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A Note on the Reversibility of the Two Stage Assembly Scheduling Problem*

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ABSTRACT

This paper is concerned with proving a conjecture that the two stage assembly system is reversible in deterministic makespan scheduling context. The reversibility means that a job sequence in the assembly system has the same makespan as that of its reverse sequence in the disassembly system which is the reversal of the assembly system. The proposed conjecture shows that the reversibility of serial flowshops can be extended to non-serial and synchronized shops.

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1. Introduction

The two stage assembly system consists of multiple fabrication machines in the first stage and a final assembly machine in the second stage. Each fabrication machine produces its own type of component independently of the other machines. The assembly machine can start its processing for a final product only when all the components of the product are available from the precedent fabrication machines. This shop model often appears in producing more larger volume of products than any product produced in serial flowshops. For example, the body and the chassis, which are the components of a fire engine, can be manufactured independently and then brought into the assembly machine. Production of the body and the chassis can take place in parallel, but the final assembly machine cannot start its processing until the body and the chassis are delivered to the assembly machine (referring to Lee *et al.* [6]). Sun *et al.* [10] have suggested another application of flexible manufacturing cell for machining various components and assembling them into many different kinds of products in small lot. Similarly, in computation work, two or more sub-programs (tasks) are independently processed first at their own parallel processors and then gathered at a main processor for final data-processing, where they either wait for all of their siblings to finish processing or are put together when processing is done on all the siblings. See Potts *et al.* [9], Hariri and Potts [4], Yoon and Sung [11] and Duda and Czachorski [1] for more industrial applications and the related scheduling researches in the two stage assembly system.

This paper is concerned with proving a conjecture that the two stage assembly system is reversible in deterministic makespan scheduling context. The reversibility means that a job sequence in the assembly system has the same makespan as that of its reverse sequence in the reversal of the assembly system, which begins with the assembly machine and ending with the associated multiple fabrication machines. The reversal of the assembly system corresponds to a disassembly system providing applications in industry such as splitting products into their constituent components in a nondestructive manner, which frequently appears in operation of waste handling and repair facilities [2]. When a mishap to a large product occurs, the product may be overhauled through the work sequence of a main disassembly station and its subsequent diagnosis/repair shops for the disassembled sub-parts. For example, the overhaul of aircraft involves a disassembly configuration [5]. Individual aircraft is disassembled into major components and these components are repaired simultaneously in separate phases. This aspect requires splitting and parallel operational phases to be introduced into the overall overhaul model. In the phases, even if the subsequent parallel shops are idle, they cannot start processing until arrival of a new aircraft completing its disassembly operation at the main splitting station. We can also see O'Shea *et al.* [8] and Gungor and Gupta [3] for reviews of various disassembly planning problems. By the reversibility property, one of the two makespan scheduling problems concerned with assembly and disassembly systems can be transferred to the other one, so that both problems have the same complexity.

This paper is motivated by the reversibility for serial flowshops in deterministic scheduling area [7] and also for queueing systems in assembly/disassembly systems, where three machine assembly system and three machine flowshop markov queueing models have the same steady-state distribution [2]. However, nobody has investigated the issue yet that the reversibility can be extended to non-serial and synchronized shops in scheduling context.

2. Two Stage Assembly and Disassembly Scheduling Problems

There are n jobs $J_1, J_2, \cdots, J_n$ to be processed. Each job consists of m tasks, and it is completed by performing all its tasks which are interrelated in serial or simultaneous processing order. The objective is to find the schedule of jobs which minimizes the makespan in each of the two systems including two stage assembly and disassembly systems.

The two stage assembly system (called AS) consists of m machines, that is, $m-1$ fabrication machines, M_k for $k=1,2,\cdots,m-1$, in the first stage and an assembly machine, M_a, in the second stage. Each job is processed in such a way that each task must be processed at its assigned machine. All the $m-1$ tasks of each job individually assigned to the fabrication machines can be processed simultaneously, but the final assembly task cannot start its processing at the assembly machine until all the precedent $m-1$ fabrication tasks are completed. This implies that the makespan is measured as the completion time of the last assembly task.

In the two stage disassembly system (called DS), the disassembly machine in the first stage, M_d, performs the disassembly task to split the job into $m-1$ parallel tasks, and then $m-1$ subsequent machines in the second stage, M_k, $k=1,2,\cdots,m-1$, process the tasks in one to one pattern. It follows that the completion time of each job is the maximum completion time of its parallel tasks and the makespan is the maximum value of the completion times of all jobs.

For convenience, the following notations are introduced;

$p_{i,k}$: task processing time of job J_i on machine M_k,
p_k : set of processing times on machine k, $p_k = \{p_{1,k}, p_{2,k}, \cdots, p_{n,k}\}$,
S : a complete schedule, and
$C_\alpha(S)$: makespan of S in α system, $\alpha \in \{\text{AS, DS}\}$.

Two scheduling problems for minimizing the makespan in the two stage assembly and disassembly systems are denoted by ASC and DSC, respectively.

3. Proof for the Reversibility

It is noted that the proposed problems are as hard combinatorial as common scheduling problems. Specifically, Lee *et al.* [6] and Potts *et al.* [9] have proved that the ASC problem is NP-complete in strong sense. Nevertheless, the following Lemma 1 shows that it is sufficient to consider only $n!$ permutation sequences for the ASC and DSC problems. In the verification, a pairwise interchange operation will be used. Given a job sequence on M_k, an operation $PI(J_i, J_j, M_k)$ will generate a new sequence, in which two jobs J_i and J_j are interchanged each other in their positions but with all other jobs kept the same positions as in the original sequence.

Lemma 1. For the m-machine ASC and DSC problems, only permutation sequences need to be considered.

Proof. For the case of the ASC problem, the proof has been given in Potts *et al.* [9]. Thus, only the case of the DSC problem needs to be proved. Without loss of generality, consider a schedule having different job sequences on a machine pair (M_d, M_k)

where M_k is a second stage machine. In this case, there exists two jobs J_i and J_j such that J_i is positioned immediately before J_j on M_k but J_j before J_i on M_d. In the schedule, an operation $PI(J_i, J_j, M_k)$ does not affect the sequences on any other second stage machines and it does not increase the completion time of any other jobs on M_k. This implies that if the interchange process is repeated for any other differently-positioned jobs on M_k, then it will end up with generating a permutation sequence on (M_d, M_k) which is no worse than the original sequence.

It should be noted that the result of Lemma 1 can be applied to the problems with not only makespan measure but also any regular measure of performance.

Lemma 2. For the m-machine ASC and DSC problems, schedules without any inserted idle time are dominant.

Proof. Given a permutation schedule, S, with idle time inserted at the time when a waiting job exists, it is always possible to generate a new schedule better than S by eliminating the idle time.

In the ASC problem, the makespan is the sum of total idle time and total processing time on M_a so that the optimal schedule is obtained by minimizing the total idle time. That is,

$$C_{AS}(S) = \sum_{i=1}^{n} I_{[i],a} + T_a \quad \text{and} \quad C_{AS}(S^*) = \min\left\{ \sum_{i=1}^{n} I_{[i],a} \middle| S \in \Omega \right\} + T_a,$$

where $I_{[i],k}$: idle time immediately prior to the i th positioned job $J_{[i]}$ on M_k,

Ω : set of all permutations,

S^* : an optimal schedule, and

T_a : total processing time on M_a, $T_a = \sum_{i=1}^{n} p_{i,a}$.

In DSC problem, the makespan is the greatest value among the completion times on second stage machines where each completion time is the sum of total idle time and total processing time at the associated machine. That is,

$$C_{DS}(S) = \max\left\{ \sum_{i=1}^{n} I_{[i],k} + T_k \,\middle|\, k = 1, 2, \cdots, m-1 \right\} \text{ and}$$

$$C_{DS}(S^*) = \min\left\{ \max\left\{ \sum_{i=1}^{n} I_{[i],k} + T_k \,\middle|\, k = 1, 2, \cdots, m-1 \right\} \,\middle|\, S \in \Omega \right\},$$

where T_k : sum of all the processing times on M_k, $T_k = \sum_{i=1}^{n} p_{i,k}$.

Then, let us now prove the reversibility of two scheduling problems. Let the reverse sequence of a permutation be defined as the sequence resulted by inverting the order of the permutation. For example, the reverse sequence of $J_3 J_4 J_1 J_2$ is $J_2 J_1 J_4 J_3$.

Theorem 1. A permutation sequence in the ASC problem with the associated processing times $(p_1, p_2, \cdots, p_{m-1}, p_a)$ has the same makespan as that of the reverse sequence in the DSC problem with $(p_d, p_1, p_2, \cdots, p_{m-1})$ where $p_d = p_a$.

Proof. Given an instance of the ASC problem, $(p_1, p_2, \cdots, p_{m-1}, p_a)$, and a permutation sequence S, the idle time immediately prior to job $J_{[i]}$ on M_a is derived as

$$I_{[i],a} = \max\left\{ \max\left\{ \sum_{j=1}^{i} p_{[j],k} \,\middle|\, k = 1, 2, \cdots, m-1 \right\} - \sum_{j=1}^{i-1} I_{[j],a} - \sum_{j=1}^{i-1} p_{[j],a}, 0 \right\} \text{ for } i = 2, 3, \cdots, n,$$

$$I_{[i],a} = \max\{ p_{[1],k} \,|\, k = 1, 2, \cdots, m-1 \}.$$

Then, by summation of the idle times,

$$\begin{aligned} I_{[1],a} + I_{[2],a} &= \max\{ \max\{ p_{[1],k} + p_{[2],k} \mid k = 1, 2, \cdots, m-1 \} - p_{[1],a}, I_{[1],a} \} \\ &= \max\{ p_{[1],k} + p_{[2],k} - p_{[1],a}, p_{[1],k} \mid k = 1, 2, \cdots, m-1 \}, \end{aligned}$$

$$\begin{aligned} \sum_{j=1}^{3} I_{[j],a} &= \max\left\{ \max\left\{ \sum_{j=1}^{3} p_{[j],k} \,\middle|\, k = 1, 2, \cdots, m-1 \right\} - p_{[1],a} - p_{[2],a}, I_{[1],a} + I_{[2],a} \right\} \\ &= \max\left\{ \sum_{j=1}^{3} p_{[j],k} - p_{[1],a} - p_{[2],a}, p_{[1],k} + p_{[2],k} - p_{[1],a}, p_{[1],k} \,\middle|\, k = 1, 2, \cdots, m-1 \right\}. \end{aligned}$$

Continuing the summation up to the last job $J_{[n],a}$, the following expression is finally derived,

$$\sum_{j=1}^{n} I_{[j],a} = \max\left\{ \sum_{j=1}^{i} p_{[j],k} - \sum_{j=1}^{i-1} p_{[j],a} \;\middle|\; i = 1, 2, \cdots, n,\ k = 1, 2, \cdots, m-1 \right\}.$$

Therewith, the makespan of S can be expressed as

$$C_{AS}(S) = \sum_{j=1}^{n} I_{[j],a} + T_a = \max\left\{ \sum_{j=1}^{i} p_{[j],k} + \sum_{j=i}^{n} p_{[j],a} \;\middle|\; i = 1, 2, \cdots, n,\ k = 1, 2, \cdots, m-1 \right\}.$$

Now, let us consider another permutation sequence S' in the instance of the DSC problem, $(p_d, p_1, p_2, \cdots, p_{m-1})$. Then the corresponding idle times are derived as

$$I_{[i],k} = \max\left\{ \sum_{j=1}^{i} p_{[j],d} - \sum_{j=1}^{i-1} I_{[j],k} - \sum_{j=1}^{i-1} p_{[j],k}, 0 \right\} \text{ for } i = 1, 2, \cdots, n,\ k = 1, 2, \cdots, m-1,$$

$$I_{[i],k} = p_{[1],d} \text{ for } k = 1, 2, \cdots, m-1.$$

By computing in a similar way to the idle times of the ASC problem, we obtain

$$\sum_{j=1}^{n} I_{[j],k} = \max\left\{ \sum_{j=1}^{i} p_{[j],d} - \sum_{j=1}^{i-1} p_{[j],k} \;\middle|\; i = 1, 2, \cdots, n \right\}, \text{ for } k = 1, 2, \cdots, m-1.$$

Therewith, the makespan of S' is derived as

$$\begin{aligned} C_{DS}(S') &= \max\left\{ \sum_{j=1}^{n} I_{[j],k} + T_k \;\middle|\; k = 1, 2, \cdots, m-1 \right\} \\ &= \max\left\{ \sum_{j=1}^{i} p_{[j],d} + \sum_{j=1}^{n} p_{[j],k} \;\middle|\; i = 1, 2, \cdots, n,\ k = 1, 2, \cdots, m-1 \right\}. \end{aligned}$$

By introducing two new notations as

$$Y_{i,k}(S) = \sum_{j=1}^{i} p_{[j],k} + \sum_{j=1}^{n} p_{[j],a} \quad \text{and} \quad \tilde{Y}_{i,k}(S') = \sum_{j=1}^{i} p_{[j],d} + \sum_{j=1}^{n} p_{[j],k},$$

the respective makespans of the two problems can be restated as

$$C_{AS}(S) = \max\{ Y_{i,k}(S) \mid i = 1, 2, \cdots, n,\ k = 1, 2, \cdots, m-1 \} \text{ and}$$
$$C_{DS}(S') = \max\{ \tilde{Y}_{i,k}(S') \mid i = 1, 2, \cdots, n,\ k = 1, 2, \cdots, m-1 \}.$$

Therefore, if S' is the reverse sequence of S, then the hypothesis, $p_d = p_a$, proves the equality, $Y_{i,k}(S) = \tilde{Y}_{n-i+1,k}(S')$ for $i = 1, 2, \cdots, n$.

This completes the proof of the equality $C_{AS}(S) = C_{DS}(S')$.

Corollary 1. If S^* is an optimal sequence for an instance of the ASC problem, $(p_1, p_2, \cdots, p_{m-1}, p_a)$, then the reverse sequence of S^* is the optimal sequence for the DSC problem with $(p_d = p_a, p_1, p_2, \cdots, p_{m-1})$.
Proof. Recall that $n!$ permutations are dominant for the two problems. Moreover, a permutation of one problem corresponds to the reverse sequence of the other problem by Theorem 1.

The result of Corollary 1 can be used to characterizing the complexity of the DSC problem as in the following Corollary 2.

Corollary 2. The DSC problem is NP-complete in strong sense.
Proof. The result follows as the consequence of Corollary 1 and the NP-complete result for the ASC problem [6, 9].

Based on Theorem 1, a dominant sequence property can be derived as in the following Corollary 3.

Corollary 3. If a partial sequence, denoted by σ, is optimal in one of the ASC and DSC problems, then the reverse sequence of σ, denoted by σ', is optimal in the other problem.
Proof. Suppose that Π and Δ represent two partial sequences containing different jobs but excluding any jobs in the partial sequence σ. Without loss of generality, let $S = \Pi\sigma\Delta$ denote a full sequence for the ASC problem. Then, the given hypothesis implies that

$$C_{AS}(S) \leq C_{AS}(\Pi\tilde{\sigma}\Delta) \text{ for all possible partial sequences } \tilde{\sigma}, \ \Pi \text{ and } \Delta,$$

where $\tilde{\sigma}$ represents a different partial sequence including the same jobs as σ. Moreover, from Theorem 1, it follows that

$$C_{AS}(S) \leq C_{DS}(\Delta'\sigma'\Pi') \text{ and } C_{AS}(\Pi\tilde{\sigma}\Delta) = C_{DS}(\Delta'\tilde{\sigma}'\Pi'),$$

where Δ' and Π' represent the reverse sequences of Δ and Π, respectively. This leads the relation, $C_{DS}(\Delta'\sigma'\Pi') \leq C_{DS}(\Delta'\tilde{\sigma}'\Pi')$ for all possible partial sequences $\tilde{\sigma}'$, Δ' and Π'.

4. Concluding Remarks

This paper investigated the reversibility property of the two stage assembly system in deterministic makespan scheduling context. The reversibility implies that two scheduling problems in the assembly and disassembly systems are the same problem. Each problem has its own industrial applications and the related scheduling researches are important. By virtue of the reversibility, if any further research finds some solution properties or develops efficient heuristics for one of the two makespan problems, then the results can be directly applied to the other one.

It is necessary to check whether the method used to prove the reversibility of two stage systems works well for the multi-stage systems with non-serial configuration. This work remains in our further research.

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