologies, *Computer Networks Journal*, **44**, 737-755

Chen, Q., Chang, H., Govindan, R., Jamin, S., Shenker, S. J. and Willinger, W. (2002), The Origin of Power-Laws in Internet Topologies Revisited, *INFOCOM.*

Chou, H. (2000) A Note on Power-Laws of Internet Topology, *e-printcs. NI/0012019.*

Chung, F. K. R. (1997), Spectral Graph Theory, 92 of Regional Conference Series in Mathematics, *American Mathematical Society*, providence, RI.

Claffy, K. C., Monk, T. E. and Mcrobb, D. (1999), Internet Tomography, *Nature*, http:// www.caida.org/ tools/ measurement/ skitter/.

Dimitropoulos X. et al. (2005), Inferring AS Relationships: Dead End or Lively Beginning?, *4th Workshop on Efficient and Experimental Algorithm.*

Doar, M. (1996), A Better Model for Fenerating Test Networks, *Proceedings of Globecom'96.*

Doar, M. and Leslie, I. (1993), How Bad Is Naive Multicasting Routing?, *Proc. IEEE INFOCOM'93*, 82-89.

Dorogovtsev, S. N., Networks with Given Correlations, http:// arxiv.org/ abs/cond-mat/ 0308336v1.

Erdos, P. and Renyl, A. (1959), On Random Graph 1, *Publicationes, Mathematicae*, 6, 290-297.

Erlbach, T., Hall, A., and Schank, T. (2002), Classifying Customer-Provider Relationships in the Internet, *In Proceedings of the IASTED.*

Fabrikant, A., Koutsoupias, E., and Papadimitriou, C. H. (2002), Heuristically Optimized Trade-Offs : A New Paradigm for Power Laws in the Internet, *ICALP 2002,* LNCS *2380*, 110-122.

Faloutsos, M., Faloutsos, P., and Faloutsos, C. (1999), On Power-Law Relationships of the Internet Topology, *ACM SIGCOMM*, 251-262, Sep 1-3, Cambridge MA.

Floyd, S. and Paxson, V. (2001), Difficulties in Simulating the Internet, *IEEE/ACM Transactions on Networking,* **9**(4), 392-403.

Gaertler, M. and Patrignani, M. (2004), Dynamic Analysis of the Autonomous System Graph, *IPS.*

Gao, L. (2001), On Inferring Autonomous System Relationship in the Internet," *IEEE/ACM Transactions on Networking*, **9** (6), 733-745

Gkantsidis, C., Mihail, M. and Saberi, A. (2003), Conductance and Congestion in Power Law Graph, *ACM SICMETRICS*.

Gkantsidis, C., Mihail, M. and Zegura, E. (2003), Spectral Analysis of Internet Topologies, *IEEE INFOCOM.*

Gkantsidis, C., Mihail, M. and Zegura, E. (2003) The Markov Chain Simulation Method for Generating Connected Power-Law Random Graphs, *Proc. 5th Workshop on Algorithm Engineering and Experiment, SIAM.*

Govindan, R. and Reddy, A. (1997), An analysis of Internet Inter-Domain Topology and Route Stability, *Proc. IEEE INFOCOM'97*, Kobe, Japan.

Guillaume, J. L., Latapy, M. (2005), Relevance of Massively Distributed Explorations of the Internet Topology : Simulation Results, *IEEE INFOCOM.*

Internet Routing Registries, http:// www.irr.net/.

IRR, List of Routing Registries, http://www.irr.net/docs/list.html.

Jaiswal, S., Rosenberg, A. L. and Towsley, D. (2004), Comparing the Structure of Power-Law Graphs and Internet AS graph, *Proceedings of the 12th IEEE ICNP*.

Jin, C., Chen, Q. and Jamin, S. (2000), Inet: Internet Topology Generator, *Technical Report UM CSE-TR-433-00.*

Li, L., Alderson, D., Willinger, W. and Doyle, J.(2004), A First-Principles Approach to Understanding the Internets Router-Level Topology, *ACM SIGCOMM'04*, Portland.

Mahadevan, P. *et al*. (2005), Lessons from Three Views of the Internet Topology," *CAIDA-TR-2005-02*.

Mahadevan, P. *et al*. (2005), Impact of Degree Correlations on Topology Generators, *SIGCOMM.*

Mahadevan, P. *et a*l. (2006), The Internet AS Level Topology : Three Dara Sources and One Definitive Metric, *ACM SIGCOMM Computer Communication Review*, **36**(1).

Magoni, D. and Pansiot, J. J. (2001), Influences of Network Topology on Protocol Simulation, *IEEE ICN'01*, 762-770.

Magoni, D. and Pansiot, J. J. (2002), Internet Topology Modeler Based on Map Sampling, *Proc. of the 7th ISCS.*

Median, A., Lakhina, A., Matta, I. and Byers, J. (2001), Brite: An Approach to Universal Topology Generation.

Medina, A., Matta, I. and Byers, J. (2000), On the Origin of Power Laws in Internet Topologies, *ACM SIGCOMM Computer Communication Review*, **30**(2), 18-34.

Mihail, M., Gkantsidis, C., Saberi A. and Zegura, E. (2002), On The Semantics of Internet Topologies, *TR-GIT-CC-02-07,* College of Computing, GIT.

Merit Network, Internet Routing Registry, http://www.irr.net.

Newman, M. E. J. (2002), Assortative Mixing in Network, *Physical Review Letters*, **89**(20), 208701.

Palmer, R. and Steffan, J. G. (2000), Generating Network Topologies That Obey Power Laws, *GLOBECOMM 2000*, 434-438

Park, K. and Lee, H. (2001), On The Effectiveness of Route-Based Packet Filtering for Distributed Dos Attack Prevention in Power-Law Internets, *ACM SIGCOMM*, San Diego.

Park, S. T., Pennock, D. M. and Giles, C. L. (2004), Comparing Static and Dynamic Measurements and Models of The Internet's AS Topology, *IEEE INFOCOM.*

Paxon, V. and Floyd, S. (1997), Why We Don't Know How To Simulate The Internet, *Proceedings of The 1997 Winter Simulation Conference*, 1037-1044.

Phillips, G., Shenken, S. and Tangmunarunkit, H. (1999), Scaling of Multicast Tree : Comments On The Chuang-Sirbu Scaling Law, *ACM SIGCOMM.*

Sagie, G. and Wool, A. (2003), A Clustering Approach For Exploring The Internet Structure, *TR EES2003-7*, Dept of EE, Tel Aviv Univ.

Satorras, R., Vazquez, A. and Vespignani, A. (2001), Dynamical and Correlation Properties of The Internet, *Physical Review Letters,* **87**(25), 258701.

Siganos, G., Faloutsos, M., Faloutsos, P. and Faloutsos, C. (2003), Power-Laws and The AS-Level Internet Ttopology, *ACM/IEEE Trans. on Networking*, **11**(4) 514-524.

Subramanian, L., Agarwal, S., Rexford J. and Katz, R. H. (2002), Characterizing The Internet Hierarchy From Multiple Vantage Points, *IEEE INFOCOM.*

Tangmunarunkit, H., Govindan, R., Jamin, S., Shenker, S. and Willinger, W. (2002), Network Topology Generators : Degree-Based Vs. Structural, *ACM SIGCOMM.*

Tauro, L., Palmer, C., Siganos, G. and Faloutsos, M. (2001), A Simple Conceptual Model For The Internet Topology, *Global Internet*, San Antonio.

Traceroute.org, Public Route Server and Looking Glass Site List, http://www.traceroute.org/.Traceroute, http://www.traceroute.org/.

University of Oregon RouteViews Project, http://www.routeviews.org/.

Vazquez, A., Pastor-Satorras, R. and Vespignani, A. (2002), Large-Scale Topological and Dynamical Properties of Internet, *Phys. Rev. E*, **65**, 066130.

Vukadinovic, D., Huang, P. and Erlebach, T. (2001), Spectral Analysis of Internet Topology, *Technical Report TIK-NR*, **118**, ETH.

Watts, J. and Strogatz, S. H. (1998), Collective Dynamics of 'Small-World' Networks, *Nature*, **393**, 440-442.

Waxman, M. (1988), Routing of Multipoint Connections, *IEEE JSAC*, 1617-1622.
Willinger, W. (2004), Internet Topology : Measurements, Models, and Model Validation, AT&T Labs-Research.
Winick, J. and Jamin, S. (2002), Inet-3.0 : Internet Topology Generator, *Technical Report UM-CSE-TR-456-02*, University of Michigan.
Xia, J. and Gao, L. (2005), On The Evaluation of As Relationship Inferences, *Technical Report*, Dept. of Electrical and Computer Engineering, Univ. of Massachusetts.
Yook, S. H., Jeong, H. and Barabasi, A. L. (2001), Modeling The Internet's Large Scale Topology, Dept. of Physics, Univ. of Norte Dame.
Zegura, W., Calvert, K. L. and Donahoo, M. J. (1997), A Quantitative Comparison of Graph-Based Models For Internet Topology, *IEEE/ACM Trans*. On Networking, **5**(6), 770-783
Zegura, E. W. (2002), Modeling Internet Topology, *2002 IPAM Tutorial Workshop*.
Zhou, S. and Mondragon, R. J. (2004), Accurately Modeling The Internet Topology, *Physical Review E*, **70**, 066108, http://arxiv.org/ abs/ cs.NI/ 0402011.
Zhou, S. and Mondragon, R. J. (2004), The Rich-Club Phenomenon In The Internet Topology, *IEEE Communications Letters*, **8**(3), 180-182.