A Study on Wavelet-based Denoising Algorithm for Signal Reconstruction in Mixed Noise Environments

Sang-Bum Bae and Nam-Ho Kim*, *Member, KIMICS*

***Abstract*—**In the process of the acquisition, storage, transmission of signals, noises are generated by various causes and the degradation phenomenon by noises tends to generate serious errors for the signal with information. So, in order to analyze and remove these noises, studies on numerous mathematical methods such as the Fourier transform have been implemented. And recently there have been many ongoing wavelet-based denoising algorithms representing excellent characteristics in time-frequency localization and multiresolution analysis, but the method to remove additive white Gaussian noise (AWGN) and the impulse noise simultaneously was not given. So, to reconstruct the corrupted signal by noises, in this paper a novel wavelet-based denoising algorithm was proposed and using signal-to-noise ratio (SNR) this method was compared to conventional methods.

***Index Terms*—**AWGN, denoising, impulse noise, multiresolution, time-frequency localization, wavelet

I. INTRODUCTION

Due to the fast development of multimedia and communication technology, the society has been in an advanced digital information period. In order to use the information signals more efficiently, it takes digitalized processing as in signal acquisition, storing, and transmission. However, in the process of signals, the degradation phenomenon is often generated by internal and external causes and the main cause of the degradation is noise. The degradation of signals caused by noise, declines the recognition of images and sounds and occurs errors during the data transmission.

Therefore, a great many mathematical methods for noise analysis and removal have been researched. According to generation causes and forms, there are many kinds of noises and the additive white Gaussian noise (AWGN) and the impulse noise are representative. AWGN generates in all frequency range, has random value and also continuous.

In order to remove AWGN the mean filter is the simplest method, but in the region of edge, the blurring phenomenon is occurred according to remove the noise by smoothing the signal [1]. And in a wide range of applications, to analyze the signal and remove noise, Fourier transform has been used and the filter design using this method could make better frequency properties according to conditions and methods [12], [13]. However, according to analyze signal only in the frequency axis, time information when the specific frequency component generates was not given. So to make up for this weak point and to represent concurrently time and frequency properties, the short time Fourier transform (STFT) was proposed, but according to fix the size of the window function, time-frequency localization represents tradeoff. And, in methods to remove impulse noise and remains the edge components, the nonlinear filter as the median filter or the order statistics filter have been proposed and many studies on modified median filters and order statistics filters have been implemented [14]-[16].

To overcome this limitation of conventional noise removal methods by linear and nonlinear type filters, wavelet-based methods representing time-frequency localization and excellent properties in multiresolution analysis are actively studied [1]-[3], [8]-[11]. For denoising AWGN, Donoho and Johnstone proposed an algorithm which is based on the threshold. From then on, many studies on values of thresholds and methods for optimal application are performed [1]-[3]. And the undecimated discrete wavelet transform (UDWT) has wavelet coefficients of same number in all scales and shows excellent characteristics in removing AWGN [4], [5]. Besides, in order to remove impulse noise, an algorithm using the B-wavelet was proposed in wavelet [6], [7].

However, because the conventional wavelet denoising algorithms only reflect the individual features of AWGN and impulse noise, in mixed noise environments, conventional noise removal methods using wavelet could not separate edges of signals with noise. So, for reconstructing the corrupted signals by noise, a new wavelet-based denoising algorithm was proposed in this paper and also signal-to-noise ratio (SNR) was used to compare the proposed method and conventional methods.

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Asterisk indicates corresponding author.

Sang-Bum Bae is currently pursuing the Ph.D. degree in the School of Electrical and Control Engineering, Pukyong National University, Busan, Korea (E-mail : blueskybsb@ pknu.ac.kr)

*Nam-Ho Kim is a professor in the School of Electrical and Control Engineering, Pukyong National University, Busan, Korea (E-mail : nhk@pknu.ac.kr)

II. WAVELET TRANSFORM

If a signal $f(t) \in L^2(R)$, the continuous wavelet transform (CWT) defines as

$$W(a,b) = \int_{-\infty}^{\infty} f(t)\psi^*_{a,b}(t)\, dt \qquad (1)$$

where, * is a complex conjugation and the baby wavelet function $\psi_{a,b}(t)$ is represented as

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \qquad (2)$$

From the equation, a is a scale factor and b is a translation factor. In addition, a wavelet function $\psi(\cdot)$ satisfies to admissibility condition as in (3), and $\Psi(\omega)$ is the Fourier transform of $\psi(t)$.

$$C = \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{\omega}\, d\omega < \infty \qquad (3)$$

where, $0<C<\infty$ and $f(t)$ is reconstructed by the inverse wavelet transform of (4).

$$f(t) = \frac{1}{C}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{|a|^2} W(a,b)\, \psi_{a,b}(t)\, da\, db \qquad (4)$$

And in (3), in order that C does not have unlimited value in $\omega=0$, it is satisfied $\Psi(0)=0$. So, (3) contains the following

$$\int_{-\infty}^{\infty} \psi(t)\, dt = 0 \qquad (5)$$

In the time-scale domain, a wavelet function by dyadic sampling such as parameters $a=2^j$, $b=2^j n$, is represented as

$$\psi_{j,n}(t) = 2^{-j/2}\psi(2^{-j}t-n) \qquad (6)$$

Furthermore, the discrete wavelet transform or the detail coefficients for signal $f(t)$ is presented as

$$\begin{aligned} d_{j,n} &= \langle f(t), \psi_{j,n}(t)\rangle \\ &= 2^{-j/2}\int_{-\infty}^{\infty} f(t)\psi(2^{-j}t-n)\, dt \end{aligned} \qquad (7)$$

From the equation, $j, n \in Z$, and $\langle\cdot,\cdot\rangle$ denotes as L^2-inner product [1]. And in multiresolution analysis (MRA), the signal $f(t) \in L^2(R)$ is represented as

$$f(t) = \sum_{n\in Z} c_{J,n}\,\phi_{J,n}(t) + \sum_{j=J}^{\infty}\sum_{n\in Z} d_{j,n}\,\psi_{j,n}(t) \qquad (8)$$

where, J means an initial resolution, $\phi_{j,n}(\cdot)$ presents the scaling function, and the approximation coefficients $c_{j,n}$ denotes as

$$c_{j,n} = 2^{-j/2}\int_{-\infty}^{\infty} f(t)\,\phi(2^{-j}t-n)\, dt \qquad (9)$$

And in the process of reconstruction using wavelet about the signal, in a scale j the approximation coefficients are calculated from (10).

$$c_{j,n} = \sum_{l\in Z} g_l\, c_{j+1,\,2n-l} + \sum_{l\in Z} h_l\, d_{j+1,\,2n-l} \qquad (10)$$

where, g_l and h_l represent the coefficients of low-pass filter and band-pass filter, respectively.

III. DENOISING METHODS

3.1 Conventional methods

At the present time, in order to remove noises effectively from noisy signals numerous methods using the wavelet have been proposed until now. Among these methods, in the AWGN environment the threshold-based method is representative and the algorithm of this method which has the excellent characteristics of noise removal is simple. This method obtains the wavelet coefficients from noisy signals. And a threshold value λ is applied to each scale and in this process, more optimal methods and levels have to be chosen. Finally, in order to get an estimated signal, the inverse wavelet transform is implemented. Fig. 1 shows this process. And in orthogonal wavelet transform (OWT) the soft-threshold expressed as (11) is applied.

$$\eta^{soft}(d_{j,n}) = \begin{cases} d_{j,n} - \mathrm{sgn}(d_{j,n})\cdot\lambda, & if\ |d_{j,n}| \ge \lambda \\ 0, & if\ |d_{j,n}| < \lambda \end{cases} \qquad (11)$$

where, $\mathrm{sgn}(\cdot)$ represents the sign function and in

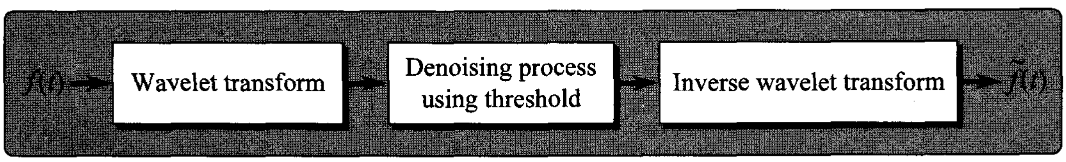


Fig. 1 Threshold-based denoising process.

the universal threshold applies $\lambda = \sigma\sqrt{2\log N}$. In addition, N is the length of the signal, and σ is the standard deviation of noise and is estimated as

$$\sigma = MAD / 0.6745 \qquad (12)$$

where, MAD is the median absolute deviation obtained by the wavelet coefficients of the finest scale. The UDWT taking the wavelet coefficients of same number in all scales represents very excellent capabilities for AWGN removal and applies a hard thresholds as

$$\eta^{hard}(d_{j,n}) = \begin{cases} d_{j,n}, & if\ |d_{j,n}| \geq \lambda \\ 0, & if\ |d_{j,n}| < \lambda \end{cases} \qquad (13)$$

From the statistical characteristics of AWGN, the UDWT applies $\lambda = 3\sigma \sim 4\sigma$ [5].

3.2 Proposed method

From the characteristics of signals, the statistical mean of the AWGN, which represents the normal distribution of standard deviation σ, is around 0. And the impulse noise represents the large variation of signals in a location but has very short time of duration and is a discontinuous point returning the previous value soon. However, in every wavelet detail coefficients the edge component has a relatively large magnitude compared to the AWGN. Also at the edge of the signal, the magnitude of the approximation coefficients tends to vary drastically and keeps a very long duration time. Therefore, using these characteristics compared to noise, components of noise could be removed effectively from corrupted signals in mixed noise environments.

From the wavelet decomposition and reconstruction, in the same scale j and location n the approximation coefficients and detail coefficients represent information on the same location of the signal. So, in order to remove noises component from the wavelet detail coefficients we could use information on the approximation coefficients in each scale j. In each scale j, the accumulation function corresponding to the approximation coefficients $c_{j,n}$ could be represented as

$$F_{j,n} = F_{j,n-1} + c_{j,n}, \quad n = 1, 2, \cdots, N \qquad (14)$$

where, $F_{j,n} = 0$ in relation to $n \leq 0$. And in order to remove noise components from noisy signals, the error function which is represented as (15) is used.

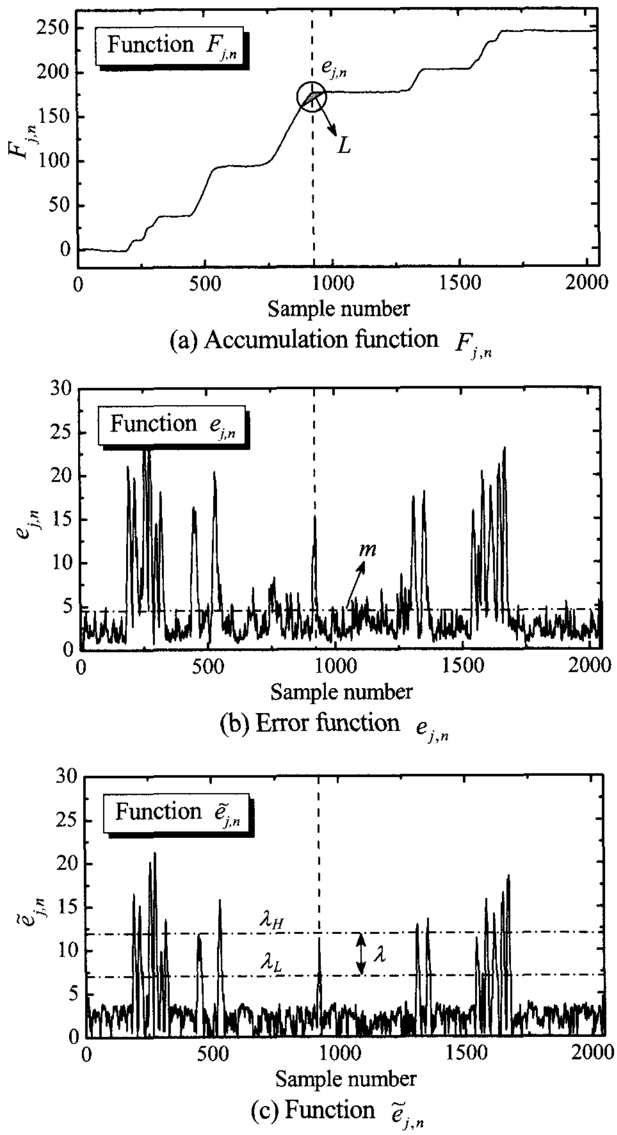
$$e_{j,n} = \sum_{i=n-k}^{n+k} |F_{j,i} - L_i| \qquad (15)$$

From the equation, a function L is a linear function satisfying $F_{j,n-k}$ and $F_{j,n+k}$. And k is a range constant to obtain the error function.

Fig. 2 is the process determining the threshold value λ from the accumulation function $F_{j,n}$ and the error function $e_{j,n}$. Fig. 2(a) is the accumulation function $F_{j,n}$ about the approximation coefficients $c_{j,n}$ and Fig. 2(b) is the error function $e_{j,n}$ obtained by (15). And in order to observe the distribution characteristics of the error data, Fig. 2(c) is the function $\tilde{e}_{j,n}$ represented by the following (16).

$$\tilde{e}_{j,n} = |\, e_{j,n} - m \,| \qquad (16)$$

From the equation, m is the mean value of the error function $e_{j,n}$.



(a) Accumulation function $F_{j,n}$

(b) Error function $e_{j,n}$

(c) Function $\tilde{e}_{j,n}$

Fig. 2 The accumulation function and the error function for a noisy signal.

From the figure, the error function $e_{j,n}$ expressed as the sum of range about the absolute error shows the evident distribution characteristics at the mean m. That is, almost error of accumulation function by noise components is contained in the deviation m at mean value m. Therefore, in Fig. 2(c) representing the absolute error of the function $e_{j,n}$ about mean m, the error by noise components is uniformly distributed in the specific magnitude range. So in most cases, a threshold value λ is chosen in limits of $[\lambda_L, \lambda_H]$ and $\lambda_L = 2.5m$, $\lambda_H = 3.5m$. Finally, in each scale after removing the noise component by appling the hard threshold, the signal is reconstructed from the estimated wavelet detail coefficients.

IV. SIMULATION AND RESULTS

In order to reconstruct signals from the mixed noise environment, a novel noise removal algorithm based on wavelet transform has been proposed in this paper. We used Bumps signal and speech signal / i / as the test signal to evaluate the noise removal performance. Moreover, the sampling frequency for the speech acquisition is 44.1[kHz], and the signal length is 2048 samples. And, the range constant k is applied to 10 and as the threshold λ, the value of $3.5m$ is used. Moreover, we used corrupted signals by impulse noise, which has different the magnitude and sign as Fig. 3, and AWGN with SNR 2[dB].

The Fig. 4 and Fig. 5 represent the simulation result of the Bumps signal and a speech signal / i /. The Fig. 4 (a) is true Bumps signal, and Fig. 4 (b) is the mixed noise signal incorporated with impulse noise in Fig.3 and AWGN of SNR 2[dB], whose the total SNR is 1.15[dB]. Besides, from Fig. 4 (c) to Fig. 4(e) present the reconstructed signals after noises are removed by OWT, UDWT, and the proposed method respectively. From figures, OWT and UDWT method represent SNR of 9.14[dB], 8.48[dB], respectively. And the proposed method denotes SNR of 10.97[dB].

Fig. 5(a) is a true speech signal / i / and Fig. 5(b) is a noisy signal with SNR 0.59[dB] corrupted by the mixed noise. Reconstruction results about the superposed speech signal / i / by noise are from Fig. 5(c) to 5(e).

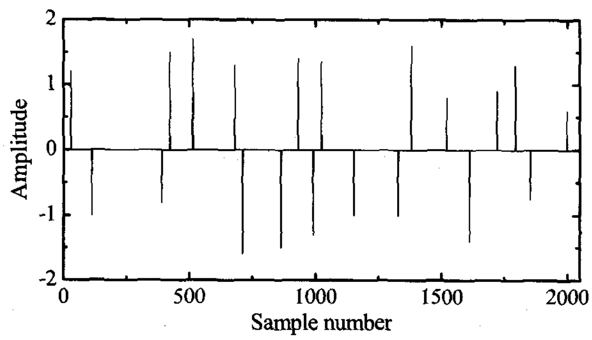
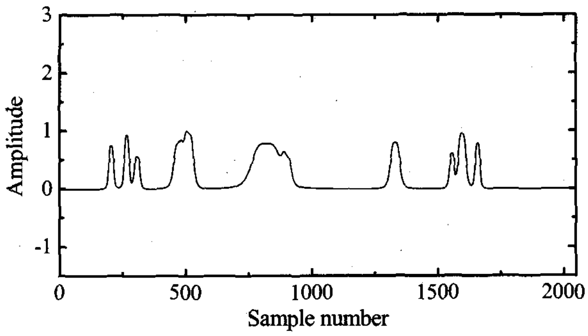
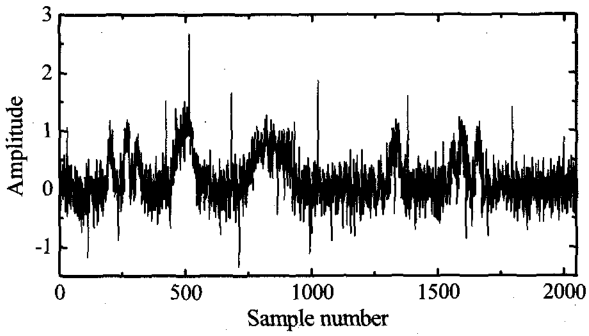


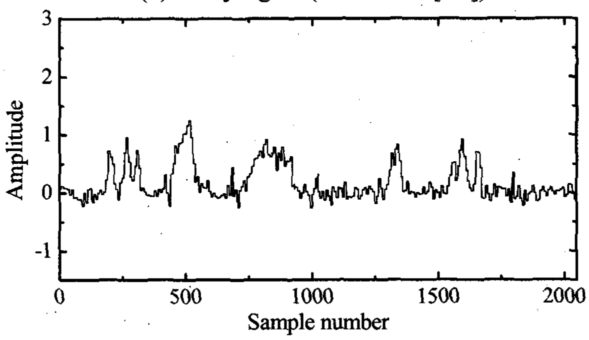
Fig. 3 Impulse noise.



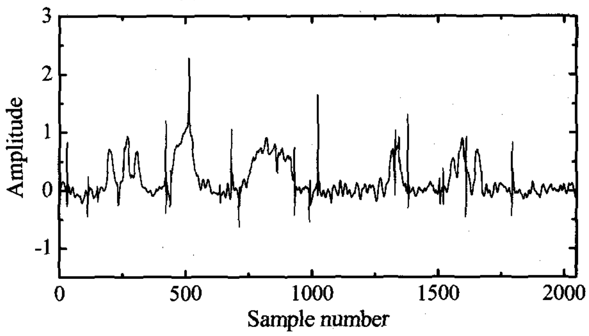
(a) Bumps signal



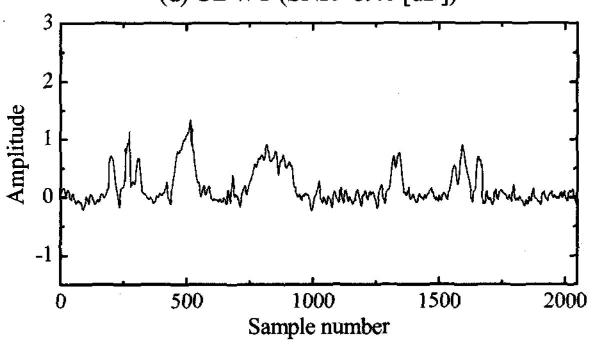
(b) Noisy signal (SNR=1.15 [dB])



(c) OWT (SNR=9.14 [dB])

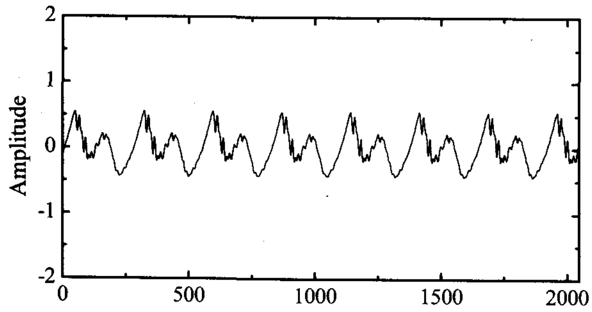


(d) UDWT (SNR=8.48 [dB])

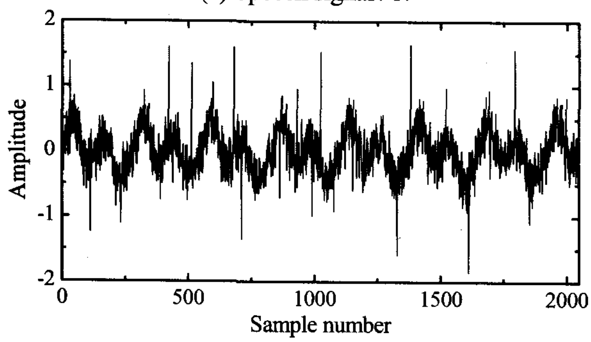


(e) Proposed method (SNR=10.97 [dB])

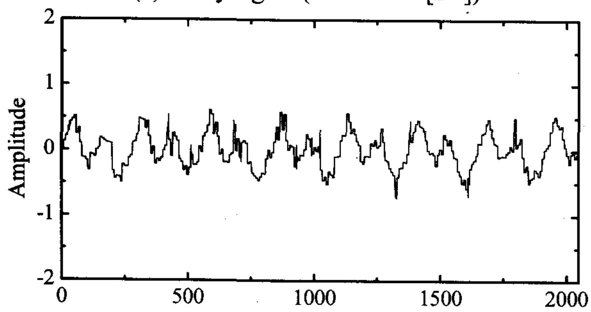
Fig. 4 Reconstruction of Bumps signal.



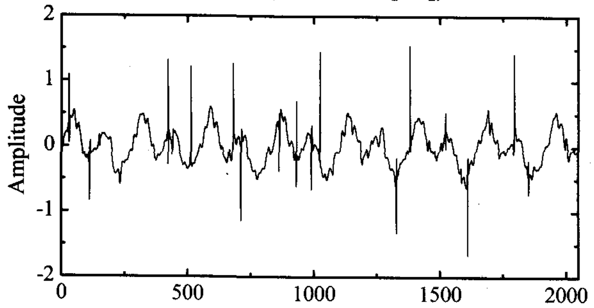
(a) Speech signal / i /



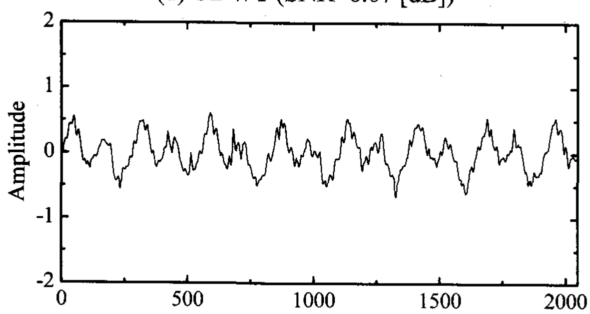
(b) Noisy signal (SNR=0.59 [dB])



(c) OWT (SNR=7.91 [dB])



(d) UDWT (SNR=6.07 [dB])



(e) Proposed method (SNR=10.13 [dB])

Fig. 5 Reconstruction of speech signal / i /.

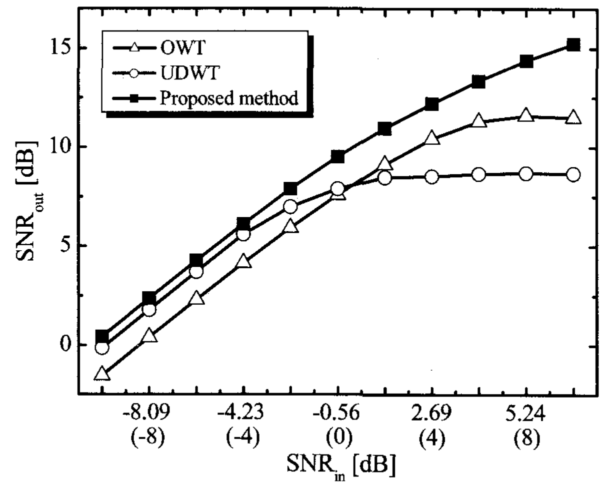


Fig. 6 SNR of reconstructed Bumps signal in mixed noise environments.

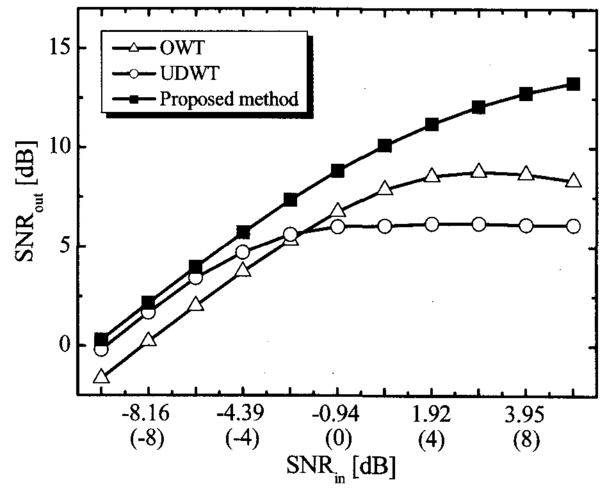


Fig. 7 SNR of reconstructed speech signal / i / in mixed noise environments.

From the figures, OWT and UDWT method show SNR of 7.91[dB], 6.07[dB], respectively while the proposed algorithm shows SNR of 10.13[dB]. From results of Fig. 4 and Fig. 5, OWT exhibits visual artifacts by subsampling and according not to separate and detect the impulse noise and edge components of signals, UDWT conserves intactly numerous the impulse noise components.

Fig. 6 and Fig. 7 represent the comparison of SNR by each algorithm with the Bumps signal and speech signal / i / in the mixed noise environment. From the figures, SNRin is SNR of the noisy signal, SNRout means SNR of the reconstructed signal, and (·) shows SNR of the AWGN.

From the simulation results, the conventional OWT and UDWT show low SNR quality in the mixed noise environment by recognized the impulse noise as edge. But the proposed algorithm, which was presented in this paper, shows superior noise removal result than other conventional method in all SNR domains for the separation ability to edge and impulse noise.

V. CONCLUSION

In this paper, in order to reconstruct signal corrupted by mixed noises, a novel wavelet-based noise removal algorithm was proposed and compared with other conventional algorithms.

From the simulation result, OWT exhibits visual artifacts by the reason of subsampling. Though UDWT shows a very good noise removal performance in the AWGN environment but in the mixed noise environment, the UDWT recognized many impulse noises as the edge. So for the high SNR domain where taken more effects by impulse noise, UDWT shows a lower performance than OWT. But the proposed algorithm, which was presented in this paper, using the error function obtained from the accumulation function of the wavelet approximation coefficients, shows a very good signal reconstruction performance in the mixed noise environment. Therefore, according as the proposed algorithm has an advanced ability in noise removal, it may also be widely used in diverse types of signal processing.

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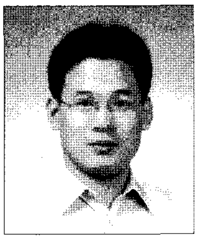
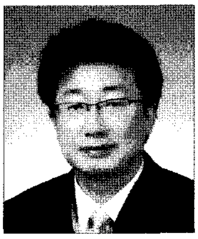


Sang-Bum Bae

He received the B.S. and M.S. degrees in control and instrumentation engineering from Pukyong National University (PKNU), Korea, in 1999 and 2002, respectively. He is currently pursuing the Ph.D. degree in the control and instrumentation engineering, PKNU, under the supervision of Prof. N. H. Kim. His research interests include circuits and systems, digital communication, and signal processing with wavelet.



Nam-Ho Kim

Corresponding Author

He received the B.S., M.S., and Ph.D. degrees in electronics engineering from Yeungnam University, Korea in 1984, 1986 and 1991, respectively. Since 1992, he has been with Pukyong National University (PKNU), Korea, where he is currently a professor in the School of Electrical and Control Engineering. From 2004 to 2006, he was Vice Dean of the College of Engineering, PKNU. His research interests include circuits and systems, high-frequency measurement, sensor systems, image and signal processing with wavelet and adaptive filters, and communications theory.