

# MIMO Takagi-Sugeno 퍼지 모델을 위한 모델참조 적응 퍼지 제어기의 설계

## A model reference adaptive fuzzy control for MIMO Takagi-Sugeno fuzzy model

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### Abstract

In this paper, a direct model reference adaptive fuzzy control (MRAFC) scheme is developed for the plant model whose structure is represented by the MIMO Takagi-Sugeno fuzzy model. The MRAFC scheme is proposed to provide asymptotic tracking of a reference signal for the systems with uncertain or slowly time-varying parameters. The developed control law and adaptive law guarantee that all signals in the closed-loop system are bounded. In addition, the plant state tracks the state of the reference model asymptotically with time for any bounded reference input signal.

Key Words : Adaptive fuzzy control, TS fuzzy model. Model reference adaptive control.

### 1. Introduction

In some control tasks, the controlled system has uncertain or time-varying parameters. Unless parameter uncertainty is gradually reduced on-line by an appropriate adaptation or estimation mechanism, it may cause inaccuracy or instability in the control system. The adaptive control has been a main theme in control engineering, and related researches are expected to continue. Although sufficient systematic theories exist for the adaptive control for linear systems, existing adaptive techniques can treat a special class of nonlinear systems having linearly parameterizable dynamics.

In recent years, in order to deal with the uncertainties of nonlinear systems in the fuzzy control system literature, a lot of effort has been put to adaptive fuzzy control system such as neural network based approaches [3-4], and the TS model based approaches [5-7]. The main advantages of adaptive fuzzy control over non-adaptive fuzzy control are: (1) better performance is usually achieved because the adaptive fuzzy controller can adjust itself to the changing environment, and (2) less information about the plant is required because the adaptation law can help to learn the dynamics of the plant during real-time operation. However, these approaches still have some problems. The adaptive control

scheme proposed by Wang [3] guarantees the uniform boundedness of all signals of the control system but it is applicable only to single-input single-output system.

In many applications, the structure of the model of the plant may be known, but its parameters may be unknown and/or change with time. Since the Takagi-Sugeno fuzzy model[8] was proposed, significant researches on the approximation of the nonlinear systems with the TS model have been reported, and adaptation schemes based on the parameter estimation of the TS model have been proposed to cope with the uncertain parameters and modeling inaccuracy.[6][11]

In this paper, a direct Model Reference Adaptive Fuzzy Control (MRAFC) scheme is proposed to provide asymptotic tracking of a reference signal for the systems having uncertain or slowly time-varying parameters. This paper presents the design and analysis of on-line parameter adaptation for the plant model whose structure is represented by the Takagi-Sugeno model. The adaptation law adjusts the controller parameters on-line so that the plant output tracks the reference model output. The developed control law and adaptive law guarantee the boundedness of all signals in the closed-loop system. In addition, the plant state tracks the state of the reference model asymptotically with time for any bounded reference input signal.

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## 2. Takagi-Sugeno Model Based Fuzzy Control

Consider the continuous-time nonlinear system described by the Takagi-Sugeno fuzzy model.[8] The  $i$ th rule of continuous-time TS model is of the following form:

$$R^i: \text{If } x_1(t) \text{ is } M_1^i \text{ and } \dots \text{ and } x_n(t) \text{ is } M_n^i. \\ \text{then } \dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t) \quad (1)$$

where  $\mathbf{x}^T(t) = [x_1(t), \dots, x_n(t)]$ ,  
and  $\mathbf{u}^T(t) = [u_1(t), \dots, u_m(t)]$ .

Given a pair of input  $(\mathbf{x}(t), \mathbf{u}(t))$ , the final output of the fuzzy system is inferred as follows:

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^l \omega_i(\mathbf{x}(t)) \{ \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t) \}}{\sum_{i=1}^l \omega_i(\mathbf{x}(t))} \quad (2)$$

where  $\omega_i(\mathbf{x}(t)) = \prod_{j=1}^n M_j^i(x_j(t))$ , and  $M_j^i(x_j(t))$  is the grade of membership of  $x_j(t)$  in  $M_j^i$ .

Utilizing the concept of parallel distributed compensation (PDC)[9], we can obtain the fuzzy controller stabilizing the fuzzy system (2). The premise part of the PDC controller is constructed by sharing the same fuzzy sets with the fuzzy model (2), that is, the PDC controller is of the following form:

$$R^i: \text{If } x_1(t) \text{ is } M_1^i \text{ and } \dots \text{ and } x_n(t) \text{ is } M_n^i. \\ \text{then } \mathbf{u}(t) = -\mathbf{K}_i \mathbf{x}(t) \quad (3)$$

where  $\mathbf{x}^T(t) = [x_1(t) \ x_2(t) \ \dots \ x_n(t)]$  and  $i = 1, \dots, l$ .

Given a state feedback  $\mathbf{x}(t)$ , the final output of the fuzzy PDC controller (3) is inferred as follows:

$$\mathbf{u}(t) = \frac{\sum_{i=1}^l \omega_i(\mathbf{x}(t)) \mathbf{K}_i \mathbf{x}(t)}{\sum_{i=1}^l \omega_i(\mathbf{x}(t))} \quad (4)$$

where  $\omega_i(\mathbf{x}(t)) = \prod_{j=1}^n M_j^i(x_j(t))$ . By substituting the controller (4) into the model (2), we can construct the closed-loop fuzzy control system as following:

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(t) \omega_j(t) \{ \mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j \} \mathbf{x}(t)}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(t) \omega_j(t)} \quad (5)$$

A sufficient condition for ensuring the stability of the closed-loop fuzzy system (5) is given in Theorem 1, which was derived in [9].

**Theorem 1.** The equilibrium of a fuzzy control system (5) is asymptotically stable in the large if there exists a common positive definite matrix  $\mathbf{P}$  such that

$$\mathbf{G}_{ij}^T \mathbf{P} + \mathbf{P} \mathbf{G}_{ij} = -\mathbf{Q}_{ij} \quad (6)$$

for all  $i, j = 1, 2, \dots, l$  where  $\mathbf{G}_{ij} = \mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j$  and  $\mathbf{Q}_{ij}$  is a positive definite matrix.

The design problem of model based fuzzy control is to select  $\mathbf{K}_j (j = 1, \dots, l)$  which satisfy the stability condition (6). In [10], the common  $\mathbf{P}$  problem was solved efficiently via convex optimization techniques for LMI's (Linear Matrix Inequality). However, the fuzzy control (4) does not guarantee the stability of system in the presence of parameter uncertainty. Moreover, the design of the control parameters is not possible for the systems whose parameters are unknown. To overcome these drawbacks, in this research, an adaptive control scheme is developed for the plant models whose structures are known but parameters unknown.

## 3. Direct Model Reference Adaptive Fuzzy Control

In this section, a direct Model Reference Adaptive Fuzzy Control (MRAFC) scheme for TS model is developed. Consider again the nonlinear system represented by the TS model (1) or (2), where state  $\mathbf{x} \in R^n$  is available for measurement,  $\mathbf{A}_i \in R^{n \times n}$  and  $\mathbf{B}_i \in R^{n \times q}$  ( $i = 1, 2, \dots, l$ ) are unknown constant matrices and  $(\mathbf{A}_i, \mathbf{B}_i)$  are controllable. The control objective is to choose the input vector  $\mathbf{u} \in R^q$  such that all signals in the closed-loop plant are bounded and the plant state  $\mathbf{x}$  follows the state  $\mathbf{x}_m \in R^n$  of a reference model specified by the system

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}(t)) \mu_j(\mathbf{x}(t)) \{ (\mathbf{A}_m)_{ij} \mathbf{x}_m + (\mathbf{B}_m)_{ij} \mathbf{r} \}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}(t)) \mu_j(\mathbf{x}(t))} \quad (7)$$

where  $(\mathbf{A}_m)_{ij} \in R^{n \times n} (i, j = 1, \dots, l)$  satisfy the stability condition of fuzzy system given in Theorem 1,  $(\mathbf{B}_m)_{ij} \in R^{n \times q}$ , and  $\mathbf{r} = R^q$  is a bounded reference input vector. The reference model and input  $\mathbf{r}$  are chosen so that  $\mathbf{x}_m$  represents a desired trajectory that  $\mathbf{x}$  has to follow.

### 3.1 Control Law

If the matrices  $A_i$  and  $B_i$  were known, we could apply the control law

$$u(t) = - \frac{\sum_{j=1}^l \mu_j(x(t))(-K_j^*x(t) + L_j^*r(t))}{\sum_{j=1}^l \mu_j(x(t))} \quad (8)$$

where  $\mu_j(x) = \omega_j(x)$ , and obtain the closed-loop plant

$$\dot{x}(t) = \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x(t))\mu_j(x(t))\{(A_i - B_iK_j^*)x + B_iL_j^*r\}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x(t))\mu_j(x(t))} \quad (9)$$

Hence, if  $K_j^* \in R^{q \times n}$  and  $L_j^* \in R^{q \times q}$  are chosen to satisfy the algebraic equations

$$A_i - B_iK_j^* = (A_m)_{ij}, \quad B_iL_j^* = (B_m)_{ij}, \quad (10)$$

then the transfer matrix of the closed-loop plant is the same as that of the reference model and  $x(t) \rightarrow x_m(t)$  exponentially fast for any bounded reference input signal  $r(t)$ . We should note that given the matrices  $A_i, B_i, (A_m)_{ij}, (B_m)_{ij}$ , no  $K_j^*, L_j^*$  may exist to satisfy the matching condition (10) indicating that the control law (8) may not have enough structural flexibility to meet the control objective. In some cases, if the structure of  $A_i, B_i$  is known,  $(A_m)_{ij}, (B_m)_{ij}$  may be designed so that (10) has a solution for  $K_j^*, L_j^*$ .

Let us assume that  $K_j^*, L_j^*$  in (10) exist, i.e., that there is sufficient structural flexibility to meet the control objective, and propose the control law

$$u(t) = - \frac{\sum_{j=1}^l \mu_j(x(t))(-K_j(t)x(t) + L_j(t)r(t))}{\sum_{j=1}^l \mu_j(x(t))} \quad (11)$$

where  $K_j(t), L_j(t)$  are the estimates of  $K_j^*, L_j^*$ , respectively, to be generated by an appropriate adaptive law.

### 3.2 Adaptive Law

By adding and subtracting the desired input term,

$$\sum_{j=1}^l \mu_j(x) \{-B_i(K_j^*x - L_j^*r)\} / \sum_{j=1}^l \mu_j(x). \quad (12)$$

equation (2) and using (10), we obtain

$$\dot{x} = \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)(A_m)_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)} x + \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)(B_m)_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)} r$$

$$+ \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)B_i(K_j^*x - L_j^*r + u)}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)} \quad (13)$$

Furthermore, by adding and subtracting the estimated input term multiplied by  $\sum_{i=1}^l \omega_i(x)B_i / \sum_{i=1}^l \omega_i(x)$ , that is,

$$\left. \frac{\sum_{i=1}^l \omega_i(x)B_i}{\sum_{i=1}^l \omega_i(x)} \left\{ \frac{\sum_{j=1}^l \mu_j(x)(K_j(t)x - L_j(t)r)}{\sum_{j=1}^l \mu_j(x)} + u \right\} \right\} \quad (14)$$

in the reference model (7), we obtain

$$\begin{aligned} \dot{\hat{x}}_m = & \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)(A_m)_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)} \hat{x}_m + \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)(B_m)_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)} r \\ & + \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)B_i(K_j(t)x - L_j(t)r + u)}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)} \end{aligned} \quad (15)$$

By using the reference model (15), we can express (13) in terms of the tracking error defined as  $e \equiv x - x_m$ , i.e.,

$$\begin{aligned} \dot{e} = & \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)(A_m)_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)} e \\ & + \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)B_i(-\tilde{K}_j x + \tilde{L}_j r)}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)}, \end{aligned} \quad (16)$$

where  $\tilde{K}_j = K_j(t) - K_j^*$  and  $\tilde{L}_j = L_j(t) - L_j^*$ .

In the tracking error dynamic equation (16),  $B_i$  is unknown. We assume that  $L_j^*$  is either positive definite or negative definite, and define  $\Gamma_j^{-1} = L_j^* \text{sgn}(l_j)$ , where  $l_j = 1$  if  $L_j^*$  is positive definite and  $l_j = -1$  if  $L_j^*$  is negative definite. Then  $B_i = (B_m)_{ij} L_j^{*-1}$  and (16) becomes

$$\begin{aligned} \dot{e} = & \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)(A_m)_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)} e \\ & + \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)(B_m)_{ij} L_j^{*-1}(-\tilde{K}_j x + \tilde{L}_j r)}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x)\mu_j(x)}. \end{aligned} \quad (17)$$

Now, by using the tracking error dynamics (17), we

derive the adaptive law for updating the control parameters  $\mathbf{K}_j(t)$  and  $\mathbf{L}_j(t)$  so that the closed-loop plant model (13) follows the reference model (7). We assume that the adaptive law has the general structure

$$\dot{\mathbf{K}}_j(t) = \mathbf{F}_j(\mathbf{x}, \mathbf{x}_m, \mathbf{e}, \mathbf{r}), \quad \dot{\mathbf{L}}_j(t) = \mathbf{G}_j(\mathbf{x}, \mathbf{x}_m, \mathbf{e}, \mathbf{r}). \quad (18)$$

where  $\mathbf{F}_j$  and  $\mathbf{G}_j$  ( $j=1, \dots, l$ ) are functions of known signals that are to be chosen so that the equilibrium

$$\mathbf{K}_{je} = \mathbf{K}_j^*, \quad \mathbf{L}_{je} = \mathbf{L}_j^*, \quad \mathbf{e}_e = 0 \quad (19)$$

of (17) and (18) has some desired stability properties.

We propose the following Lyapunov function candidate

$$V(\mathbf{e}, \tilde{\mathbf{K}}_j, \tilde{\mathbf{L}}_j) = \mathbf{e}^T \mathbf{P} \mathbf{e} + \sum_{j=1}^l \text{tr}(\tilde{\mathbf{K}}_j^T \Gamma_j \tilde{\mathbf{K}}_j + \tilde{\mathbf{L}}_j^T \Gamma_j \tilde{\mathbf{L}}_j) \quad (20)$$

where  $\mathbf{P} = \mathbf{P}^T > 0$  is a common positive definite matrix of the Lyapunov equations  $(\mathbf{A}_m)_{ij}^T \mathbf{P} + \mathbf{P} (\mathbf{A}_m)_{ij} < -\mathbf{Q}_{ij}$  for all  $\mathbf{Q}_{ij} = \mathbf{Q}_{ij}^T > 0$  ( $i, j=1, \dots, l$ ) whose existence is guaranteed by the stability assumption for  $\mathbf{A}_m$ . Then, after some straightforward mathematical manipulations, we obtain the time derivative  $\dot{V}$  of  $V$  along the trajectory of (17) and (18) as

$$\begin{aligned} \dot{V} = & -e^T \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x}) \mathbf{Q}_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x})} e \\ & + 2 \text{tr} \left\{ \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x}) \tilde{\mathbf{K}}_j^T \Gamma_j (\mathbf{B}_m)_{ij}^T \text{sgn}(l_j)}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x})} \mathbf{P} \mathbf{e} \mathbf{x}^T + \sum_{j=1}^l \tilde{\mathbf{K}}_j^T \Gamma_j \dot{\tilde{\mathbf{K}}}_j \right\} \\ & + 2 \text{tr} \left\{ \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x}) \tilde{\mathbf{L}}_j^T \Gamma_j (\mathbf{B}_m)_{ij}^T \text{sgn}(l_j)}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x})} \mathbf{P} \mathbf{e} \mathbf{r}^T + \sum_{j=1}^l \tilde{\mathbf{L}}_j^T \Gamma_j \dot{\tilde{\mathbf{L}}}_j \right\} \end{aligned} \quad (21)$$

In the last two terms of (21), if we let

$$\sum_{j=1}^l \tilde{\mathbf{K}}_j^T \Gamma_j \dot{\tilde{\mathbf{K}}}_j = \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x}) \tilde{\mathbf{K}}_j^T \Gamma_j (\mathbf{B}_m)_{ij}^T \text{sgn}(l_j)}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x})} \mathbf{P} \mathbf{e} \mathbf{x}^T, \quad (22a)$$

$$\sum_{j=1}^l \tilde{\mathbf{L}}_j^T \Gamma_j \dot{\tilde{\mathbf{L}}}_j = \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x}) \tilde{\mathbf{L}}_j^T \Gamma_j (\mathbf{B}_m)_{ij}^T \text{sgn}(l_j)}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x})} \mathbf{P} \mathbf{e} \mathbf{r}^T, \quad (22b)$$

we can make  $\dot{V}$  to be negative, i.e.,

$$\dot{V} = -e^T \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x}) \mathbf{Q}_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(\mathbf{x}) \mu_j(\mathbf{x})} e \leq 0. \quad (23)$$

Hence, the obvious choice for adaptive law to make  $\dot{V}$  negative is

$$\dot{\mathbf{K}}_j(t) = \dot{\tilde{\mathbf{K}}}_j = \left\{ \frac{\sum_{i=1}^l \omega_i(\mathbf{x}) (\mathbf{B}_m)_{ij}^T}{\sum_{j=1}^l \omega_j(\mathbf{x})} \right\} \left\{ \frac{\mu_j(\mathbf{x})}{\sum_{j=1}^l \mu_j(\mathbf{x})} \right\} \text{sgn}(l_j) \mathbf{P} \mathbf{e} \mathbf{x}^T, \quad (24a)$$

$$\dot{\mathbf{L}}_j(t) = \dot{\tilde{\mathbf{L}}}_j = \left\{ \frac{\sum_{i=1}^l \omega_i(\mathbf{x}) (\mathbf{B}_m)_{ij}^T}{\sum_{j=1}^l \omega_j(\mathbf{x})} \right\} \left\{ \frac{\mu_j(\mathbf{x})}{\sum_{j=1}^l \mu_j(\mathbf{x})} \right\} \text{sgn}(l_j) \mathbf{P} \mathbf{e} \mathbf{r}^T, \quad (24b)$$

#### 4. Implementation and Analysis

Figure 1 illustrates the configuration of the model reference adaptive fuzzy control system. The reference model is used to specify the ideal response that the fuzzy control system should follow.

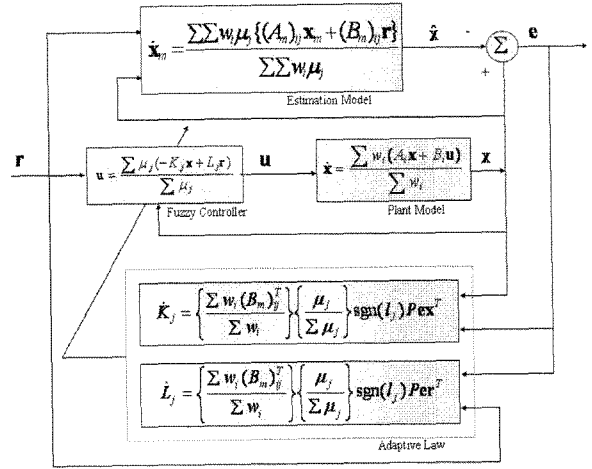


Fig. 1 The overall control scheme of the MRAFC

The plant is assumed to contain unknown parameters, but its structure is known. The fuzzy controller is constructed from fuzzy systems whose parameters are adjustable. The adaptation law adjusts the control parameters  $\mathbf{K}(t)$  and  $\mathbf{L}(t)$ , on-line such that the state  $\mathbf{x}$  of plant tracks the state  $\mathbf{x}_m$  of reference model, which allows plant output to follow the reference model output.

Using arguments previously discussed, we establish the following theorem which shows the properties of the MRAFC. The control law (11) together with the adaptive law (24) guarantees that all signals in the closed-loop system are bounded. In addition, the plant state  $\mathbf{x}$  tracks the state of the reference model  $\mathbf{x}_m$  asymptotically with

time for any bounded reference input signal  $r$ .

**Theorem 2. Stability of the MRAFC**

Consider the plant model (2) and the reference model (7) with the control law (11) and adaptive law (13). Assume that the reference input  $r$  and the state  $x_m$  of the reference model are uniformly bounded. Then the control law (11) and the adaptive law (24) guarantee that  $\tilde{K}(t), \tilde{L}(t), e(t)$  are bounded and  $e(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

**Proof.** From (20) and (23), it directly follows that  $V$  is a Lyapunov function for the system (17), (18), which implies that the equilibrium given by (19) is uniformly stable, which, in turn, implies that the trajectory  $\tilde{K}(t), \tilde{L}(t), e(t)$  are bounded for all  $t$ . Because  $e = x - x_m$  and  $x_m \in \mathcal{L}_\infty$  we have that  $x \in \mathcal{L}_\infty$ . From (11) and  $r \in \mathcal{L}_\infty$ , we also have that  $u \in \mathcal{L}_\infty$ ; therefore, all signals in the closed-loop are bounded.

Now, let us show that  $e \in \mathcal{L}_2$ . From (20) and (23), we conclude that  $V$  has a limit, i.e.,

$$\lim_{t \rightarrow \infty} V(e(t), \tilde{K}_j(t), \tilde{L}_j(t)) = V_\infty \quad (25)$$

because  $V$  is bounded from below and it is non-increasing with time. From (23) and (25), it follows that

$$\int_0^\infty e^T \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x) \mu_j(x) Q_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x) \mu_j(x)} e d\tau = - \int_0^\infty \dot{V} d\tau = V_0 - V_\infty, \quad (26)$$

where  $V(0) = V(e(0), \tilde{K}_j(0), \tilde{L}_j(0))$ . On the other hand, from  $0 \leq \omega_i(x(t)) \leq 1$ ,  $0 \leq \mu_j(x(t)) \leq 1$ , and  $\lambda_{\min}(Q_{ij}) \|e\|^2 \leq e^T Q_{ij} e \leq \lambda_{\max}(Q_{ij}) \|e\|^2$ , we have

$$\{\lambda_{\min}(Q_{ij})\}_{\min} \|e\|^2 \leq e^T \left( \frac{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x) \mu_j(x) Q_{ij}}{\sum_{i=1}^l \sum_{j=1}^l \omega_i(x) \mu_j(x)} \right) e \leq (27)$$

$$\{\lambda_{\max}(Q_{ij})\}_{\max} \|e\|^2$$

where  $\{\lambda_{\min}(Q_{ij})\}_{\min} = \min\{\lambda_{\min}(Q_{11}), \dots, \lambda_{\min}(Q_{ll})\}$ ,  $\{\lambda_{\max}(Q_{ij})\}_{\max} = \max\{\lambda_{\max}(Q_{11}), \dots, \lambda_{\max}(Q_{ij})\}$ .

After inserting (27) into (26), and straightforward manipulation, we have

$$\frac{V_0 - V_\infty}{\{\lambda_{\min}(Q_{ij})\}_{\min}} \int_0^\infty \|e\|^2 d\tau \leq \frac{V_0 - V_\infty}{\{\lambda_{\max}(Q_{ij})\}_{\max}} \quad (28)$$

which implies that  $e \in \mathcal{L}_2$ . Because  $e, \tilde{K}_j, \tilde{L}_j, r \in \mathcal{L}_\infty$ , it follows from (17) that  $\dot{e} \in \mathcal{L}_\infty$ , which, together with  $e \in \mathcal{L}_2$ , implies that  $x(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

## 5. Conclusions

In this paper, we have developed a direct model reference adaptive control scheme for the Takagi-Sugeno model whose structure is assumed to be known but the parameters unknown. We have constructed the control system by utilizing the PDC as a fuzzy compensator where the estimated control parameters were used instead of the desired design parameters. The adaptation law estimating the control parameters constructing the desired reference model was developed via Lyapunov function. The adaptation law adjusts the controller parameters on-line so that the plant output tracks the reference model output. The developed adaptive law guarantees the boundedness of all signals in the closed-loop system and ensures that the plant state tracks the state of the reference model asymptotically with time for any bounded reference input signal.

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