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ON NCI RINGS

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ABSTRACT. We in this note introduce the concept of NCI rings which is a generalization of NI rings. We study the basic structure of NCI rings, concentrating rings of bounded index of nilpotency and von Neumann regular rings. We also construct suitable examples to the situations raised naturally in the process.

1. Introduction

Throughout this note every ring is associative with identity unless otherwise stated. Let R be a ring (possibly without identity). We use $N_*(R)$, $N^*(R)$, and $N(R)$ to denote the prime radical (i.e., the intersection of all prime ideals), the nilradical (i.e., the sum of all nil ideals), and the set of all nilpotent elements in R, respectively. Note $N_*(R) \subseteq N^*(R) \subseteq N(R)$. The n by n matrix ring over R is denoted by $Mat_n(R)$. The n by n upper and lower triangular matrix rings over R are denoted by $UTM_n(R)$ and $LTM_n(R)$, respectively.

A ring will be called *nil-simple* if it has no nonzero nil ideals. Kim et al. [12] called such a ring nil-semisimple, but nil-simple may be more suitable for this situation. Nil-simple rings are clearly semiprime, but semiprime rings need not be nil-simple as can be seen by [11, Example 1.2 and Proposition 1.3]. Due to Rowen [19, Definition 2.6.5], an ideal P of a ring R is called *strongly prime* if P is prime and R/P is nil-simple. Maximal ideals are clearly strongly prime. Nil-simple rings need not be prime as can be seen by direct products of reduced rings; and prime rings also need not be nil-simple by [11, Example 1.2 and Proposition 1.3].

An ideal P of a ring R is called *minimal strongly prime* if P is minimal in the space of strongly prime ideals in R. $N^*(R)$ of a ring R is the unique maximal nil ideal of R by [19, Proposition 2.6.2], and with the help of [19, Proposition 2.6.7] we have

$$N^*(R) = \{a \in R \mid RaR \text{ is a nil ideal of } R\}$$
$$= \bigcap\{P \mid P \text{ is a strongly prime ideal of } R\}$$

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$$= \bigcap\{P \mid P \text{ is a minimal strongly prime ideal of } R\}.$$

A ring is called *reduced* if it has no nonzero nilpotent elements. Due to Marks [16], a ring R is called *NI* if $N^*(R) = N(R)$. Note that R is NI if and only if $N(R)$ forms an ideal if and only if $R/N^*(R)$ is reduced. A ring R is called *2-primal* if $N_*(R) = N(R)$, according to Birkenmeier et al. [2]. It is obvious that R is 2-primal if and only if $R/N_*(R)$ is reduced. 2-primal rings are almost completely characterized by Marks [17]. Note that a ring R is reduced if and only if R is nil-simple and NI if and only if R is semiprime and 2-primal. It is obvious that 2-primal rings are NI, but the converse need not hold by Birkenmeier et al. [3, Example 3.3], Marks [16, Example 2.2], or [11, Example 1.2]. Note that $Mat_n(R)$ is always not NI for any ring R with identity and $n \geq 2$.

We now introduce the following concept that is a generalization of NI rings: a ring R is called *NCI* provided that $N(R)$ contains a nonzero ideal of R whenever $N(R) \neq 0$.

Lemma 1.1. (1) *For any ring A (possibly without identity), $UTM_n(A)$ and $LTM_n(A)$ are NCI for $n \geq 2$.*

(2) *For a ring A, $Mat_n(A)$ is not NI for $n \geq 2$.*

(3) *Let A be a ring (possibly without identity) with $N(A) \neq 0$. Then A is NCI if and only if $N^*(A) \neq 0$.*

(4) *If a ring A is NCI with a nilpotent nonzero ideal then $Mat_n(A)$ is NCI.*

(5) *For a simple ring A, $Mat_n(A)$ is not NCI for $n \geq 2$.*

(6) *For a reduced ring A, $Mat_n(A)$ is not NCI for $n \geq 2$.*

(7) *Let A be a ring. If $Mat_n(A)$ is NCI then so is A.*

Proof. (1) $UTM_n(A)$ and $LTM_n(A)$ have the nonzero nilpotent ideals

$$\begin{pmatrix} 0 & 0 & \cdots & 0 & A \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A & 0 & \cdots & 0 & 0 \end{pmatrix},$$

respectively. Thus they are NCI when $n \geq 2$.

(2) By the existence of the nonzero nilpotent $\begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$, $Mat_n(A)$ cannot be NI for $n \geq 2$.

The proof of (3) is obtained from the definition since $N^*(A)$ is the sum of all nil ideals of A.

(4) Let I be a nilpotent nonzero ideal of A. Then $Mat_n(I)$ is a nonzero nilpotent ideal of $Mat_n(A)$ and so $Mat_n(A)$ is NCI by (3).

(5) Since A is simple, $N^*(Mat_n(A)) = 0$. But $N(Mat_n(A)) \neq 0$ and so $Mat_n(A)$ is not NCI by (3).

(6) Assuming that $Mat_n(A)$ is NCI, then $N^*(Mat_n(A)) \neq 0$ and so there exists a nonzero nil ideal I of A such that $N^*(Mat_n(A)) = Mat_n(I)$, a contradiction to the reducedness of A.

(7) It suffices to compute the case of $n \geq 2$. If $Mat_n(A)$ is NCI then $N^*(Mat_n(A)) \neq 0$ and so there exists a nonzero nil ideal I of A such that $N^*(Mat_n(A)) = Mat_n(I)$. □

NI rings are clearly NCI but the converse need not be true by the following example. The *index* (*of nilpotency*) of a nilpotent element x in a ring R is the least positive integer n such that $x^n = 0$. The *index* (*of nilpotency*) of a subset I of R is the supremum of the indices of all nilpotent elements in I. If such a supremum is finite, then I is said to be *of bounded index of nilpotency.*

Example 1.2. (1) Let T be a ring, $S = UTM_2(T)$, and $R = Mat_2(S)$. Then R is not NI by Lemma 1.1(2). Consider the ideal

$$I = \begin{pmatrix} \begin{pmatrix} 0 & T \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & T \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & T \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & T \\ 0 & 0 \end{pmatrix} \end{pmatrix} \text{ of } R.$$

Then $I^2 = 0$ and $0 \neq I$, so R is NCI by Lemma 1.1(4).

(2) Let $F\{X, Y\}$ be the free algebra on X, Y over a field F and $I = (X^2)^2$, where (X^2) is the ideal of $F\{X, Y\}$ generated by X^2. Consider the ring $R = F\{X, Y\}/I$. Then $N_*(R) = Rx^2R$ and $N(R) = xRx + Rx^2R + Fx$ by the computation in [9, Example 1], where $x = X + I$. Moreover we get $N_*(R) = Rx^2R = N^*(R)$. If not, $N^*(R)/N_*(R)$ is a nonzero nil ideal of $S = R/N_*(R) \cong F\{X, Y\}/(X^2)$. But S is a prime ring with $N(S) = N_2(S)$ by the computation in [9, Example 1], where $N_2(S)$ denotes the set of all nilpotent elements of index 2 in S. It then follows that $\bar{a}^2 = 0$ for any $0 \neq \bar{a} \in N^*(R)/N_*(R)$, and so $\bar{a}S\bar{a} = 0$ by [10, Lemma 11]. But since S is prime, we get $\bar{a} = 0$, a contradiction. Hence $0 \neq N_*(R) = N^*(R) \subsetneq N(R)$. Therefore R is NCI by Lemma 1.1(3) but R is not NI.

When given a ring R is of bounded index of nilpotency, R is NI if and only if R is 2-primal by [11, Proposition 1.4]. So one may conjecture that R is NI if and only if R is NCI when R is of bounded index of nilpotency. However the answer is negative by Example 1.2(2). In Example 1.2(2), the ring R has index 4 since S has index 2, but it is not NI. But if given rings are of bounded index of nilpotency and semiprime then we get an affirmative answer as follows.

Proposition 1.3. *Suppose that a ring R is of bounded index of nilpotency. Then the following conditions are equivalent:*

(1) *R is reduced;*
(2) *R is 2-primal and semiprime;*
(3) *R is NI and semiprime;*
(4) *R is NCI and semiprime.*

Proof. It suffices to show (4)$\Rightarrow$(1) since reduced rings are semiprime. Since R is of bounded index of nilpotency, we get $N_*(R) = N^*(R)$ by the proof of [11, Proposition 1.4] based on Levitzki [8, Lemma 1.1] or Klein [13, Lemma 5]. Now suppose that R is both NCI and semiprime. Then we have $0 = N_*(R) = N^*(R)$ from the semiprimeness of R, and $N(R) = 0$ follows from the NCIness of R. □

The ring R in Example 1.2(2), that is NCI but not NI, is not semiprime. The condition "of bounded index of nilpotency" is not superfluous as can be seen by the semiprime NI ring (that is not 2-primal) in [3, Example 3.3].

A ring R is called *von Neumann regular* if for each $a \in R$ there exists $x \in R$ such that $a = axa$. A ring R is called *strongly regular* if for each $a \in R$ there exists $x \in R$ such that $a = a^2x$. A ring is called *abelian* if every idempotent is central. A ring is strongly regular if and only if it is abelian and von Neumann regular [5, Theorem 3.5]. A ring is called *right* (resp. *left*) *duo* if every right (resp. left) ideal is two-sided.

Proposition 1.4. *Let R be a von Neumann regular ring. Then the following conditions are equivalent:*
(1) *R is right (left) duo;*
(2) *R is reduced;*
(3) *R is abelian;*
(4) *R is 2-primal;*
(5) *R is NI;*
(6) *R is NCI.*

Proof. The equivalences of the conditions (1), (2), and (3) are proved by [5, Theorem 3.2]. (2)$\Rightarrow$(4), (4)$\Rightarrow$(5), and (5)$\Rightarrow$(6) are straightforward.

(6)$\Rightarrow$(2): Let R be NCI and assume $N(R) \neq 0$. Then R contains a nonzero nil ideal, say I. Take $0 \neq a \in I$. Since R is von Neumann regular, there exists $b \in R$ such that $a = aba$. Then we get $a = aba = ababa = abababa = \cdots$. But $ab \in I$, say $(ab)^n = 0$. Consequently we have $0 \neq a = aba = \cdots = (ab)^n a = 0$, a contradiction. □

From Proposition 1.4 we obtain a similar result to [5, Theorem 3.5].

Corollary 1.5. *A ring is strongly regular if and only if it is both NCI and von Neumann regular.*

2. Basic structure and examples of NCI rings

In this section we study the basic structure of NCI rings and extend the class of NCI rings.

Lemma 2.1. *The class of NCI rings is closed under direct sums and direct products.*

Proof. Suppose that R_i is an NCI ring for each i in a nonempty index set I, and let D be the direct sum of R_i's. If every R_i is reduced then so is D, and so we assume that not every R_i is reduced. Then the direct sum of $N^*(R_i)$'s is a nonzero nil ideal of D since every R_i is NCI; hence D is also NCI. The proof for direct products is similar. □

Remark. (1) The direct sums in Lemma 2.1 are considered as rings possibly without identity.

(2) The converse of Lemma 2.1 need not be true. Let $R_1 = Mat_2(A)$ and $R_2 = UTM_2(A)$ where A is a simple ring. The direct sum $D = R_1 \oplus R_2$ has nonzero nil ideal $0 \oplus \begin{pmatrix} 0 & A \\ 0 & 0 \end{pmatrix}$, and so D is NCI by Lemma 1.1(3). But R_1 is not NCI by Lemma 1.1(5).

Subrings (possibly without identity) of NI (resp. 2-primal) rings are also NI (resp. 2-primal) by [11, Proposition 2.4(2)] (resp. [2, Proposition 2.2]). However the class of NCI rings is not closed under subrings by the following.

Example 2.2. (1) Let A be a simple ring and $S = Mat_n(A)$ with $n \geq 2$. Then S is not NCI by Lemma 1.1(5). Consider the ring $R = UTM_m(S)$ with $m \geq 2$. Then R is NCI by Lemma 1.1(1), but the subring $\begin{pmatrix} S & 0 & \cdots & 0 & 0 \\ 0 & S & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & S \end{pmatrix} \cong$ $\oplus_{i=1}^m S_i$ of R is not NCI, where $S_i = S$ for all i.

(2) The ring D in Remark(2) of Lemma 2.1 is NCI but the subring $R_1 \oplus 0$ is not NCI.

Factor rings and ideals of NCI rings need not be NCI as can be seen by $\dfrac{D}{0 \oplus R_2} \cong R_1$ and $R_1 \oplus 0$ in Remark(2) of Lemma 2.1.

Given a ring R and a proper ideal I of R, it is proved that R is NI (resp. 2-primal) when R/I and I are both NI (resp. 2-primal) by [11, Proposition 2.4(1)] (resp. [2, Proposition 2.4]), where I is considered as a subring without identity. However this result need not hold for NCI rings by the following.

Example 2.3. Let $\mathbb{Z}$ be the ring of integers. Set $R = Mat_n(\mathbb{Z}) \oplus \mathbb{Z}$ for $n \geq 2$. Taking $I = 0 \oplus 4\mathbb{Z}$, then $\dfrac{R}{I} = Mat_n(\mathbb{Z}) \oplus \dfrac{\mathbb{Z}}{4\mathbb{Z}}$ is NCI with the nonzero nil ideal $0 \oplus \dfrac{2\mathbb{Z}}{4\mathbb{Z}}$, and I is reduced (so NCI). However R is not NCI by Lemma 1.1(6).

But if the NCI ideal I has nonzero $N^*(I)$ then R is also NCI as follows.

Proposition 2.4. *Let R be a ring and I be a nonzero proper ideal of R. If I is NCI with $N(I) \neq 0$ as a subring without identity then R is NCI.*

Proof. Let I be NCI with $N(I) \neq 0$. Then $N^*(I) \neq 0$, say J. Take $0 \neq a \in J$ and consider the ideal RaR. Note $RaR \subseteq I$. Let $x = \sum_{\text{finite}} ras \in RaR$. Then

$$x^3 = \sum \sum \sum (rasr)a(sras) \in IaI \subseteq J,$$

hence x^3 is nilpotent, forcing $x \in N(R)$. Thus R is NCI. □

A ring R is called *directly finite* if $ab = 1$ implies $ba = 1$ for $a, b \in R$. NI (resp. 2-primal) ring is directly finite by [11, Proposition 2.7(1) (resp. [2, Proposition 2.10]). However this result cannot be extended to the class of NCI rings as follows.

Example 2.5. Let F be a field and R be the column finite infinite matrix ring over F. Let $a \in R$ be the matrix with $(i, i+1)$-entry 1 and zero elsewhere, and $b \in R$ be the matrix with $(i+1, i)$-entry 1 and zero elsewhere, where $i = 1, 2, \ldots$. Then $ab = 1$ but $ba \neq 1$; hence R is not directly finite.

Next consider $U = UTM_n(R)$ for $n \geq 2$. Then U is NCI by Lemma 1.1(1). But $xy = 1$ and $yx \neq 1$ with the help of the computation above where $x, y \in U$ are scalar matrices with diagonals a, b respectively. Thus U is not directly finite.

Given a ring R the polynomial ring and the (formal) power series ring over R are denoted by $R[X]$ and $R[[X]]$ respectively, where X is any set of commuting indeterminates over R: if $X = \{x\}$ then write $R[x]$ and $R[[x]]$ in place of $R[\{x\}]$ and $R[[\{x\}]]$ respectively.

Birkenmeier et al. [2, Proposition 2.6] proved that polynomial rings over 2-primal rings are also 2-primal. While there exists an NI ring over which the polynomial ring need not be NI by Smoktunowicz [20]. We do not know whether polynomial rings over NCI rings are NCI. But for rings of bounded index of nilpotency, the NCIness can go up to polynomial rings.

Proposition 2.6. *Let R be a ring of bounded index of nilpotency. Then the following conditions are equivalent:*

(1) *R is NCI;*
(2) *$R[X]$ is NCI;*
(3) *$R[[X]]$ is NCI.*

Proof. (1)⇒(2): If R is reduced then so is $R[X]$, so we assume $N(R) \neq 0$. If R is NCI then $N^*(R) \neq 0$. Then $N^*(R)$ contains a nonzero nilpotent ideal of R by [8, Lemma 1.1] or [13, Lemma 5], say N. Then $R[X]$ has a nonzero nilpotent ideal $N[X]$, proving that $R[X]$ is NCI.

(2)⇒(1): If $R[X]$ is reduced then so is R, so we assume $N(R[X]) \neq 0$. If $R[X]$ is NCI then $N^*(R[X]) \neq 0$, say that $0 \neq f(X) \in N^*(R[X])$ and $a \in R$ is the nonzero coefficient of a term of the least degree in $f(X)$. Then RaR is a nonzero nil ideal of R since $R[X]f(X)R[X]$ is a nil ideal of $R[X]$. Thus R is NCI.

The proofs of (1)⇒(3) and (3)⇒(1) are similar to the preceding one. □

Remark. The proofs of (2)⇒(1) and (3)⇒(1) are obtained without the condition "of bounded index of nilpotency".

Given a ring R, an endomorphism σ of R, and a σ-derivation δ of R, the *Ore extension* $R[x; \sigma, \delta]$ of R is the ring obtained by giving the polynomial ring

$R[x]$ the new multiplication $xr = \sigma(r)x + \delta(r)$ for all $r \in R$. If $\delta = 0$ then we write $R[x;\sigma]$ and call it a *skew* polynomial ring. If $\sigma = 1$ then we write $R[x;\delta]$ and call it a *differential* polynomial ring. It is also natural to check whether NCIness can go up to skew and differential polynomial rings. However there exist counterexamples to these cases as follows.

Example 2.7. There exists an NCI ring such that the skew polynomial ring over it is not NCI.

Proof. The proof is essentially due to [11, Example 4.5]. Given a division ring D let $R = D \oplus D$, then R is NCI obviously. Define $\sigma : R \to R$ by $\sigma(s,t) = (t,s)$. Then σ is an automorphism of R. Let $S = R[x;\sigma]$ be the skew polynomial ring over R by σ. S is semiprime by the argument in [11, Example 4.5].

On the other hand, S is right Noetherian by [18, Theorem 2.9] because R is right Noetherian and σ is an automorphism. Then every nil subring of S is nilpotent by [14] and so S has no nonzero nil right or left ideals because S is semiprime. Thus $N^*(S) = 0$, but $N(S) \neq 0$ as cab be seen by $((1,0)x)((1,0)x) = 0$. Thus S is not NCI. □

Example 2.8. There exists an NCI ring such that the differential polynomial ring over it is not NCI.

Proof. The proof is essentially due to [1, Example 11], [6, Proposition 1.14], and [11, Example 4.6]. Let $R = F[x]/(x^2)$ and define $\delta : A \to A$ by $\delta(x + (x^2)) = 1 + (x^2)$, where F is a field of characteristic 2 and $(x^2) = F[x]x^2$. Then R is NI (hence NCI) since $N^*(R) = F[x]x$ and $\dfrac{R}{F[x]x} \cong F$. Next let $S = R[x;\delta]$.

Then S is a simple ring by the [6, Proposition 1.14] and [11, Example 4.6], obtaining $N^*(R) = 0$. But $x + (x^2)$ is a nonzero nilpotent element of R and so R is not NCI. □

For skew and differential polynomial rings we have similar results to Proposition 2.6 when given rings are of bounded index of nilpotency.

Proposition 2.9. *Let R be a non-reduced NCI ring. If R is of bounded index of nilpotency, then the skew and differential polynomial rings are NCI.*

Proof. Since R is non-reduced and NCI, we have $N^*(R) \neq 0$. But $N^*(R)$ contains a nonzero nilpotent ideal of R by [8, Lemma 1.1] or [13, Lemma 5], say N. Then $N[x;\sigma]$ and $N[x;\delta]$ are nonzero nilpotent ideals in $R[x;\sigma]$ and $R[x;\delta]$, respectively. □

(1)⇒(2) and (1)⇒(3) in Propositions 2.6, and Proposition 2.9 also hold under the condition "$N^*(R)$ is nilpotent" in place of "R is of bounded index of nilpotency". Thus the results also hold under the following conditions.

Remark. $N^*(R)$ is nilpotent when a ring R satisfies each of the following cases:

(1) If R is a ring with right Krull dimension (in the sense of Gabriel and Rentschler [6]) then $N^*(R)$ is nilpotent by [15];

(2) If a ring R is right Goldie or satisfies ascending chain condition on both right and left annihilators then $N^*(R)$ is nilpotent by [14] and [4, Theorem 1.34] respectively.

Let R be an algebra over a commutative ring S. Recall that the *Dorroh extension* of R by S is the ring $R \times S$ with operations $(r_1, s_1) + (r_2, s_2) = (r_1 + r_2, s_1 + s_2)$ and $(r_1, s_1)(r_2, s_2) = (r_1r_2 + s_1r_2 + r_1s_2, s_1s_2)$, where $r_i \in R$ and $s_i \in S$.

Proposition 2.10. *Suppose that R is an algebra over a commutative ring S and let D be the Dorroh extension of R by S. Then R is NCI if and only if so is D.*

Proof. Let R be NCI. If $N(R) \neq 0$ then $N(D) \neq 0$ and D contains the nonzero nil ideal $N^*(R) \times 0$; hence D is NCI.

Next let $N(R) = 0$. If $N(S) = 0$ then D is also reduced, concluding the NCIness of D. So assume $N(S) \neq 0$. Then since commutative rings are 2-primal (i.e., $N(S) = N_*(S)$), we get the nonzero nil ideal $0 \times N_*(S) = 0 \times N^*(S) = 0 \times N(S)$ of D. Thus D is NCI.

Conversely suppose that D is NCI. Let $N(S) = 0$. If R is reduced then we are done and so we assume $N(R) \neq 0$. Then $N(D) \neq 0$, and we get $N^*(R) \neq 0$ since D is NCI and S is reduced. Thus R is NCI. Next let $N(S) \neq 0$. Then $N(S)R$ is a nonzero nil ideal of R since R is an algebra over S. □

We end this note with raising following question:

Is $R[x]$ NCI when R is an NCI ring?

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