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A note on T-sum of bell-shaped fuzzy intervals

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Abstract

The usual arithmetic operations on real numbers can be extended to arithmetical operations on fuzzy intervals by means of Zadeh's extension principle based on a t-norm T. Dombi and Győrbíró proved that addition is closed if the Dombi t-norm is used with two bell-shaped fuzzy intervals. Recently, Hong [Fuzzy Sets and Systems 158(2007) 739-746] defined a broader class of bell-shaped fuzzy intervals. Then he study t-norms which are consistent with these particular types of fuzzy intervals as applications of a result proved by Mesiar on a strict t-norm based shape preserving additions of LR-fuzzy intervals with unbounded support. In this note, we give a direct proof of the main results of Hong.

Key Words : Fuzzy number, t-norm, t-norm-based addition, bell-shape fuzzy interval, shape preserving.

1. Preliminaries

Recall that a triangular norm (briefly t-norm) is a binary operation T on the unit interval [0,1] which is commutative, associative, monotone and has 1 as neutral element.

A t-norm T is Archimedean if and only if $T(x,x)<x$ for all $x\in(0,1)$. Every continuous Archimedean t-norm T can be represented by a continuous and strictly decreasing function $f:[0,1]\to[0,\infty]$ with $f(1)=0$ and

$$T(x,y)=f^{[-1]}(f(x)+f(y)).$$

where $f^{[-1]}$ is the pseudo-inverse of f, defined by

$$f^{[-1]}(y)=\sup\{x\in[0,1]\,|\,f(x)>y\}.$$

The function f is called the additive generator of a t-norm T.

If f is bounded, then it can be chosen uniquely so that $f(0)=1$ and the corresponding t-norm T is called a nilpotent t-norm. If f is unbounded, the corresponding t-norm T is called a strict t-norm.

Let T be a given t-norm and let A_i, $i=1,2,\cdots,n$ and A_2 be fuzzy intervals. Then their T-sum $C=A_1\oplus\cdots\oplus A_n$ is defined by the generalized extension principle of Zadeh [5] as

$$C(z)=\sup_{x_1+x_2+\cdots+x_n=z}T(A_1(x_1),\cdots,A_n(x_n)),\ z\in R.$$

If T is a strict continuous Archimedean t-norm with the additive generator f then

$$C(z)=f^{-1}\left(\inf_{x_1+x_2+\cdots+x_n=z}(f(A_1(x_1)),\cdots,f(A_n(x_n)))\right).$$

Definition 1.1([1,2]) Let f be the additive generator of a strict t-norm T. If $f\circ B^f_{c,d}(x)=((x-c)/d)^2$, then we call $B^f_{c,d}(x)=f^{-1}(((x-c)/d)^2)$ the bell-shape membership function with respect to T, where c is the center and d is the width of the function.

2. The T-sum of bell-shape fuzzy intervals

In the following section the T-sum of generalized bell-shaped fuzzy intervals where T is a continuous Archimedean t-norm will be calculated. As a direct application of the result of Mesier [3,4] on strict t-norm based shape preserving additions of the LR-fuzzy intervals with unbounded support, Hong [Fuzzy Sets and Systems 158(2007) 739-746] has the following shape-preserving addition of bell-shaped fuzzy intervals.

Theorem 2.1 Let T be a strict t-norm with additive generator, f, and let $B^f_{c_i,d_i}(x)$, $i=1,2,\cdots$ be a sequence of the bell-shaped membership functions. Then the addition $\oplus_T$ based on T preserves the bell-shape of fuzzy intervals and

$$B^f_{c_1,d_1}\oplus_T\cdots\oplus_T B^f_{c_n,d_n}=B^f_{c_1+\cdots+c_n,(d_1^2+\cdots+d_n^2)^{1/2}}.$$

Additionally, if $\sum_{i=1}^{\infty}c_i=C$ and $\sum_{i=1}^{\infty}d_i^2=D$, then

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$$\lim_{n\to\infty} B^f_{c_1,d_1} \oplus_T \cdots \oplus_T B^f_{c_n,d_n} = B^f_{C,D^{1/2}}.$$

In this note, we give a direct proof of above result of Hong. Before proving the main theorem we consider the following lemma.

Lemma 2.2 For positive β_1, β_2,

$$\inf_{x+y=z}\left(\frac{x}{\beta_1}\right)^2+\left(\frac{y}{\beta_2}\right)^2=\left(\frac{z}{(\beta_1^2+\beta_2^2)^{1/2}}\right)^2.$$

Proof. Let us denote $h(x)=\left(\frac{x}{\beta_1}\right)^2+\left(\frac{z-x}{\beta_2}\right)^2$ and find the minimal value of h for $x\in R$. The derivative of the function h is:

$$h'(x)=\frac{2}{\beta_1}\left(\frac{x}{\beta_1}\right)-\frac{2}{\beta_2}\left(\frac{z-x}{\beta_2}\right).$$

The only point x for which $h'(x)=0$ is

$$x_0=\frac{z}{1+\lambda},\text{ where }\lambda=\left(\frac{\beta_2}{\beta_1}\right)^{\frac{2}{3}}.$$

It is easy to show that h has its minimum at the point x_0 and minimal value of h is

$$h(x_0)=\left[\frac{z}{\beta_1\beta_2(1+\lambda)}\right]^2(\beta_2^2+\beta_1^2\lambda^2)=\left(\frac{z}{(\beta_1^2+\beta_2^2)^{1/2}}\right)^2.$$

Proof of Theorem 2.1 It is enough to show that for $n=2$. As mentioned in Section 1, the investigated membership function is

$$B^f_{c_1,d_1}\oplus_T B^f_{c_2,d_2}(z)$$
$$=f^{-1}\left(\inf_{x_1+x_2=z} f\left(B^f_{c_1,d_1}(x_1)\right)+f\left(B^f_{c_2,d_2}(x_2)\right)\right).$$

Since $B^f_{c_i,d_i}(x)=B^f_{0,d_i}(x-c_i)$ for all x, we have for each z,

$$B^f_{c_1,d_1}\oplus_T B^f_{c_2,d_2}(z)$$
$$=f^{-1}\left(\inf_{x_1+x_2=z} f\left(B^f_{c_1,d_1}(x_1)\right)+f\left(B^f_{c_2,d_2}(x_2)\right)\right)$$
$$=f^{-1}\left(\inf_{y_1+y_2=z-c} f\left(B^f_{0,d_1}(y_1)\right)+f\left(B^f_{0,d_2}(y_2)\right)\right)$$
$$=B^f_{0,d_1}\oplus_T B^f_{0,d_2}(z-c)$$

where $c=c_1+c_2$. Now,

$$B^f_{0,d_1}\oplus_T B^f_{0,d_2}(z)$$
$$=f^{-1}\left(\inf_{x_1+x_2=z} f\left(B^f_{0,d_1}(x_1)\right)+f\left(B^f_{0,d_2}(x_2)\right)\right)$$
$$=f^{-1}\left(\inf_{x_1+x_2=z}\left(\frac{x_1}{d_1}\right)^2+\left(\frac{x_2}{d_2}\right)^2\right)$$
$$=f^{-1}\left(\left(\frac{z}{(d_1^2+d_2^2)^{1/2}}\right)^2\right)$$
$$=f^{-1}\left(f\left(B^f_{0,(d_1^2+d_2^2)^{1/2}}(z)\right)\right)$$
$$=B^f_{0,(d_1^2+d_2^2)^{1/2}}(z),$$

where the third equality comes from Lemma 2.2. Therefore we have

$$B^f_{c_1,d_1}\oplus_T B^f_{c_2,d_2}(z)$$
$$=B^f_{0,(d_1^2+d_2^2)^{1/2}}(z-c)=B^f_{c_1+c_2,(d_1^2+d_2^2)^{1/2}}(z). \quad (1)$$

and using (1) we have

$$\lim_{n\to\infty} B^f_{c_1,d_1}\oplus_T\cdots\oplus_T B^f_{c_n,d_n}(z)$$
$$=\lim_{n\to\infty} B^f_{c_1+\cdots+c_n,(d_1^2+\cdots+d_n^2)^{1/2}}(z)$$
$$=\lim_{n\to\infty} f^{-1}\left(\left(\frac{z-(c_1+\cdots+c_n)}{(d_1^2+\cdots+d_n^2)^{1/2}}\right)^2\right)$$
$$=\lim_{n\to\infty} f^{-1}\left(\left(\frac{z-C}{D^{1/2}}\right)^2\right)$$
$$=B^f_{C,D^{1/2}}(z)$$

which completes the proof.

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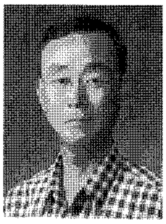

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