Journal of
Ship&Ocean
Technology
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SOTECH Vol. 11, No. 3, pp. 1 ~ 13, 2007

Low Frequency Roll Motion of a Semi-Submersible Moored in Irregular Waves

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Abstract

A semi-submersible drilling rig is regarded as one of the typical offshore structures operated in the field with moderate environments such as the Gulf of Mexico, Brazil, and West Africa. Its typical roll and pitch natural periods are around 30 seconds, which avoids prevailing regions of the wave energy spectrum, and their responses in waves are quite acceptable for common operation conditions. But large roll and pitch motions can be induced by wave difference frequency energy spectrum if the metacentric heights of a semi-submersible decrease to small values in some loading conditions, and it is because the roll and pitch natural periods increase and approach to the region where the spectral density of the low frequency wave drift moment has significant value.

This paper describes the low frequency roll motion of a semi-submersible that are excited by the wave 2nd order difference frequency energy by a series of model experiments. From the model tests with several different initial metacentric heights (GM), it was observed that a semi-submersible can experience large roll motion due to the wave group spectrum.

Keywords: low frequency drift force, low frequency roll motion, semi-submersible, 2nd order wave QTF, instability, truncation method

1 Introduction

It is more than 40 years since semi-submersibles were introduced, initially to provide a working platform with minimum wave-induced motions for use in drilling operations. The evolution of the semi-submersible has been fostered by the need to overcome a number of problems arising from offshore exploration activities. There have been hundreds of semi-submersible of various sizes and configurations with the twin-pontoon design that has been the most popular design, and today many new design concepts of semi-submersible are introduced and some of them are realized or now in operation. In addition to drilling duties, the features that made the semi-submersible popular as a drilling rig have made it equally attractive for other applications such as pipe-laying, firefighting, heavy weight lifting, even for the production itself.

In spite of their remarkable developments, many factors still require attention and related problems still need to be solved. Restriction on the motion performance in the harsh environmental condition needs the development of new type semi-submersibles with better station-keeping ability. Challenges to the new design of semi-submersibles are now ongoing by many oil companies or offshore engineering institutes.

Low frequency drift force and motions have been extensively studied by many researchers (Nakamura 2000, Takezawa 1984). But their effect on the motion response of a moored semi-submersible is still very important, not only for the horizontal motions but also for the vertical motions.

This paper presents an extensive and detailed experimental study on the low frequency vertical motion by carrying out model tests with a delicate modeling of the mooring system by using "truncation method." The results of model tests with various loading conditions clearly show the significant effect of low frequency drift force on the roll motion response of a semi-submersible

2 Model Test

2.1 Test condition

Table 1 Main particulars of the semi-submersible

Scale ratio	unit	Design: survival condition	
80		Real	Model
GMT	m	7.78	0.097
Roll gyradius	m	32.51	0.406
Yaw gyradius	m	38.58	0.482
Heave natural period	sec	22.20	2.482
Roll natural period	sec	30.30	3.388

Table 2: Design irregular wave condition

Wave condition	100 year GoM Hurricane	
Hs [m]	12.2	0.153
Tp [sec]	14.2	1.588
Spectrum type	JONSWAP	
γ	2.0	

Model experiments were carried out in *Samsung Ship Model Basin* (SSMB) with the L×B×T dimensions of 400m×14m×7m.

A semi-submersible operating in the Gulf of Mexico with the design water depth of 5,000 ft (=1,524 m) is used and a 1/80 scale model is manufactured for the tests. Table 1 shows the main particulars of the semi-submersible model.

The main data in the table are the designed value when the semi-submersible is free floating in designed draft of 36.0 meters. Note that the GMT value is varied in the following several test conditions in order to change the roll natural period of the model.

Table 2 shows the design environment condition for the tests, which is the GoM 100 year return period wave incoming with 135.0 degrees heading angle, and wind and current are not considered in the tests.

2.2 Measurements

Table 3: Measurements and sensors

Measurements		Ch#	Sensor
Wave elevation	center	1	Capacity-type wave probe
	front	2	
	side	3	
6 DOF Motions	Surge	4	RODYM 6DMM
	Sway	5	
	Heave	6	
	Roll	7	
	Pitch	8	
	Yaw	9	
Air-gap	Front (AG_fwd)	10	Resistance-type wave probe
	Center (AG_c)	11	
	Aft (AG_aft)	12	
Run-up of columns	Column1 (RU1)	13	Resistance-type wave probe
	Column3 (RU3)	14	
Mooring line top tension	Column1 (TN1)	15	Strain-type load cell
	Column2 (TN2)	16	
	Column3 (TN3)	17	
	Column4 (TN4)	18	
Acceleration of Deck center	Accel. X	19	3-channel Accelerometer
	Accel. Y	20	
	Accel. Z	21	

Table 3 shows the list of measured signals, and Figure 1 shows the sensor installation on the semi-submersible model.

For the measurement of the incoming wave elevation, two capacity-type wave probes are installed at the front and side of the model, and six degree-of-freedom motions are measured by using a non-contact optical dynamic motion measuring system with three CCD cameras and three LED targets. Top tensions at the ends of the mooring lines are measured by the load cells installed between the fairleads and mooring lines.

Five resistance-type wave probes are installed to measure the relative motions ; *air-gap* to measure the relative motion between the free surface and main deck and *run-up* to measure the relative wave heights coming up along the column surface.

One 3-channel accelerometer is installed to measure and verify the acceleration on the deck center. Figure 2 shows the overview of model setup for the tests.

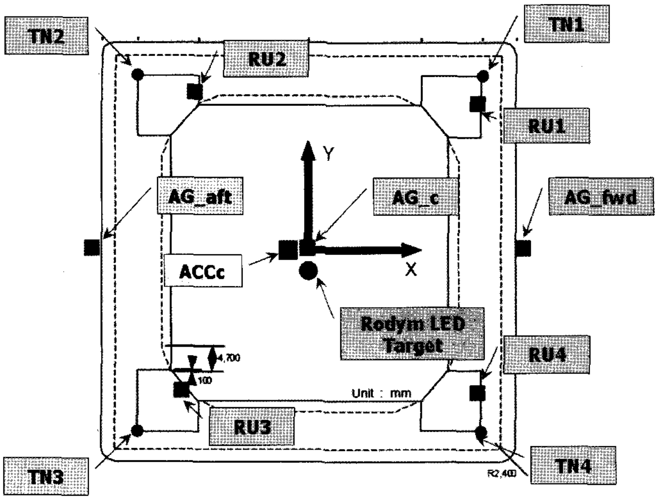

Figure 1: Measuring sensors on the model

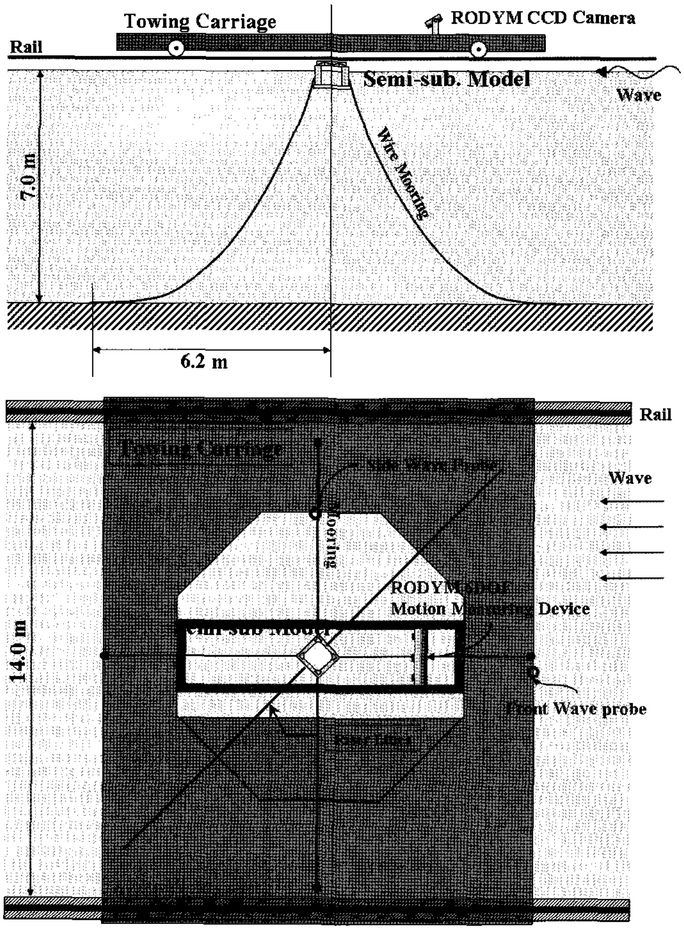

Figure 2: Model test setup

2.3 Mooring system (Truncation method)

Table 4: Mooring line modeling

General			Full scale	Model scale
No. of Lines		EA	8	4
Hor. Distance		m	1524	6.200
Ver. Depth		m	1524	7.000
Total wet weight /Line		kgf	182180	0.712
Segments				
Bottom Chain	Length	m	222.5	Not modeled
	Diameter	m	0.114	
	W_water	kgf/m	229.3	
Wire	Length	m	1918.2	9.425
	Diameter	m	0.108	0.0014
	W_water	kgf/m	59.5	0.0755
Top Chain	Length	m	70.1	Not modeled
	Diameter	m	0.118	
	W_water	kgf/m	242.9	

As described in Section 2.1, the design water depth for the prototype semi-submersible is 5000ft (=1524m in full scale, 19.05m in 1/80 model scale), and is impossible to be scaled down to 7.0m water depth of the SSMB experimental tank with the scale ratio of 80, which means that the mooring lines cannot be modeled properly in the model test.

This situation is the usual problem for the model tests with a mooring system for the deep or ultra-deep water depth in most of model facilities. One technique to resolve this problem is the "truncation method" to remodel the mooring system for the model test itself to be fit with the water depth of the test basin. For the tests with this technique, the following several main parameters that affect the horizontal motion of the semi-submersible model should be considered to be the same with those of prototype mooring system:

1) Static offset curve:
   - Horizontal displacement vs. restoring force curve
2) Total wet weight of mooring lines
3) Mooring line pretension
4) Mooring line damping

Actually, it is far more difficult to realize the mooring line damping in the model test than other three requirements, because the dynamic line damping is strongly dependent on the viscous force on the lines, that is to say, *Re* and *Kc* number. Hence in this model tests, we focused on satisfying the requirements 1) ~ 3) in the modeling of the "truncated mooring lines," and only the effective diameter of the mooring lines are scaled down from the prototype to model for the consideration of line damping.

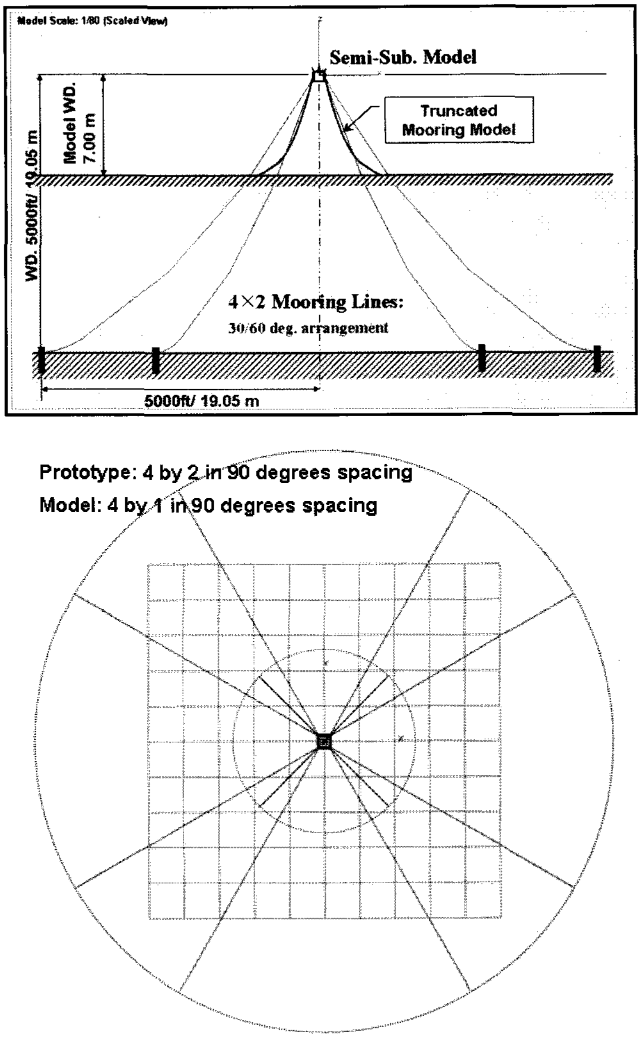

Figure 3: Modeling of truncated mooring lines

Figure 3 shows the truncated mooring line models. Modeling of the mooring lines with "truncation method" is carried out by using the FEM based hull/mooring/riser coupled analysis program *"HARP,"* which is developed by *Texas A&M University and Offshore Dynamics, Inc.* The results of the design for the tests also with the prototype mooring system are shown in Table 4.

The detailed data, such as horizontal and vertical distance from the fairlead to anchor, total wet weight of each line, of mooring line model in Table 4 are obtained by trial-and-error. That is, coupled static analyses by *HARP* is carried out until the modeled mooring system gives the same horizontal stiffness (displacement vs. restoring force) curve of the floater/mooring system with that of the prototype mooring system.

In this mooring model, it is hard to expect that the mooring system will have the same line damping or line added mass with the prototype mooring system. The diameter of the model mooring line in table 4 (=1.35[mm]) is just the value of prototype mooring line diameter divided by the scale ratio of 80, and no dynamic effects due to the line truncation and line lumping from 8 lines to 4 lines are considered. Therefore, the mooring line dynamics of the model test may be different from that of the prototype, which may results in the discrepancy of line tensions, horizontal motion response between the model and real prototype. In order to get the proper estimation of the horizontal motion and line tension characteristics of the system, further analysis should be carried out for the fully modeled prototype system with the proper hydrodynamic coefficients that may be decided by the model tests.

Semi-taut mooring system of the prototype semi-submersible without touch down zone on the seabed consists of 8 mooring lines with 4×2 arrangements by 30/60 degrees spacing, and is modeled to 4 equivalent truncated mooring lines as shown in Figure 3.

3 Results

3.1 Static offset test

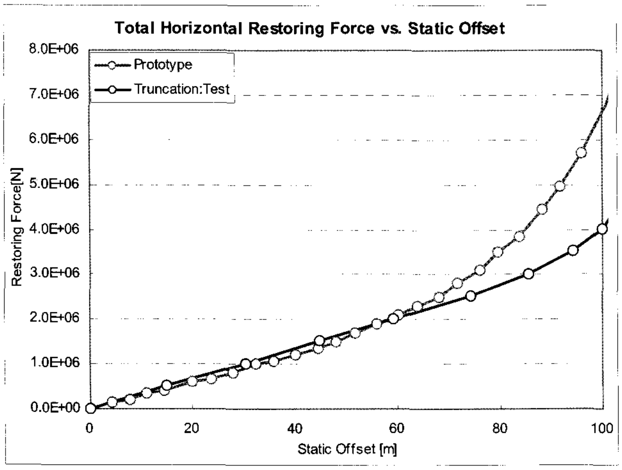

Figure 4: Static offset curve

The load-displacement relation in horizontal planar motion is measured by the static offset test and the result is shown in Figure 4. The slope of the curve represents the stiffness of horizontal motion. As can be seen in the figure, the restoring force is not linearly proportional to the horizontal displacement because the force is produced from the change of mooring line configurations and resulting sum of the line top tensions. As the horizontal displacement becomes larger than about 60m, the slope of the curve shows rapid increase, also with the difference between the prototype and model test. But the horizontal motion response does not exceed 60 meters for the most severe wave condition, so it can be said that the truncated mooring model is well describing the horizontal stiffness characteristics of the prototype mooring lines.

3.2 Heeling and free decay tests

Table 5: Heeling and free decay test results

Condition ID	GMT [m]	Roll T_N [s]	Description
GM10	7.78	30.4	Free floating
GM11	8.53	29.8	Mooring connected
GM12	8.96	29.2	Mooring/Riser connected
GM22	5.48	37.0	
GM32	3.81	45.4	
GM42	2.64	54.1	

* GMij: i for different loading conditions, j for line connection (0: free floating, 1: mooring, 2: mooring and riser)

To measure the metacentric height (GM) of the model, heeling tests are carried out by moving one of the ballast weights on the deck and measuring the heeling angles after the static equilibrium. GM10 condition is when the semi-submersible model is free floating without any connection of mooring lines or risers, and GM11 is with the connection of only mooring lines. Design GM value for the semi-submersible model at survival condition is 7.78m and was realized for the free floating GM10 condition. With the mooring lines and riser connection, *the effective GM* increased because of the additional roll restoring moment produced by the line members. In GM12 ~ GM42 conditions, two riser models with the diameter and wet weight that are scaled down from the prototype to model scale by the scale ratio 80, is connected at the pontoon center of the semi-submersible model. For the change of VCG position, some of the weights are moved in vertical direction without changing the total displacement of the model.

Also free decay tests are carried out to find the natural period and damping of the model in 6 DOF motions. The tests are carried out by making a model initial displacement and letting it loose and move freely.

Table 5 shows the results of heeling tests and free roll decay tests.

3.3 Regular wave tests: Mathieu Instability

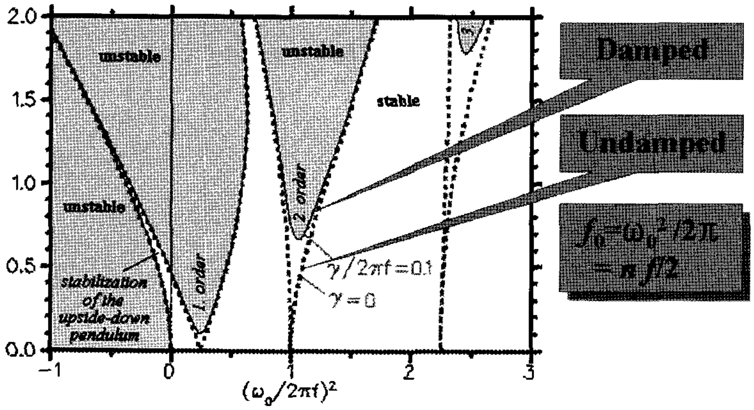

Figure 5: Mathieu stability diagram for *n*= 0, 1 and 2 (Elmer 1998)

As shown in Table 1 and Table 5, the roll natural period is nearly twice of the heave natural period for GM32 condition, and the Mathieu instability problem may happen if the dynamic amplitude of the GM change is sufficiently large.

According to the Mathieu stability diagram (Figure 5) and damped Mathieu equation, the semi-submersible can experience unstable roll motion when the following condition matches (Hong et al. 2005, Lee et al. 2004):

$$\frac{\delta GM}{GM_0} = 4\zeta_{44} \tag{1}$$

From the free roll decay test for GM32 condition, it is found that the roll damping coefficient is 2.6% of the critical roll damping. From the regular wave test, the measured amplitude of run-up at columns, relative motion of the model and the wave elevation at column surface, was 4.2 meters and corresponding amplitude of GM change, which is the same as the change of the vertical center of volume (VCB), was 1.5 meters.

$$\left(\frac{\delta GM}{GM_0} = \frac{1.5}{3.8} = 0.395\right) >> \left(4\varsigma_{44} = 4\times 0.026 = 0.104\right) \tag{2}$$

Equation (2) shows that the model satisfies the Mathieu-type instability condition sufficiently, but the result of regular wave test shows no instable roll motion as shown in Figure 6. To ensure the stability of roll motion, roll disturbance about 3 degrees is made externally at 1500s after the measurements, but the roll motion become stable quickly again.

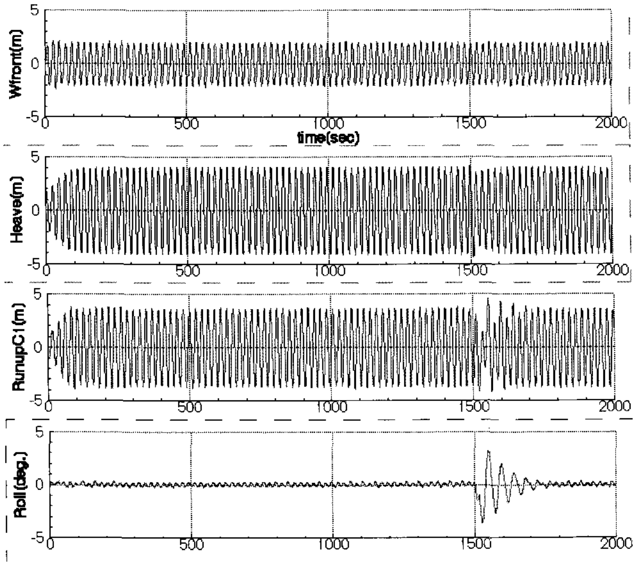

Figure 6: Time history of the semi-submersible model in regular wave (Wave amplitude=2m, T=22.2s)

The reason for the stable motion from this test needs to be discussed further, but it is clear that the Mathieu instability due to the double relation between roll and heave natural periods was not found in the heave resonant regular wave test.

3.4 Irregular wave tests: Low frequency roll and pitch motion

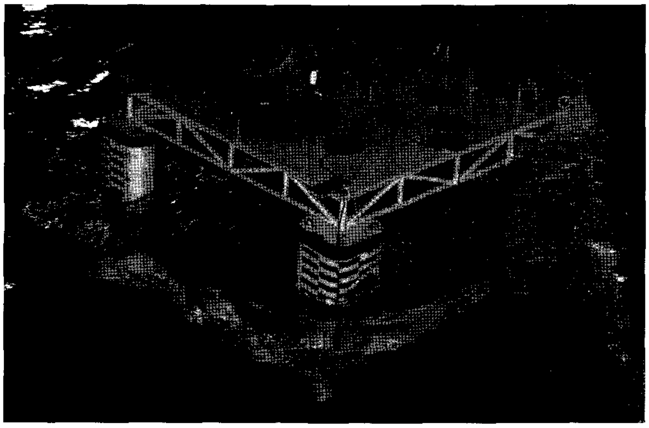

Figure 7: Irregular wave test (GoM 100yr Hurricane, 135 deg., GM12 condition)

Model tests for the design wave condition (GoM 100 year Hurricane wave) are carried out to measure and evaluate the motion responses, accelerations, mooring line top tensions and relative motions (air-gap, run-up). Test and wave conditions are described in Section 2.1. Figure 7 shows a snapshot of irregular wave test for the GM12 condition.

Throughout the paper, maximum motion range is defined as the motion maximum minus motion minimum measured from the three hour (in real scale) model test.

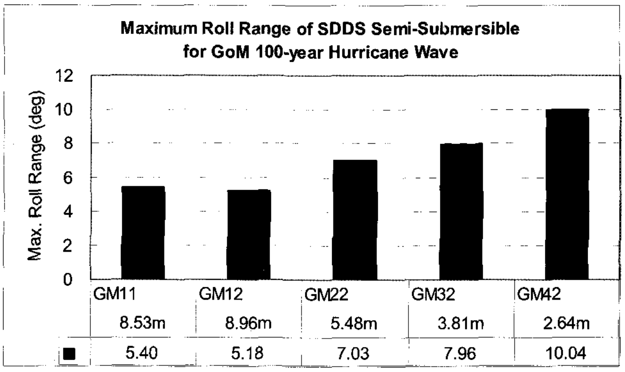

Figure 8: Maximum roll range (motion maximum-minimum) for the GoM 100 year Hurricane waves

Figure 8 shows that the change of GM value affects the maximum roll range significantly.

For GM11 (GMT=8.53m) condition, the maximum roll range is 5.4 degrees and corresponding roll natural period is 29.8 seconds.

For the case of GM12 (8.96m) condition, the roll natural period is 29.2 seconds and the maximum roll range reduces to 5.18 degrees, and it is because of the two riser models. The addition of the two SCR riser model gives the external load due to the riser tension at the riser porch, and induces additional roll restoring moment on the semi-submersible model.

To investigate the effect of smaller GM and longer roll natural period on the roll motion, three more loading conditions (GM22, GM32 and GM42) are prepared for the same irregular wave condition (the GoM 100yr Hurricane wave) with the mooring and riser connected condition. Corresponding roll natural periods are 37.0s, 45.4s and 54.1s respectively (Table 5).

As shown in Figure 8, it is clear that maximum roll range of the semi-submersible increases as the GM decreases and natural period increases. For the GM42 condition of which roll period is 54.1 seconds, maximum roll range is 10.04 degrees, which is larger than the GM11 loading condition by about 4.6 degrees.

The main reason for these phenomena can be easily explained by the consideration of wave drift force and moment on the semi-submersible model.

Roll motion of a semi-submersible may be expressed by the following simple equation if we ignore the coupling effect from other motions and assume the linear damping;

$$(I_{xx} + A_{44})\ddot{\phi} + B_{44}\dot{\phi} + MgGM_T\phi = M(t) \tag{3}$$

where

I_{xx} = Mass moment of inertia in roll [kg.m^2]
A_{44} = Added mass moment of inertia in roll [kg.m^2]
B_{44} = Roll damping [kg.m^2/s]
$M(t)$ = Wave exiting moment in roll

And the RMS (root-mean-square) of the low frequency component of the roll motion approximately can be found from the following simple form (Journee, 2001);

$$\phi_{RMS} = \int_0^\infty \left|\frac{\phi_a}{M_a}(\mu)\right|^2 \cdot S_f(\mu)\cdot d\mu \tag{4}$$

where

μ = difference frequency.

$\left|\frac{\phi_a}{M_a}(\mu)\right|$ = Roll RAO to wave exciting moment

ϕ_a = Amplitude of roll motion

M_a = Amplitude of roll exciting moment

$S_f(\mu)$ = Spectral density of the wave drift force

$$= 8\int_0^\infty S_\zeta(\omega) S_\zeta(\omega+\mu)\left|T(\omega,\omega+\mu)\right|\cdot d\omega \tag{5}$$

$T(\omega,\omega+\mu)$ = Amplitude of roll QTF

Figure 9 shows the spectral density of the GoM 100 year Hurricane waves and resulting wave drift moment with the measured roll response spectrum of the model. In the figure,

2^{nd} order wave drift QTF (quadratic transfer function. $T(\omega, \omega + \mu)$) of the semi-submersible is calculated by the 3D panel program "*WAMIT*."

If the roll natural period moves away from the wave frequency range, the semi-submersible model can avoid the large roll motion due to the resonant reaction to the 1^{st} order wave energy because there is nearly no wave energy at the roll natural period, but still there is the effect of the wave drift moment from the 2^{nd} order difference frequency component of the waves, and the roll motion response is decided by the combination of group wave spectral density and the amplitude of quadratic transfer function as shown in the equation (4).

In the Figure 9, it can be easily found that the spectral density of the wave drift moment at the low frequency zone increases and corresponding maximum roll motion increases.

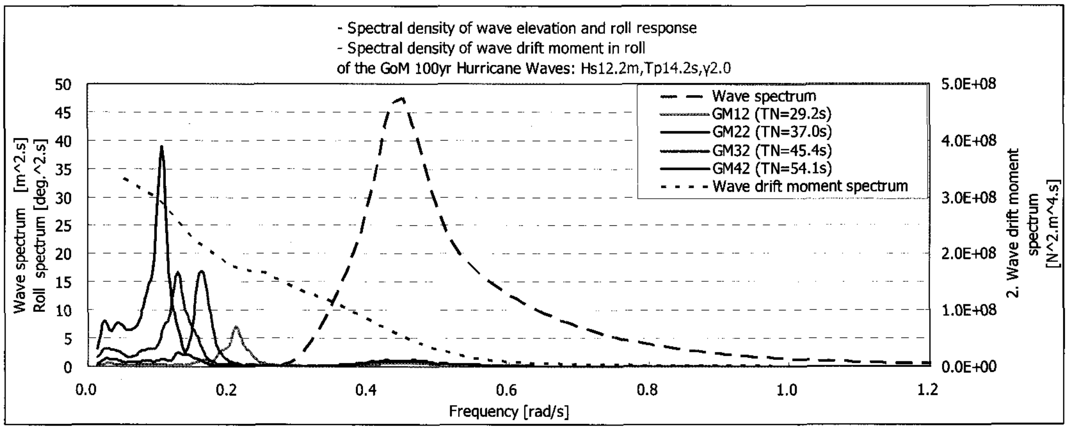

Figure 9: Spectral density of waves, wave drift moment and roll response of the model in the GoM 100 year Hurricane waves

5 Conclusion

In this paper, a simple and well-known phenomenon of low frequency roll motion of a semi-submersible is confirmed in detail by the experiments with several different loading conditions.

The low frequency motion due to the wave drift force is usually important for the horizontal planar motion such as surge and sway due to their very long natural periods, and it is because of the relatively small stiffness made by the mooring system. A typical moored offshore structure have hundreds of seconds as surge and sway natural periods with small restoring forces, hence the low frequency drift motion dominates the overall horizontal motion response.

The same phenomena can be of problem for the vertical motion such as roll and pitch, if their natural periods become too long for some special loading conditions. As shown in this paper, a semi-submersible can experience large roll motion due to the low frequency wave drift moment if the GM is small and the corresponding roll natural period moves to the low frequency range.

For the development of a new semi-submersible with a good motion performance, the first problem to solve is how to design the hull form, to be sure, but the decision of the design loading condition and possible loading conditions may considerably affect the performance of roll motion responses. The roll and pitch natural periods should be better, of course, to be away from the wave frequency range, but too long natural period will induce the large motion responses due to the low frequency wave drift force and moment. Hence, the designer is recommended to check every possible loading condition and avoid the too long roll and pitch natural periods.

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