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Reliability Analysis of Slope Stability with Sampling Related Uncertainty

통계오차를 고려한 사면안정 신뢰성 해석

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요 지

다양한 불확실성을 체계적으로 반영하는 신뢰성 기반 해석기법을 사면안정 해석의 한 형식으로 제시한다. 통계오차, 공간 변동성, 그리고 공간 평균의 효과를 고려할 수 있는 지반특성 표현식이 사용되었다. 여러 가지 형식의 지반특성 표현식을 이용하여 사면안정 신뢰성 해석을 수행한 결과 통계오차, 공간적 상관성, 그리고 조건부 해석기법을 사용할 경우가 기존의 단순 확률변수 기법에 비해 상당히 작은 파괴확률을 제시한다는 사실이 밝혀졌다. 이 결과는 사면안정 해석에서 공간적 변동성과 통계오차가 합리적으로 고려되어야 한다는 점을 제시한다.

Abstract

A reliability-based approach that can systematically model various sources of uncertainty is presented in the context of slope stability. Expressions for characterization of soil properties are developed in order to incorporate sampling errors, spatial variability and its effect of spatial averaging. Reliability analyses of slope stability with different statistical representations of soil properties show that the incorporation of sampling error, spatial correlation, and conditional simulation leads to significantly lower probability of failure than that obtained by using simple random variable approach. The results strongly suggest that the spatial variability and sampling error have to be properly incorporated in slope stability analysis.

Keywords : Conditional approach, Random field, Reliability, Slope stability, Spatial averaging

1. Introduction

The estimation of key soil properties and subsequent quantitative assessment of the associated uncertainties has always been an important issue in geotechnical engineering. The most common reliability-based approach has been to assume that soil properties could be modeled as simple random variables (e.g., Christian et al. 1994; Christian and Urzua 1998; Duncan 2000). This approach implicitly assumes that a given soil property is perfectly correlated with the seemingly homogeneous segment of soil profile, and is the same at all locations within that segment. It is, however, known that the assumption of perfect correlation can lead to an overestimate of the failure probability of a geotechnical structure, since it usually overestimates the level of uncertainty. In general, the stochastic nature of spatially varying soil properties can be treated in the framework of a random field (e.g. Vanmarcke, 1977a). Since in typical applications the whole domain of interest is discretized into smaller elements, the random field property has to be represented by a suitable spatial average of each element. Over the

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last couple of decades various spatial averaging methods have been proposed for slope stability analysis (Vanmarcke 1977a, 1977b; Li and White 1987a, 1987b; Li and Lumb 1987).

Kim (2003) reported the importance of geotechnical variability in the analysis of earthquake-induced slope deformations. This paper reports a follow-up analytical study that investigated the influence of inherent variability and sampling errors in the analysis of static slope stability. Kim (2003) briefly described some of the findings from that study and this paper reports all the remaining findings in details. In this work, the author is particularly interested in the development of the statistics of the local average of the material property of a certain portion of the space, since soils generally exhibit plastic behavior and the stability of a soil slope tends to be controlled by the averaged soil properties rather than the properties at a particular location along the potential slip surface (i.e., Li and White 1987a, Tang et al. 1976, Vanmarcke 1977b). In this regard the author extends the previous applications of the spatial averaging method presented by Vanmarcke (1977a, 1977b) and Li and White (1987a, 1987b) by accounting for the sampling errors and the locations of measurements. Both unconditional and conditional simulation approaches are described and illustrated. Reliability-based computational techniques are then used to obtain a probability of slope failure. The results are compared with the conventional random variable approach to illustrate the influence of the various assumptions on the computed probability of failure.

2. Statistical Spatial Averaged Soil Properties

A random field is defined as a family of random variables at points with coordinates $\mathbf{x} = (x_1, \cdots, x_n)$ in an n-dimensional parameter space (Vanmarcke 1983). If the random field is Gaussian, then the random field $\upsilon(\mathbf{x})$ can be completely described by its mean function $\mu(\mathbf{x})$, variance function $\sigma^2(\mathbf{x})$, and the correlation $\rho(\mathbf{x}, \mathbf{x}')$. Non-Gaussian random fields, in general, need more information beyond second moment statistics to completely describe them. One special case is when the random fields are defined by the Nataf multivariate distribution (Liu and Der Kiureghian 1986), which is adopted in this study when modeling non-Gaussian random fields.

The spatial average $\bar{\upsilon}$ of a random soil property $\upsilon(\mathbf{x})$ of the certain element Ω_e (i.e., discretized zone of interest) can be defined as the stochastic integral (Vanmarcke 1983). For the sake of simplicity the author will first consider that the (sample) data are error free as:

$$\bar{\upsilon} = \frac{1}{V}\int_{\Omega_e} \upsilon(\mathbf{x})\,d\mathbf{x} \;\; ; \;\; V = \int_{\Omega_e} d\mathbf{x} \;\; ; \;\; \mathbf{x} \in \Omega_e \tag{1}$$

where Ω_e is an elementary volume, area or line in the three-, two- and one-dimensional cases, respectively.

The first and second moment statistics (i.e., mean, variance, and covariance) of spatial average $\bar{\upsilon}$ can then be manipulated in terms of the statistics of point random property $\upsilon(\mathbf{x})$ as:

$$E[\bar{\upsilon}] = E\left[\frac{1}{V}\int_{\Omega_e}\upsilon(\mathbf{x})\,d\mathbf{x}\right] = \frac{1}{V}\int_{\Omega_e}\mu(\mathbf{x})\,d\mathbf{x} \tag{2}$$

$$\mathrm{var}[\bar{\upsilon}] = E\left\{[\bar{\upsilon} - E(\bar{\upsilon})]^2\right\} = \frac{1}{V^2}\iint_{\Omega_e}\sigma(\mathbf{x})\sigma(\mathbf{x}')\rho(\mathbf{x},\mathbf{x}')\,d\mathbf{x}d\mathbf{x}' \tag{3}$$

where the random property can be described with trend and random components such that $\upsilon(\mathbf{x}) = \mu(\mathbf{x}) + \varepsilon(\mathbf{x})$. Similarly, covariance of the spatial average $\bar{\upsilon}$ between Ω_e and Ω_e' is given as:

$$\begin{aligned}\mathrm{cov}[\bar{\upsilon},\bar{\upsilon}'] &= E\{[\bar{\upsilon} - E(\bar{\upsilon})][\bar{\upsilon}' - E(\bar{\upsilon}')]\} \\ &= \frac{1}{VV'}\int_{\Omega_e}\int_{\Omega_e'}\sigma(\mathbf{x})\sigma(\mathbf{x}')\rho(\mathbf{x},\mathbf{x}')\,d\mathbf{x}d\mathbf{x}'\end{aligned} \tag{4}$$

If the field is a weakly stationary field, $\sigma^2(\mathbf{x})\sigma^2(\mathbf{x}')$ and $\rho(\mathbf{x}, \mathbf{x}')$ can be replaced by σ^2 and $\rho(\mathbf{r})$ respectively, where $\mathbf{r}$ is a lag distance vector between the $\mathbf{x}$ and $\mathbf{x}'$ such as $\mathbf{r} = (x_1 - x_1', x_2 - x_2', \cdots, x_n - x_n')$. Then the variation and covariance of the spatial average can further be simplified as:

$$\mathrm{var}[\bar{\upsilon}] = \sigma^2 \cdot \gamma(\Omega_e) \tag{5}$$

Where $\gamma(\Omega_e) = \frac{1}{V^2}\iint_{\Omega_e}\rho(\mathbf{r})\,d\mathbf{x}d\mathbf{x}'$ and $\mathbf{r} = \mathbf{x} - \mathbf{x}'$. Similarly,

$$\mathrm{cov}[\bar{\upsilon},\bar{\upsilon}'] = \sigma^2 \cdot \gamma(\Omega_e, \Omega_e') \qquad (6)$$

Where $\gamma(\Omega_e, \Omega_e') = \frac{1}{VV'} \int_{\Omega_e} \int_{\Omega_e'} \rho(\mathbf{r})\, d\mathbf{x} d\mathbf{x}'$.

In the above formulations, $\gamma(\Omega_e)$ and $\gamma(\Omega_e, \Omega_e')$ may be called the variance and covariance reduction factors, respectively (Vanmarcke 1983). It should be noted that the reduction factors are dependent only on the correlation function (i.e., scale of fluctuation, etc.) and geometry of the domain of interest and independent of the magnitude of the point variance. The variance reduction factor is bounded by 0 to 1 since the correlation coefficient is always equal or less than unity. Therefore, the variability of local average is always less than that of the point value and further decreases as the size of the averaged domain increases.

The first and second moment statistics of the spatial averages can be obtained once the variance and covariance reduction factors are determined. Detailed derivations of statistics of the linear and areal averages for specific geometries can be found in Vanmarcke (1977a, 1983) and Li and White (1987a).

3. Conditional and Unconditional Approach

Uncertainty in the determination of soil properties comes from various sources. One obvious source of uncertainty is the inherent randomness of the natural phenomena. Other sources of uncertainty include the inaccuracies in the estimation of the parameters and in the choice of the distribution representing the randomness, due to limited observational data. The following derivations are based on a (weakly) homogeneous random field. The author begins with the unconditional approach that does not account for the location of measurements.

3.1 Unconditional Approach

Suppose that random soil properties $\upsilon(\mathbf{x})$ have been observed at N points (or areas) inside the homogeneous zone of interest. Each measurement υ^*_i may be regarded as a realization of random properties $\upsilon(\mathbf{x})$ and statistics of random properties $\upsilon(\mathbf{x})$ can be estimated from the measured values. Let us assume, for the moment, that the observations are made at sufficiently large distance from each other so that correlation between samples can be neglected for practical purpose (statistically independent). Unbiased sample moments may be used as point estimates of the corresponding moments of population such as (e.g., Ang and Tang 1975):

$$\mu \approx \hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} \upsilon_i^* \qquad (7)$$

$$\mathrm{var}[\upsilon] \approx \sigma^{*2} \qquad (8)$$

The above estimates, however, do not convey information on the degree of accuracy of those estimates of parameters, which depends mainly on the number of the observations. The observational data υ^*_i can be conceived to be realizations of a set of independent sample random variables $V^*_i\,; i = 1, 2, \cdots, N$ among the population and then the sample mean $\hat{\mu}$ can be regarded a random variable, given as:

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} V_i^* \qquad (9)$$

Its mean value is given as:

$$E[\hat{\mu}] = \frac{1}{N} \sum_{i=1}^{N} E(V_i^*) = \frac{1}{N} \cdot N\mu = \mu \qquad (10)$$

and its variance is:

$$\mathrm{var}[\hat{\mu}] = \frac{1}{N^2} \sum_{i=1}^{N} \mathrm{var}[V_i^*] = \frac{\sigma^{*2}}{N} \qquad (11)$$

Thus, the sample mean $\hat{\mu}$ has a mean value μ (i.e., unbiased estimator) and standard deviation (or error) $\sigma^* / \sqrt{N}$.

Now, the first two moments of spatial average can be estimated based on the observational data, accounting for not only point estimates but also the degree of accuracy of those estimations. Here we first define the spatial average over the element domain Ω_e in the same way as we did in the previous sections.

$$\bar{\upsilon} = \frac{1}{V}\int_{\Omega_e} \upsilon(\mathbf{x})\,d\mathbf{x}$$
$$= \frac{1}{V}\int_{\Omega_e} [\mu(\mathbf{x}) + \varepsilon(\mathbf{x})]\,d\mathbf{x} \;\; ; \;\; V = \int_{\Omega_e} d\mathbf{x} \;\; ; \;\; \mathbf{x} \in \Omega_e \quad (12)$$

For a homogeneous random field, $\mu(\mathbf{x}) = \mu$ and $E[\varepsilon(\mathbf{x})\varepsilon(\mathbf{x}')] = \sigma^2 \rho(\mathbf{r})$. The expected value of the spatial average can be evaluated by replacing μ with sample mean $\hat{\mu}$ as:

$$E[\bar{\upsilon}] \approx E\left[\frac{1}{V}\int_{\Omega_e} (\hat{\mu} + \varepsilon(\mathbf{x}))\,d\mathbf{x}\right] = E[\hat{\mu} + \varepsilon(\mathbf{x})] = \mu \quad (13)$$

Similarly, the variance of the spatial average is estimated (see the Appendix for the derivation):

$$\mathrm{var}[\bar{\upsilon}] \approx \frac{\sigma^{*2}}{N} + \sigma^2 \gamma(\Omega_e) \quad (14)$$

Finally, the covariance between two spatial averages is given (see the Appendix for the derivation):

$$\mathrm{cov}[\bar{\upsilon}, \bar{\upsilon}'] \approx \frac{\sigma^{*2}}{N} + \sigma^2 \gamma(\Omega_e, \Omega_e') \quad (15)$$

The first term in the above two solutions (Equations 14 and 15) represents sampling errors (i.e., uncertainties in the estimation of the sample mean) while the second term is the reduced inherent variance due to the spatial average. It should be noted that the equations explicitly separate spatial correlation and sampling-related uncertainty. In the special case of $\mathrm{var}(\hat{\mu}) = 0$, which happens when $N \rightarrow \infty$, the above two solutions (Equations 14 and 15) become identical with Equations 5 and 6. Ang and Tang (1984) and Tang (1984) reported a relationship that is similar to Equation 14. Their proposed formula combines various individual sources of uncertainties in determining the c.o.v. (i.e., σ / μ) of the spatial average soil property. Their formula, however, is based on the first order approximation of various sources of uncertainties that are factored (i.e., in a multiplicative form), and therefore may not be applicable to a problem with sources of large uncertainty. Li and White (1987a) also reported similar relationships.

Although the above formulation provides a simple and yet systematic tool to quantify the absolute and relative uncertainty in the determination of soil properties, in some situations as described in the following section, more general approach may be necessary to consider the correlation between the measurements.

3.2 Conditional Approach

An important and highly desirable characteristic of a random field simulation is that the random field simulation reproduces the observed values at their respective sampling locations. Conditional simulation has this very desirable property and it has been extensively used in many different applications, particularly mineral exploration (see e.g. Krige 1966; Matheron 1967; Journel 1989).

When a prior estimate of the mean value of a property is not available, as is usually the case in most field exploration problems, a linear estimator may be expressed as a weighted linear combination of the observed values in the form:

$$\hat{\upsilon}_a = \hat{\upsilon}(\mathbf{x}_a) = \sum_{j=1}^{N} w_{aj} \upsilon_j^* \quad (16)$$

Requirements that the estimator be unbiased and the expected value of its squared error be minimal yield the following conditions for weights w_{aj}:

$$\sum_{j=1}^{N} w_{aj} = 1 \quad (17)$$

$$\sum_{j=1}^{N} w_{aj}\sigma_{jk} - \lambda_a = \sigma_{ak}; \;\; k = 1, 2, \ldots, N \quad (18)$$

where λ_a is a Lagrange multiplier. The above two equations are a system of N+1 linear equations with N unknowns w_{aj} and λ_a. A measure of the error in the estimation can be given in terms of the expected value of the squared error at the minimum condition (sometimes called "ordinary kriging variance"):

$$\sigma_{OK,a}^2 = E[(\hat{\upsilon}_a - \upsilon_a)^2] = \sigma_a^{\,2} - \sum_{j=1}^{N} w_{aj}\sigma_{ja} + \lambda_a \quad (19)$$

The second term in the right hand side of Equation 19 represents the reduction in the variance of the estimator from the ensemble (or point) variance as a result of spatial correlation. If all of the observation points and the point to be estimated are separated far enough to be $\sigma_{ja} \approx 0$; $\sigma_{ij} \approx 0$ for $i \neq j$, the estimate may equal to the arithmetic mean, and the ordinary kriging variance may reduce to:

$$\sigma^2_{OK,a} \approx \sigma_a^{\ 2} + \lambda_a \approx \sigma^2\gamma(\Omega_a) + \frac{\sigma^{*2}}{N} \qquad (20)$$

because $w_{aj}\sigma_{jj} \approx w_{aj}\,\mathrm{var}[\upsilon_j^*] = w_{aj}\sigma^{*2} \approx \lambda_a$; $w_{aj} \approx \frac{1}{N}$ from Equations 17 and 18.

Similarly, the covariance between two values $\hat{\upsilon}_a$ and $\hat{\upsilon}_b$ is given by:

$$\sigma_{OK,ab} = E[(\hat{\upsilon}_a - \upsilon_a)(\hat{\upsilon}_b - \upsilon_b)] = \sigma_{ab} - \sum_{k=1}^{N} w_{ak}\sigma_{kb} + \lambda_a \qquad (21)$$

alternatively,

$$\sigma_{OK,ab} = E[(\hat{\upsilon}_a - \upsilon_a)(\hat{\upsilon}_b - \upsilon_b)] = \sigma_{ab} - \sum_{k=1}^{N} w_{bk}\sigma_{ka} + \lambda_b \qquad (22)$$

Again when $\sigma_{ja} \approx 0$; $\sigma_{ij} \approx 0$ for $i \neq j$, the ordinary kriging covariance reduces to:

$$\sigma_{OK,ab} \approx \sigma_{ab} + \lambda_a \approx \sigma^2\gamma(\Omega_a,\Omega_b) + \frac{\sigma^{*2}}{N} \qquad (23)$$

It should be noted that uncertainties arising from the sampling errors are implicitly included in Equations 19 and 21, because the mean value needs to be estimated based on observations. Also, Equations 20 and 23 are identical to Equations 14 and 15 (based on the unconditional evaluation) as they should be.

4. Probability of Failure

Conceptually, the performance of a structure can be described by a limit state function $g(\mathbf{x})$ such that failure is defined whenever the condition of $g(\mathbf{x}) \leq 0$ is satisfied, where $\mathbf{x}$ is the vector of model variables. The probability of failure is then given by:

$$p_f = P(g(\mathbf{x}) \leq 0) = \int_{g(\mathbf{x})\leq 0} f(\mathbf{x})\,d\mathbf{x} \qquad (24)$$

where $f(\mathbf{x})$ is the joint probability density function (PDF) of $\mathbf{x}$.

During last four decades, a number of computational methods have been developed to efficiently solve the problem. These include the mean-value first order second moment (MVFOSM or often simply FOSM) (Cornell 1969) and the first- and second-order reliability methods (FORM and SORM) (Ang and Tang 1984; Madsen et al. 1986). A variety of other computation methods, including simulation methods (Rubinstein 1981; Shinozuka 1983) and response surface methods (Faravelli 1989) are also available.

5. Example Analyses

The purpose of the example analyses is to illustrate the influence of the various assumptions on the estimated soil properties and the resulting probabilities of failure

5.1 Influence of Spatial Correlation and Averaging

The author is interested in evaluating the risk of failure of a hyperthetic cohesive slope shown in Figure 1. Three vertical borings are carried out and 10 soil samples are taken at the specific locations shown in Figure 1. Subsequent tests yield a sample mean $\hat{\mu}_c = 45\ kN/m^2$ and a sample standard deviation $s_c(=\sigma^*_c) = 13.5\ kN/m^2$ for undrained shear strength, and $\hat{\mu}_\gamma = 18\ kN/m^3$ and $s_\gamma = 0.9\ kN/m^3$ for soil density. For practical purpose, these values can be considered as point statistics (e.g., Vanmarcke

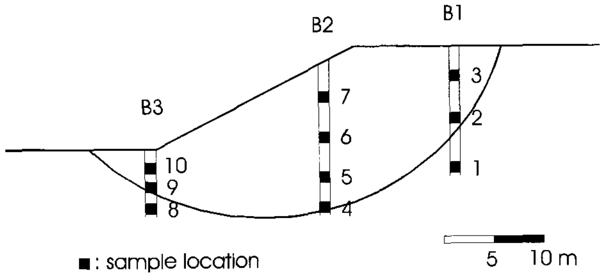
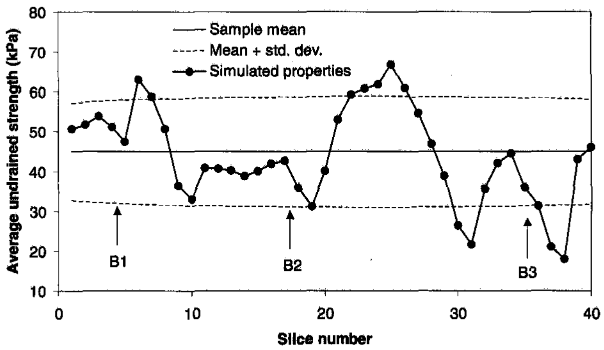


Fig. 1. Geometry and Sample Location of a Circular Slip Slope Surface

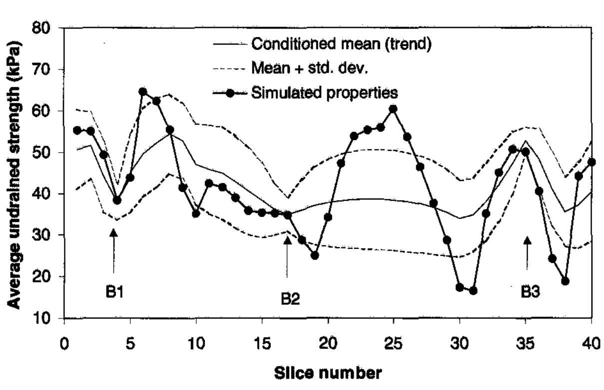
1977a). Previous experience with local geology indicates that the soil can be modeled as a homogeneous random field, and scales of fluctuation are taken as $\delta_x = 5m$ and $\delta_y = 1m$ respectively. A separable 2-D exponential autocorrelation function is employed to model the correlation (i.e., Vanmarcke 1983; Li and White 1987a). It is known that the computed statistics of the local average is generally not sensitive to the type of autocorrelation function (i.e., Vanmarcke 1977a).

For limit equilibrium analyses, the author is interested in simulating the random soil properties that represent the spatial averages of the vertical soil slices and thus we need to compute the statistics of averaged soil properties for each slice. Figures 2a and 2b show samples of simulated shear strengths over the 40 slices of the slip surface for both unconditional and conditional cases, assuming a Gaussian distribution. Scales of fluctuation used in those figures, however, are not $\delta_x = 5m$, $\delta_y = 1m$ but $\delta_x = 25m$, $\delta_y = 5m$ just for clear illustrative purpose. The undrained strengths generated by the conditional method (Figure 2b) show that the conditional mean (trend) is no longer stationary, but deviates from the sample mean and is influenced by the measured values nearby. The conditional standard deviation becomes smaller near the measurement points and gradually increases with increasing distance, ultimately reaching the value of the unconditional standard deviation. Hence the conditional approach leads to increasing variance reduction with increasing number of measurements as would be expected.

The correlation between the average shear strengths over the soil slices (Figure 3) shows that sampling error increases the correlation between the random quantities. It is interesting to observe that the conditional correlation does not decrease monotonically with distance but it even increases with increase of distance from the measurement points. That is because the conditional correlation depends not only on the lag distance between the random quantities of interest, but also depends on the distance from the sampling points to the point (or area) of interest.



(a) Unconditional Simulation (With Sampling Errors Considered)



(b) Conditional Simulation

Fig. 2. Simulation of Average Undrained Shear Strengths

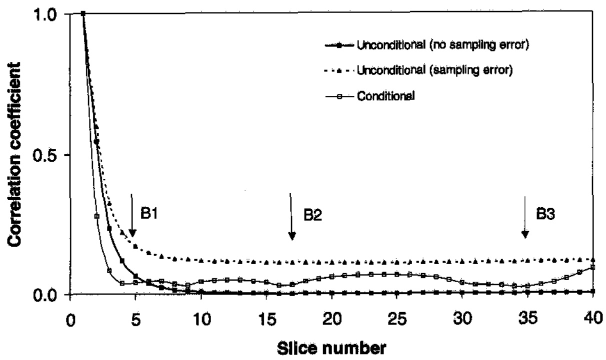


Fig. 3. Correlation Between the Line Averages with Respect to the Average of the First Slice

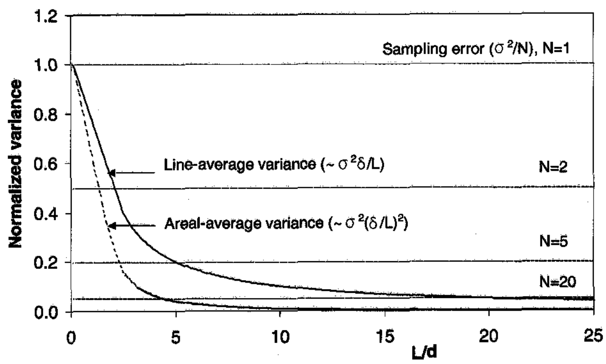


Fig. 4. Comparison of the Uncertainty Magnitudes Between the Spatial Variation (Inherent Uncertainty) and Sampling error (Statistical Uncertainty); N is the Number of Tests, L is the Scale of Averaging, and δ is the Scale of Fluctuation (Kim, 2003)

Figure 4 shows the comparison of magnitude of uncertainty between the spatial variation and sampling error. Unlike the inherent uncertainty, errors from the insufficient data and imperfect measurement do not decrease by averaging over the area of space, but depend on the number of samples.

5.2 Reliability Analysis of Slope Stability

The potential sliding mass was divided into 40 vertical soil slices of equal width for stability analyses using the Simplified Bishop method. Deterministic analyses of the static slope stability, with the mean soil properties, yielded a factor of safety 1.52. Analyses with more adverse soil properties ($\mu - \sigma$ for the shear strengths and $\mu + \sigma$ for soil density) resulted in a factor of safety 1.02.

The limit-state function for the reliability analyses was defined as:

$$g(\mathbf{x}) = FS(\mathbf{x}) - 1$$

which defined the slope to be safe for factor of safety $FS(\cdot)$ greater than one. The reliability computations were carried out with the aid of CALREL, a general-purpose structural reliability analysis program developed by Liu et al. (1989), linked to user-defined subroutine programs for static slope stability analyses including STAGLEM and GLEM developed by Kim (2001). The undrained strength of soil was modeled with both normal and lognormal distributions in order to examine the influence of uncertainty of distribution forms on the risk level of the problem. The soil properties were modeled using the same parameters as used in the previously discussed statistical analyses.

The computed distributions (densities) of the factors of safety are shown in Figure 5. The mean is essentially the same for all the cases but the shape of the distribution for the case of the sampling error added (both conditional and unconditional cases) is more dispersed due to the added uncertainty and, consequently, resulted in a higher probability of failure. It should be noted that the area underneath the density function with the factor of safety FS less than one is the probability of failure. Reflecting the variance reduction from the spatial averaging, the density of the unconditional approach (without sampling error) is relatively narrower than that of the conventional random value approach. In this particular example problem, the differences between the unconditional and conditional cases are relatively small because the additional variance reduction by conditioning is small due to the relatively small scale of fluctuation ($\delta_x = 5m$ and $\delta_y = 1m$).

The results in Table 1 also show that the risk of failure is sensitive to the choice of the distribution model of random soil properties (i.e., an order of magnitude difference in P_f). In this particular case, the analyses with soil properties assumed to have normal distributions consistently yield higher probability of failure than with lognormal distributions. That may be mainly because the normal distribution has more density (or weight) in smaller values than the lognormal distribution with the same mean and standard deviation, since the lognormal is non-negative. Thus, if the distribution form is uncertain, it may be worth examining distributions other than normal distribution.

The scale of fluctuation has significant effect on the reliability of the slope stability, especially for the unconditional approach without sampling errors (Figure 6). As expected, risk of failure significantly increases with increase

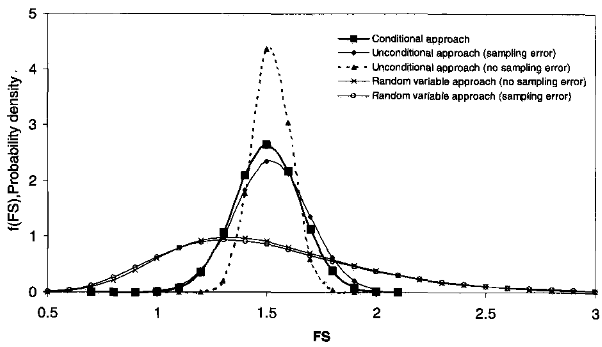


Fig. 5. Probability Distributions of Factors of Safety ($\delta_x = 5m$ and $\delta_y = 1m$) with Soil Properties Normally Distributed

Table 1. Probabilities of failure (P_f) for three different approaches with normal and lognormal distributions

Source of uncertainty	Normal	Lognormal
Spatial variation only	2.1×10^{-9}	1.2×10^{-12}
Sampling error added	1.4×10^{-3}	2.2×10^{-4}
Conditional approach	4.2×10^{-4}	6.1×10^{-5}

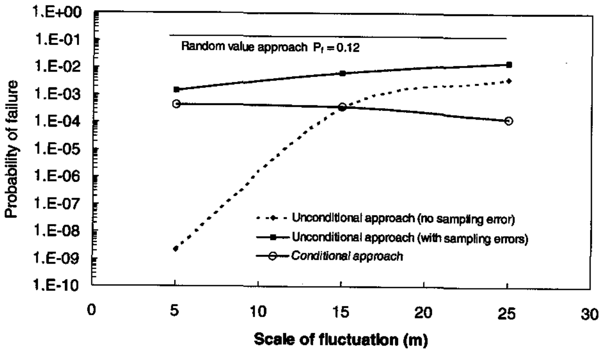


Fig. 6. Effects of the Scale of Fluctuation on the Failure Probability of the Slope

of the scale of fluctuation, since the variance reduction decreases. Uncertainty arising from sampling errors substantially contributes to the risk of failure, thus illustrating the importance of the sampling errors in the assessment of slope stability problem. In contrast, the results of the unconditional approach with sampling errors and conditional approach are relatively insensitive to the scale of fluctuation. That is partly due to the fact that with the increasing scale of fluctuation the increase in variance reduction by conditioning is offset by a decrease in the variance reduction as a result of spatial averaging. The fact that these approaches are less sensitive to the scale of fluctuation can be potentially important implication, since accurate determination of the scale of fluctuation has been problematic and requires an additional effort beyond that needed to obtain the mean and standard deviation. This example also shows that significant variance reduction can be achieved by spatial averaging, and illustrates the importance of spatial correlation with the soil property determination. The results suggest that if the size of the averaging domain is sufficiently large, the variance associated with inherent uncertainty may be practically neglected, thus allowing us to focus on the statistical uncertainty and measurement errors

6. Conclusions

The results of analyses confirm that the variability of the local average is always less than that of the point value and that it decreases with increase of the size of the averaging domain. Another important outcome of the stochastic treatment is that the variability of the local average always decreases as the dimension of a domain increases. The variance reduction due to averaging of an area or a volume can be more significant than line averaging. Thus, the uncertainty of the area or volume-averaged soil properties is consequently often far less than that of the point properties.

Unlike the inherent uncertainty, sampling-related uncertainty does not decrease by averaging of the area or space, but depends on only the number of samples.

The results suggest that if the size of the averaging domain is sufficiently large, relative to the scale of fluctuation, the variance associated with inherent uncertainty may be practically neglected, thus allowing us to focus on the sampling-related uncertainty such as the statistical uncertainty.

The conditional approach, while computationally more intensive, offers the advantage of honoring the data at the respective sampling points and it is particularly well suited in situations with large numbers of samples in a highly correlated random field.

Estimates of probability of failure obtained from reliability analyses show that the conditional approach, which accounts for the inherent spatial variability of the soil deposit and sampling errors, leads to significantly lower estimates of the probability of failure than that obtained by using simple random variable (perfectly correlated soil) approach. The proposed unconditional approach that accounts for sampling-related uncertainty also results in a good approximation to the results of more general conditional approach.

Finally, the analyses strongly suggest that the spatial variability and sampling error have to be properly incorporated in slope stability analysis.

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Appendix

The following derivations may help the reader understand the developed equations.

$$\begin{aligned} \mathrm{var}[\hat{\mu}] &= \mathrm{var}\left[\frac{1}{N}\sum_{i=1}^{N} V_i^*\right] \\ &= \frac{1}{N^2}\left[\sum_{i=1}^{N}\mathrm{var}(V_i^*)\right] \\ &= \frac{\sigma^{*2}}{N} \end{aligned} \qquad \text{Equation (11)}$$

$$\begin{aligned} \mathrm{var}[\overline{\upsilon}] &= E[(\overline{\upsilon}-\mu)^2] \approx E\left\{\left[\frac{1}{V}\int_{\Omega_e}(\hat{\mu}-\mu)\,d\mathbf{x} + \frac{1}{V}\int_{\Omega_e}\varepsilon(\mathbf{x})\,d\mathbf{x}\right]^2\right\} \\ &= \frac{1}{V^2}\int_{\Omega_e}\int_{\Omega_e} E[(\hat{\mu}-\mu)(\hat{\mu}'-\mu)]\,d\mathbf{x}d\mathbf{x}' + \frac{1}{V^2}\int_{\Omega_e}\int_{\Omega_e} E[\varepsilon(\mathbf{x})\varepsilon(\mathbf{x}')]\,d\mathbf{x}d\mathbf{x}' \\ &= \frac{1}{V^2}\int_{\Omega_e}\int_{\Omega_e}\mathrm{var}(\hat{\mu})\,d\mathbf{x}d\mathbf{x}' + \frac{1}{V^2}\int_{\Omega_e}\int_{\Omega_e}\sigma^2\rho(\mathbf{r})\,d\mathbf{x}d\mathbf{x}' \\ &= \frac{\sigma^{*2}}{N} + \sigma^2\gamma(\Omega_e) \end{aligned}$$

Equation (14)

$$\begin{aligned} \mathrm{cov}[\overline{\upsilon},\overline{\upsilon}'] &= E[(\overline{\upsilon}-\mu)(\overline{\upsilon}'-\mu)] \\ &\approx E\left\{\left[\frac{1}{V}\int_{\Omega_e}(\hat{\mu}-\mu)\,d\mathbf{x} + \frac{1}{V}\int_{\Omega_e}\varepsilon(\mathbf{x})\,d\mathbf{x}\right]\left[\frac{1}{V'}\int_{\Omega_e'}(\hat{\mu}'-\mu)\,d\mathbf{x} + \frac{1}{V}\int_{\Omega_e}\varepsilon(\mathbf{x}')\,d\mathbf{x}\right]\right\} \\ &= \frac{1}{VV'}\int_{\Omega_e'}\int_{\Omega_e} E[(\hat{\mu}-\mu)(\hat{\mu}'-\mu)]\,d\mathbf{x}d\mathbf{x}' + \frac{1}{VV'}\int_{\Omega_e'}\int_{\Omega_e} E[\varepsilon(\mathbf{x})\varepsilon(\mathbf{x}')]\,d\mathbf{x}d\mathbf{x}' \\ &= \frac{1}{VV'}\int_{\Omega_e'}\int_{\Omega_e}\mathrm{var}(\hat{\mu})\,d\mathbf{x}d\mathbf{x}' + \frac{1}{VV'}\int_{\Omega_e'}\int_{\Omega_e}\sigma^2\rho(\mathbf{r})\,d\mathbf{x}d\mathbf{x}' \\ &= \frac{\sigma^{*2}}{N} + \sigma^2\gamma(\Omega_e,\Omega_e') \end{aligned}$$

Equation (15)

References

1. Ang, A. H. S., and Tang, W. H. (1984), *Probability Concepts in Engineering Planning and Design, Volume II-Decision, Risk, and Reliability*, John Wiley & Sons, New York.
2. Ang, A. H. S., and Tang, W. H. (1975), *Probability Concepts in Engineering Planning and Design, Volume I-Basic Principles*, John Wiley & Sons, New York.
3. Christian, J. T., Ladd, C. C., and Baecher, G. B. (1994), "Reliability Applied to Slope Stability Analysis", *Journal of Geotechnical Engineering*, ASCE, Vol.120, No.12, pp.2180-2207.
4. Christian, J. T., and Urzua, A. (1998), "Probabilistic Evaluation of Earthquake-Induced Slope Failure", *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol.124, No.11, pp.1140-1143.
5. Cornell, C. A. (1969), "Structural Safety Specification Based on Second-Moment Reliability", *Symposium, International Association of Bridge and Structural Engineers*, London.
6. Duncan, J. M. (2000), "Factors of Safety and Reliability in Geotechnical Engineering", *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol.126, No.4, pp.307-316.
7. Faravelli, L. (1989), "Response Surface Approach for Reliability Analysis", *Journal of Engineering Mechanics*, ASCE, Vol.115, No.12, pp.2763-2781.
8. Journel, A. G. (1989), *Fundamentals of Geostatistics in Five Lessons*, American Geophysical Union.
9. Kim, J. (2001), *Probabilistic Approach to Evaluation of Earthquake-Induced Permanent Deformation of Slopes*, Ph.D. Dissertation, Dept. of Civil and Env. Engineering, University of California at Berkeley.
10. Kim, J. (2003), "The Importance of Geotechnical Variability in the Analysis of Earthquake-Induced Slope Deformations", *Journal of Korean Geotechnical Society*, Vol.19, No.2, pp123-133
11. Krige, D. G. (1966), "Two-Dimensional Weighted Moving Averaging Trend Surfaces for Ore Evaluation", *Proceedings of Symposium on Mathematical Statistics and Computer Applications for Ore Evaluation*, Johannesburg, S. A., pp.13-38.
12. Li, K. S., and Lumb, P. (1987), "Probabilistic Design of Slopes", *Canadian Geotechnical Journal*, Vol.24, pp.520-535.
13. Li, K. S., and White, W. (1987a), *Probabilistic Characterization of Soil Profiles*, Research Report No.19, Department of Civil Engineering, University College, The University of New South Wales, Canberra, Australia.
14. Li, K. S., and White, W. (1987b), *Probabilistic Approaches to Slope Design*, Research Report No. 20, Department of Civil Engineering, University College, The University of New South Wales, Canberra, Australia.
15. Liu, P. L., and Der Kiureghian, A. (1986), "Multivariate Distribution Models with Prescribed Marginals and Covariances", *Probabilistic Engineering Mechanics*, Vol.1, No.2, pp.105-112.
16. Liu, P. L., Lin, H. Z., and Der Kiureghian, A. (1989), *CALREL User Manual*, Report No. UCB/SEMM-89/18, Department of Civil Engineering, University of California at Berkeley, USA.
17. Madsen, H. O., Krenk, S., and Lind, N. C. (1986), *Methods of Structural Safety*, Prentice-Hall, Englewood Cliffs, NJ.
18. Matheron, G. (1967), "Kriging or Polynomial Interpolation Procedures", *Canadian Institute of Mining Bulletin*, Vol.60, pp.1041.
19. Rubinstein, R. Y. (1981), *Simulation and the Monte Carlo method*, John Wiley & Sons, New York.
20. Shinozuka, M. (1983), "Basic Analysis of Structural Safety", *Journal of Structural Engineering*, ASCE, Vol.109, No.3, pp.721-740.
21. Tang, W. H. (1984), "Principles of Probabilistic Characterization of Soil Properties", *Proceedings of the Symposium on Probabilistic Characterization of Soil Properties*, Atlanta, Georgia, pp.74-89.
22. Tang, W. H., Yucemen, M. S., and Ang, A. H. S. (1976), "Probability-Based Short Term Design of Soil Slope", *Canadian Geotechnical Journal*, Vol.13, pp.201-215.
23. Vanmarcke, E. (1983), *Random Fields: Analysis and Synthesis*, The MIT Press, Cambridge, Massachusetts.
24. Vanmarcke, E. H. (1977a), "Probabilistic Modeling of Soil Profiles", *Journal of Geotechnical Engineering Division*, ASCE, Vol.103 (GT11), pp.1227-1246.
25. Vanmarcke, E. H. (1977b), "Reliability of Earth Slopes", *Journal of Geotechnical Engineering Division*, ASCE, Vol.103 (GT11), pp.1247-1265.

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