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CHARACTERIZATIONS OF REAL HYPERSURFACES OF COMPLEX SPACE FORMS IN TERMS OF RICCI OPERATORS

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Abstract. We prove that a real hypersurface M in a complex space form $M_n(c)$, $c \neq 0$, whose Ricci operator and structure tensor commute each other on the holomorphic distribution and the Ricci operator is η-parallel, is a Hopf hypersurface. We also give a characterization of this hypersurface.

0. Introduction

A complex n-dimensional Kaeherian manifold of constant holomorphic sectional curvature c is called a *complex space form*, which is denoted by $M_n(c)$. A complete and simply connected complex space form consists of a complex projective space $P_n(\mathbb{C})$, a complex Euclidean space $\mathbb{C}^n$ or a complex hyperbolic space $H_n(\mathbb{C})$, according to $c > 0$, $c = 0$ or $c < 0$.

R. Takagi ([9]) classified all homogeneous real hypersurfaces in $P_n(\mathbb{C})$ into six model spaces A_1, A_2, B, C, D and E (see also [10]). J. Berndt ([2]) has completed the classification of homogeneous real hypersurfaces with principal structure vector fields in $H_n(\mathbb{C})$, which are divided into the model spaces A_0, A_1, A_2 and B. A real hypersurface of type A_1 or A_2 in $P_n(\mathbb{C})$ or that of A_0, A_1 or A_2 in $H_n(\mathbb{C})$ is said to be *of type A* for simplicity.

We shall denote the induced almost contact metric structure of the real hypersurface M in $M_n(c)$ by $(\phi, <,>, \xi, \eta)$. The Ricci operator of M will be denoted by S, and the shape operator or the second fundamental tensor field of M by A. If the structure vector field ξ is principal, then M is called a *Hopf hypersurface*. The *holomorphic distribution* T_0 of a real hypersurface M in $M_n(c)$ is defined by

$$T_0(p) = \{X \in T_p(M) \mid \ < X, \xi >_p = 0\},$$

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where $T_p(M)$ is the tangent space of M at $p \in M$. The Ricci operator S is said to be *η-parallel* if

$$< (\nabla_X S)Y, Z >= 0 \tag{0.1}$$

for any vector fields X, Y and Z in T_0.

Many authors have occupied themselves with the study of geometrical properties of real hypersurfaces with η-parallel Ricci operators (see [1], [3], [4], [5], [6], [7], [8] and [9]). Recently, I.-B. Kim, K. H. Kim and the present author studied real hypersurfaces in $M_n(c)$ with certain conditions related to the Ricci operator and the structure tensor field ϕ in [3]. In [4], I.-B. Kim, H. J. Park and the present author gave a characterization of the real hypersurface with a special η-parallel Ricci operators. For the conditions on the η-parallel Ricci operator, Kimura and Maeda ([5]) and Suh ([8]) proved the following.

Theorem A. *Theorem A* ([5], [8]) *Let M be a real hypersurface in a complex space form $M_n(c)$, $c \neq 0$. Then the Ricci operator of M is η-parallel and the structure vector field ξ is principal if and only if M is locally congruent to one of the model spaces of type A or type B.*

The purpose of this paper is to improve the results in the previous paper [4] and characterize the real hypersurfaces with η-parallel Ricci operator. Namely, we shall prove the followings.

Theorem 1. *Let M be a real hypersurface with η-parallel Ricci operator in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. If M satisfies*

$$< (S\phi - \phi S)X, Y >= 0, \tag{0.2}$$

for any X and Y in T_0, then M is a Hopf hypersurface.

Theorem 2. *Let M be a real hypersurface with η-parallel Ricci operator in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. If M satisfies* (0.2)*, then M is locally congruent to one of the model spaces of type A or type B.*

1. Preliminaries

Let M be a real hypersurface immersed in a complex space form $(M_n(c), <,>, J)$ of constant holomorphic sectional curvature c, and let N be a unit normal vector field on an open neighborhood in M. For a local tangent vector field X on the neighborhood, the images of X and N under the almost complex structure J of $M_n(c)$ can be expressed by

$$JX = \phi X + \eta(X)N, \qquad JN = -\xi,$$

where ϕ defines a linear transformation on the tangent space $T_p(M)$ of M at any point $p \in M$, and η and ξ denote a 1-form and a unit tangent vector field on the neighborhood respectively. Then, denoting the Riemannian metric on M induced from the metric on $M_n(c)$ by the same symbol $<,>$, it is easy to see that

$$< \phi X, Y > + < \phi Y, X >= 0, \qquad < \xi, X >= \eta(X)$$

for any tangent vector fields X and Y on M. The collection (ϕ, $<,>$, ξ, η) is called an *almost contact metric structure* on M, and satisfies

$$\begin{aligned}(1.1)\qquad &\phi^2X = -X + \eta(X)\xi, \qquad \phi\xi = 0, \qquad \eta(\phi X) = 0, \qquad \eta(\xi) = 1,\\ &< \phi X, \phi Y > = < X, Y > -\eta(X)\eta(Y).\end{aligned}$$

Let ∇ be the Riemannian connection with respect to the metric $<,>$ on M, and A be the shape operator in the direction of N on M. Then we have

$$(1.2)\qquad \nabla_X\xi = \phi AX, \qquad (\nabla_X\phi)Y = \eta(Y)AX- < AX, Y > \xi.$$

Since the ambient space is of constant holomorphic sectional curvature c, the equations of Gauss and Codazzi are given by

(1.3)

$$\begin{aligned}R(X,Y)Z =&\frac{c}{4}\{< Y,Z > X- < X,Z > Y+ < \phi Y,Z > \phi X- < \phi X,Z > \phi Y\\ &- 2 < \phi X,Y > \phi Z\}+ < AY,Z > AX- < AX,Z > AY,\end{aligned}$$

$$(1.4)\qquad (\nabla_XA)Y - (\nabla_YA)X = \frac{c}{4}\{\eta(X)\phi Y - \eta(Y)\phi X - 2 < \phi X, Y > \xi\}$$

for any tangent vector fields X, Y and Z on M, where R is the Riemannian curvature tensor field of M. Then it is easily seen from (1.3) that the Ricci operator S of M is expressed by

$$(1.5)\qquad SX = \frac{c}{4}\{(2n+1)X - 3\eta(X)\xi\} + mAX - A^2X,$$

where $m = \mathrm{trace}A$ is the mean curvature of M, and the covariant derivative of (1.5) is given by

$$\begin{aligned}(1.6)\qquad (\nabla_XS)Y =& -\frac{3c}{4}\{< \phi AX, Y > \xi + \eta(Y)\phi AX\} + (Xm)AY\\ &+ m(\nabla_XA)Y - (\nabla_XA)AY - A(\nabla_XA)Y.\end{aligned}$$

If the vector field $\phi\nabla_\xi\xi$ does not vanish, that is, the length β of $\phi\nabla_\xi\xi$ is not equal to zero, then it is easily seen from (1.1) and (1.2) that

$$(1.7)\qquad A\xi = \alpha\xi + \beta U,$$

where $\alpha = < A\xi, \xi >$ and $U = -\frac{1}{\beta}\phi\nabla_\xi\xi$. Therefore U is a unit tangent vector field on M and $U \in T_0$. If the vector field U can not be defined, then we may consider $\beta = 0$ identically. Therefore $A\xi$ is always expressed as in (1.7).

2. η-parallel Ricci operators

In this section we assume that the open subset

$$\mathcal{U} = \{p \in M \mid \beta(p) \neq 0\}$$

is not empty. Then, in the previous paper [4], we have proved the followings.

Lemma 2.1. ([4]) *Let M be a real hypersurface with the η-parallel Ricci operator S in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. If it satisfies (0.2), then we have*

$$m = \text{trace}A = \alpha + \gamma, \tag{2.1}$$

$$AU = \beta\xi + \gamma U, \tag{2.2}$$

on $\mathcal{U}$, where we have put $\gamma = < AU, U >$.

It follows from (1.1), (1.5), (1.7) and (2.2) that

$$\begin{aligned} S\xi &= (\frac{n-1}{2}c + \alpha\gamma - \beta^2)\xi, \\ SU &= (\frac{2n+1}{4}c + \alpha\gamma - \beta^2)U, \\ S\phi U &= (\frac{2n+1}{4}c + \alpha\gamma - \beta^2)\phi U. \end{aligned} \tag{2.3}$$

Differentiating the second equation of (2.3) covariantly along any vector field X in T_0, we obtain

$$(\nabla_X S)U = \{(\frac{2n+1}{4}c + \alpha\gamma - \beta^2)I - S\}\nabla_X U + X(\alpha\gamma - \beta^2)U. \tag{2.4}$$

If we take inner product of (2.4) with U and make use of (0.1) and (2.3), we get

$$X(\alpha\gamma - \beta^2) = 0 \qquad \text{for} \qquad X \in T_0. \tag{2.5}$$

We put

$$Q = (\alpha + \gamma)A - A^2 = S - \frac{c}{4}\{(2n+1)I - 3\eta \otimes \xi\}.$$

Then Q is a symmetric endomorphism on the tangent space of M. Since we see from (0.2) and (2.3,1) that $S\phi = \phi S$ on M, we have $Q\phi = \phi Q$ on M. Moreover (2.3) is equivalent to

$$Q\xi = (\alpha\gamma - \beta^2)\xi, \qquad QU = (\alpha\gamma - \beta^2)U, \qquad Q\phi U = (\alpha\gamma - \beta^2)\phi U. \tag{2.6}$$

Let k_r be an eigenvalue of Q, and $Q(k_r)$ be the eigenspace of Q associated with k_r, where $1 \leq r \leq 2n-1$. If λ is a principal curvature of M, then there is an eigenvalue k_r of Q such that $k_r = (\alpha + \gamma)\lambda - \lambda^2$. From this quadratic, we see that there are at most two distinct principal curvatures λ_1 and λ_2 of M for a given eigenvalue k_r. Therefore we have

$$Q(k_r) = \begin{cases} A(\lambda_1) \\ A(\lambda_1) \oplus A(\lambda_2) \qquad (\lambda_1 \neq \lambda_2), \end{cases} \tag{2.7}$$

where $A(\lambda_j)$ is the eigenspace of A associated with the principal curvature $\lambda_j (j = 1, 2)$ of M, and $\oplus$ indicates the direct sum of vector spaces. For a tangent vector field $X \in T_0$ such that $QX = k_r X$, we have $Q\phi X = k_r \phi X$ because of $Q\phi = \phi Q$.

Let $k_1, \ldots, k_s$ be the distinct eigenvalues of Q, and let $k_1 = \alpha\gamma - \beta^2$. Then, by (2.6) and the above results, it is easily seen that the dimension of $Q(k_1)$, denoted it by dim $Q(k_1)$, is odd and that of $Q(k_r)$ is even for $2 \leq r \leq s$. Moreover we see from (1.7) that there are two distinct principal curvatures, say λ and μ, of M such that $\xi \in A(\lambda) \oplus A(\mu)$, and hence $Q(k_1)$ is given by $Q(k_1) = A(\lambda) \oplus A(\mu)$. Since λ and μ are distinct solutions of $x^2 - (\alpha + \gamma)x - k_1 = 0$, we have

$$\lambda + \mu = \alpha + \gamma, \qquad \lambda\mu = k_1 = \alpha\gamma - \beta^2. \tag{2.8}$$

Now we shall prove

Lemma 2.2. *Under the same assumptions of Lemma 2.1, there exist unit vector fields $X \in A(\lambda)$ and $Y \in A(\mu)$ such that*

$$\xi = fX + gY, \qquad U = gX - fY, \tag{2.9}$$

where f and g are smooth functions on $\mathcal{U}$, and satisfy $f^2 + g^2 = 1$ and $fg \neq 0$.

Proof. If $A(\lambda)$ is spanned by $\{X_1, \ldots, X_u\}$ and $A(\mu)$ by $\{Y_1, \ldots, Y_v\}$, then ξ is expressed by

$$\xi = \sum_{i=1}^{u} a_i X_i + \sum_{j=1}^{v} b_j Y_j.$$

We can choose X and Y such as $\sum a_i X_i = \| \sum a_i X_i \| X$ and $\sum b_j Y_j = \| \sum b_j Y_j \|$ Y. By putting $f^2 = \| \sum a_i X_i \|^2$ and $g^2 = \| \sum b_j Y_j \|^2$, we have $\xi = fX + gY$, $f^2 + g^2 = 1$ and $fg \neq 0$.

Since we have already seen that $\xi = fX + gY$ and $\beta U = -\phi \nabla_\xi \xi$ on $\mathcal{U}$, it is easy to verify that

$$\beta U = fg(\lambda - \mu)(gX - fY)$$

by use of (1.2) and (1.7). Therefore we can choose f and g such that $U = gX - fY$. □

Lemma 2.3. *Under the same assumptions of Lemma 2.1, the dimension of $Q(k_1)$ is equal to 3 on $\mathcal{U}$.*

Proof. We have already seen that dim $Q(k_1)$ is odd, and from (2.6) that dim $Q(k_1)$ is not less than 3.

Assume that dim $Q(k_1) \geq 5$. Then, since $Q(k_1) = A(\lambda) \oplus A(\mu)$, we may consider that dim $A(\lambda) >$ dim $A(\mu)$ and dim $A(\lambda) = 2\ell + 1 (\ell \geq 1)$. For the vector fields $X \in A(\lambda)$ and $Y \in A(\mu)$ given in Lemma 2.2, we define the subspaces Σ, Ω, $\phi\Sigma$ and $\phi\Omega$ of $Q(k_1)$ by

$$\Sigma = \{X_\lambda \in A(\lambda) \mid < X_\lambda, X >= 0\}, \qquad \phi\Sigma = \{\phi X_\lambda \mid X_\lambda \in \Sigma\},$$
$$\Omega = \{Y_\mu \in A(\mu) \mid < Y_\mu, Y >= 0\}, \qquad \phi\Omega = \{\phi Y_\mu \mid Y_\mu \in \Omega\}.$$

Then we see that $Q(k_1) = \Sigma \oplus \Omega \oplus \mathrm{span}\{X, Y\}$ and dim $\Sigma >$ dim Ω.

Now we shall show that $\phi\Sigma \subset \Omega$. For any two orthogonal vector fields X_λ and Y_λ in Σ, we see from Lemma 2.2 that both X_λ and Y_λ are orthogonal to

ξ. If we differentiate $AX_\lambda = \lambda X_\lambda$ covariantly along Y_λ and make use of the equation of Coddazzi (1.4), then we obtain $X_\lambda\lambda = Y_\lambda\lambda = 0$ and

$$(A - \lambda I)[X_\lambda, Y_\lambda] = \frac{c}{2} < \phi X_\lambda, Y_\lambda > \xi. \tag{2.10}$$

Taking inner product of (2.10) with X and using (2.9), we get $< \phi X_\lambda, Y_\lambda >= 0$. This means that $\phi\Sigma \cap \Sigma = \{0\}$ and hence $\phi\Sigma \subset \Omega \oplus \text{span}\{X, Y\}$ because $\phi X_\lambda \in Q(k_1)$. Similarly, differentiating $AX_\lambda = \lambda X_\lambda$ covariantly along X and taking account of (1.4), we also have $X\lambda = 0$ and

$$(A - \lambda I)[X_\lambda, X] = \frac{c}{4}\{\eta(X)\phi X_\lambda + 2 < \phi X_\lambda, X > \xi\}.$$

Taking the inner product of the above equation with X and using (2.9) yields

$$< \phi X_\lambda, X >= 0. \tag{2.11}$$

Since we get $< \phi X_\lambda, \xi >= f < \phi X_\lambda, X > +g < \phi X_\lambda, Y >= 0$ by (2.9), it follows from (2.11) that

$$< \phi X_\lambda, Y >= 0. \tag{2.12}$$

Therefore it is easily seen from (2.11) and (2.12) that $\phi\Sigma \cap \text{span}\{X, Y\} = \{0\}$ and hence $\phi\Sigma \subset \Omega$. This shows that dim $\phi\Sigma \leq$ dim Ω, and give rise to a contradiction because dim $\Sigma =$ dim $\phi\Sigma$. Thus we have dim $Q(k_1) = 3$. □

By Lemma 2.2, it is easy to see that ϕU is orthogonal to both X and Y. Since we have $\phi U \in Q(k_1) = A(\lambda) \oplus A(\mu)$ by (2.6) and dim $Q(k_1) = 3$ by Lemma 2.3, we may consider that $\phi U \in A(\mu)$, that is,

$$A\phi U = \mu\phi U. \tag{2.13}$$

Lemma 2.4. *Under the same assumptions of Lemma 2.1, we have*

$$(\nu + \kappa) < \phi X_\nu, X_\kappa >= 0 \tag{2.14}$$

on $\mathcal{U}$, where the non-zero vector fields X_ν and X_κ are orthogonal to ξ, U and ϕU, and satisfy $AX_\nu = \nu X_\nu$ and $AX_\kappa = \kappa X_\kappa$.

Proof. By Lemmas 2.2 and 2.3, we see that the principal curvatures ν and κ of M never equal to λ and μ. Let $X_\nu \in Q(k_q)$, that is, $k_q = (\alpha + \gamma)\nu - \nu^2$. Then we see from Lemma 2.3 that $k_q \neq k_1 = \alpha\gamma - \beta^2$. Therefore, if we multiply (2.4) by X_ν and take account of (0.1), (1.5) and (2.5), then we obtain

$$< \nabla_X U, X_\nu >= 0 \qquad \text{for} \qquad X \in T_0.$$

This means that the vector field $\nabla_X U$ is expressed by a linear combination of ξ, U and ϕU only. Since we have $< \nabla_X U, \xi >= \mu < X, \phi U >$ by taking account of (1.2) and (2.13), we see that

$$\nabla_X U = \mu < X, \phi U > \xi + < \nabla_X U, \phi U > \phi U \tag{2.15}$$

on $\mathcal{U}$. Now differentiating (2.2) covariantly along X_ν and using (2.15), we obtain

$$(\nabla_{X_\nu} A)U = (X_\nu\beta)\xi + (X_\nu\gamma)U + (\gamma - \mu) < \nabla_{X_\nu} U, \phi U > \phi U + \beta\nu\phi X_\nu,$$

from which

$$<(\nabla_{X_\nu} A)X_\kappa, U>=\beta\nu<\phi X_\nu, X_\kappa>.$$

As a similar argument as the above, we also have

$$<(\nabla_{X_\kappa} A)X_\nu, U>=\beta\kappa<\phi X_\kappa, X_\nu>.$$

Therefore, from the last two equations and the equation of Coddazzi (1.4), we can verify (2.14). □

3. Proof of Theorems

In this section, we shall prove Theorems 1 and 2.

Proof of Theorem 1. We can choose a local orthonormal frame field

$$\{X_1,\ X_2, \dots,\ X_{2n-1}\} \tag{3.1}$$

on $\mathcal{U}$ such that $X_1 = X$ and $X_2 = Y$ are given in Lemma 2.2, $X_3 = \phi U$ and $AX_i = \lambda_i X_i$ for $4 \le i \le 2n-1$. For any $X_i (i \ge 4)$ in (3.1), there exists an eigenvalue $k_r (2 \le r \le s)$ of Q such that $X_i \in Q(k_r)$. Since $Q\phi = \phi Q$, we see that $\phi X_i \in Q(k_r)$. As we have already seen in (2.7) and (2.8), we see that either $Q(k_r) = A(\lambda_i)$ or $Q(k_r) = A(\lambda_i) \oplus A(\alpha + \gamma - \lambda_i)$.

Let $Q(k_r) = A(\lambda_i) \oplus A(\alpha + \gamma - \lambda_i)$. Since $\phi X_i \in Q(k_r)$, there are two non-zero vector fields $X_{\lambda_i} \in A(\lambda_i)$ and $X_{\alpha+\gamma-\lambda_i} \in A(\alpha + \gamma - \lambda_i)$ such that

$$\phi X_i = aX_{\lambda_i} + bX_{\alpha+\gamma-\lambda_i},$$

where a and b are smooth functions on $\mathcal{U}$.

If $ab \neq 0$, then we have $\lambda_i = 0$ by putting $X_\nu = X_i$ and $X_\kappa = X_{\lambda_i}$ into (2.14) of Lemma 2.4, and $\alpha + \gamma = 0$ by putting $X_\nu = X_i$ and $X_\kappa = X_{\alpha+\gamma-\lambda_i}$ into (2.14). This means that $\lambda_i = \alpha + \gamma - \lambda_i = 0$, that is, $Q(k_r) = A(0)$ and a contradiction. Therefore we have either $\phi X_i \in A(\lambda_i)$ or $\phi X_i \in A(\alpha + \gamma - \lambda_i)$.

If $\phi X_i \in A(\lambda_i)$, then we obtain $\lambda_i = 0$ by putting $X_\nu = X_i$ and $X_\kappa = \phi X_i$ into (2.14), and $Q(k_r) = A(0) \oplus A(\alpha + \gamma)$. For a non-zero vector field $X_{\alpha+\gamma} \in A(\alpha + \gamma)$, we have either $\phi X_{\alpha+\gamma} \in A(0)$ or $\phi X_{\alpha+\gamma} \in A(\alpha + \gamma)$. In each case, using (2.14), it is easily seen that $\alpha + \gamma = 0$, and a contradiction.

Thus we see that $\phi X_i \in A(\alpha+\gamma-\lambda_i)$. Putting $X_\nu = X_i$ and $X_\kappa = \phi X_i$ into (2.14), we get $\alpha + \gamma = 0$. Hence we have $Q(k_r) = A(\lambda_i) \oplus A(-\lambda_i)$. Moreover we see that the multiplicity of λ_i is equal to that of $-\lambda_i$

If $Q(k_r) = A(\lambda_i)$, then we have $\phi X_i \in A(\lambda_i)$, and hence $\lambda_i = 0$ from (2.14).

Summing up the above results, for the vector fields $X_i (4 \le i \le 2n-1)$ given in (3.1), there are two cases where all the principal curvatures λ_i associated with X_i are equal to zero on $\mathcal{U}$, and where the multiplicity of a non-zero principal curvature λ_i associated with X_i is equal to that of $-\lambda_i$ (associated with ϕX_i), and trace$A = \alpha + \gamma = 0$.

The former implies that trace$A = \alpha+\gamma = \lambda + 2\mu$, and we see from (2.8) that $\mu = 0$ identically on $\mathcal{U}$. Thus the type number at any point of $\mathcal{U}$ is not greater than 1, and this does not occur (for instance, see [7]). The latter shows that trace$A = \alpha + \gamma = \lambda + 2\mu = 0$, and from (2.8) that $\mu = 0$ and $k_1 = \alpha\gamma - \beta^2 = 0$

on $\mathcal{U}$. Therefore we have $\alpha^2 + \beta^2 = 0$ and a contradiction. Thus the subset $\mathcal{U}$ must be empty. □

Proof of Theorem 2. Theorem 2 follows from Theorem A and Theorem 1. □

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