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Reference-Intrinsic Analysis for the Ratio of Two Normal Variances

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Abstract

In this paper, we consider a decision-theoretic oriented, objective Bayesian inference for the ratio of two normal variances. Specifically we derive the Bayesian reference criterion as well as the intrinsic estimator and the credible region which correspond to the intrinsic discrepancy loss and the reference prior. We illustrate our results using real data analysis and simulation study.

Keywords : Bayesian Reference Criterion, Credible Region, Intrinsic Estimator, Intrinsic Expected Loss, Reference Prior

1. Introduction

In many problems we are interested in comparing the variabilities of two populations. Usually this can be done by inferring from their variance ratio. Decision theoretic approach to this problem was motivated by the work of Stein (1964) and Brown(1968). This approach was later extended by Gelfand and Dey (1988), Madi(1995) and Ghosh and Kundu(1996). For Bayesian inference for the variance ratio in normal populations, we may often refer to Box and Tiao(1973) and Lee(1989) among others.

Recently Bernardo(1999) and Bernardo and Rueda(2002) introduced a new model

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selection criterion, called the Bayesian Reference Criterion(BRC). Bernardo(1999) takes a decision theoretic approach to developing an objective Bayes solution to test for nested hypotheses. Furthermore, Bernardo and Juárez(2003) addressed intrinsic point estimation and Bernardo(2005) defined intrinsic credible region based on the information-theory based loss function as well as reference prior from an objective Bayesian viewpoint. The procedures merge the use of the reference algorithm (Berger and Bernardo, 1992; Bernardo, 1979) to derive noninformative priors, with the intrinsic discrepancy (Bernardo and Rueda, 2002; Bernardo and Juárez, 2003) as loss function to obtain an objective answer for statistical problem.

The purpose of this article is to develop a decision-theoretic oriented, objective Bayesian answer to the problems of sharp hypothesis testing and both point and region estimation for the ratio of two normal variances.

The contents of the remaining paper are as follows. In Section 2, we describe the reference-intrinsic methodology. Also we derive the Bayesian reference criterion, the intrinsic estimator and the credible region for the ratio of two normal variances. Comparisons with alternative approaches are carried out in Section 3, using both real data and simulated data. Some concluding remarks are given in Section 4.

2. Reference-Intrinsic Analysis

2.1. The methodology

Suppose that available data $\boldsymbol{x}$ consist of a random sample $\boldsymbol{x}=\{x_1,\cdots,x_n\}$ from the family $M\equiv\{p(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\lambda}),\boldsymbol{x}\in X,\boldsymbol{\theta}\in\Theta,\boldsymbol{\lambda}\in\Lambda\}$, where $\boldsymbol{\theta}$ is some vector of interest and $\boldsymbol{\lambda}$ is some vector of nuisance parameters.

The intrinsic discrepancy loss $\delta_{\boldsymbol{x}}\{\tilde{\boldsymbol{\theta}},(\boldsymbol{\theta},\boldsymbol{\lambda})\}$, introduced by Bernardo and Rueda (2002), is basically used to measure the “distance” between the probability density $p(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\lambda})$ and the family of probability densities $M\equiv\{p(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\lambda}),\boldsymbol{\lambda}\in\Lambda\}$, defined as

$$\delta_{\boldsymbol{x}}\{\tilde{\boldsymbol{\theta}},(\boldsymbol{\theta},\boldsymbol{\lambda})\}=inf_{\tilde{\boldsymbol{\lambda}}\in\Lambda}\delta\{p(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\lambda}),p(\boldsymbol{x}|\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\lambda}})\} \tag{2.1}$$

where $\delta\{p(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\lambda}),p(\boldsymbol{x}|\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\lambda}})\}=\min\{k(\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\lambda}}|\boldsymbol{\theta},\boldsymbol{\lambda}),k(\boldsymbol{\theta},\boldsymbol{\lambda}|\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\lambda}})\}$ and
$k(\boldsymbol{\theta},\boldsymbol{\lambda}|\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\lambda}})=\int_X p(\boldsymbol{x}|\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\lambda}})\log\frac{p(\boldsymbol{x}|\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\lambda}})}{p(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\lambda})}d\boldsymbol{x}$, that is the Kullback-Leibler divergences.
Computation of intrinsic loss functions in regular models may be simplified by

$$\delta_{\boldsymbol{x}}\{\tilde{\boldsymbol{\theta}},(\boldsymbol{\theta},\boldsymbol{\lambda})\}=\min\left\{inf_{\tilde{\boldsymbol{\lambda}}\in\Lambda}k(\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\lambda}}|\boldsymbol{\theta},\boldsymbol{\lambda}),inf_{\tilde{\boldsymbol{\lambda}}\in\Lambda}k(\boldsymbol{\theta},\boldsymbol{\lambda}|\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\lambda}})\right\}. \tag{2.2}$$

Assuming the data $x_{ij}(i=1,2;j=1,\cdots,n_i)$ are independent and normally distributed with means μ_i and unknown variances σ_i^2. We derive the Bayesian reference criterion, the intrinsic estimator and intrinsic credible region when the parameter of interest is $\theta=\sigma_2^2/\sigma_1^2$, the ratio of two normal variances.

With reparametrization $\theta=\sigma_2^2/\sigma_1^2, \lambda=(\sigma_1^2)^{n_1}(\sigma_2^2)^{n_2}$, the joint probability density function of $\boldsymbol{x}$ with parameters $(\theta,\lambda,\mu_1,\mu_2)$ is given by

$$\begin{aligned}
&p(\boldsymbol{x}|\theta,\lambda,\mu_1,\mu_2)\\
&=(2\pi)^{-\frac{n_1+n_2}{2}}\lambda^{-\frac{1}{2}}\\
&\quad\times\exp\left\{-\frac{1}{2}\theta^{\frac{n_2}{n_1+n_2}}\lambda^{-\frac{1}{n_1+n_2}}\sum_{j=1}^{n_1}(x_{1j}-\mu_1)^2-\frac{1}{2}\theta^{-\frac{n_1}{n_1+n_2}}\lambda^{-\frac{1}{n_1+n_2}}\sum_{j=1}^{n_2}(x_{2j}-\mu_2)^2\right\}\\
&=N(\boldsymbol{x}_1|\mu_1,\theta^{-\frac{n_2}{2(n_1+n_2)}}\lambda^{\frac{1}{2(n_1+n_2)}})N(\boldsymbol{x}_2|\mu_2,\theta^{\frac{n_1}{2(n_1+n_2)}}\lambda^{\frac{1}{2(n_1+n_2)}}),
\end{aligned}\tag{2.3}$$

where $\boldsymbol{x}\equiv\{\boldsymbol{x}_1,\boldsymbol{x}_2\}=\{x_{11},\cdots,x_{1n_1},x_{21},\cdots,x_{2n_2}\}$.

In our problem, the directed divergence $k\{\tilde\theta,\tilde\lambda,\tilde\mu_1,\tilde\mu_2|\theta,\lambda,\mu_1,\mu_2\}$ is

$$\begin{aligned}
&k\{\tilde\theta,\tilde\lambda,\tilde\mu_1,\tilde\mu_2|\theta,\lambda,\mu_1,\mu_2\}\\
&=\int p(\boldsymbol{x}|\theta,\lambda,\mu_1,\mu_2)\log\frac{p(\boldsymbol{x}|\theta,\lambda,\mu_1,\mu_2)}{p(\boldsymbol{x}|\tilde\theta,\tilde\lambda,\tilde\mu_1,\tilde\mu_2)}d\boldsymbol{x}\\
&=\frac{n_1}{2}\left\{\frac{1}{n_1+n_2}log\left(\frac{\tilde\lambda}{\lambda}\right)-\frac{n_2}{n_1+n_2}log\left(\frac{\tilde\theta}{\theta}\right)-1+\left(\frac{\lambda}{\tilde\lambda}\right)^{\frac{1}{n_1+n_2}}\left(\frac{\theta}{\tilde\theta}\right)^{-\frac{n_2}{n_1+n_2}}+(\mu_1-\tilde\mu_1)^2\tilde\lambda^{-\frac{1}{n_1+n_2}}\tilde\theta^{\frac{n_2}{n_1+n_2}}\right\}\\
&+\frac{n_2}{2}\left\{\frac{1}{n_1+n_2}log\left(\frac{\tilde\lambda}{\lambda}\right)+\frac{n_1}{n_1+n_2}log\left(\frac{\tilde\theta}{\theta}\right)-1+\left(\frac{\lambda}{\tilde\lambda}\right)^{\frac{1}{n_1+n_2}}\left(\frac{\theta}{\tilde\theta}\right)^{\frac{n_1}{n_1+n_2}}+(\mu_2-\tilde\mu_2)^2\tilde\lambda^{-\frac{1}{n_1+n_2}}\tilde\theta^{-\frac{n_1}{n_1+n_2}}\right\}.
\end{aligned}\tag{2.4}$$

As a function of $(\tilde\lambda,\tilde\mu_1,\tilde\mu_2)$, the directed divergence is minimized when $(\tilde\lambda,\tilde\mu_1,\tilde\mu_2)$ takes the value $\left(\lambda\left\{\frac{n_1}{n_1+n_2}\left(\frac{\theta}{\tilde\theta}\right)^{-\frac{n_2}{n_1+n_2}}+\frac{n_2}{n_1+n_2}\left(\frac{\theta}{\tilde\theta}\right)^{\frac{n_1}{n_1+n_2}}\right\}^{n_1+n_2},\mu_1,\mu_2\right)$. Thus by substituting in (2.4), the minimum directed divergence is

$$inf_{\substack{\tilde\lambda>0\\ \tilde\mu_1\in R\\ \tilde\mu_2\in R}}k\{\tilde\theta,\tilde\lambda,\tilde\mu_1,\tilde\mu_2|\theta,\lambda,\mu_1,\mu_2\}=\frac{n_1+n_2}{2}log\left\{\frac{n_1}{n_1+n_2}\left(\frac{\theta}{\tilde\theta}\right)^{-\frac{n_2}{n_1+n_2}}+\frac{n_2}{n_1+n_2}\left(\frac{\theta}{\tilde\theta}\right)^{\frac{n_1}{n_1+n_2}}\right\}.\tag{2.5}$$

Similarly, the directed divergence $k\{\theta,\lambda,\mu_1,\mu_2|\tilde{\theta},\tilde{\lambda},\tilde{\mu}_1,\tilde{\mu}_2\}$ is given by

$$k\{\theta,\lambda,\mu_1,\mu_2|\tilde{\theta},\tilde{\lambda},\widetilde{\mu_1},\widetilde{\mu_2}\}$$
$$= \frac{n_1}{2}\left\{\frac{1}{n_1+n_2}log\left(\frac{\lambda}{\tilde{\lambda}}\right)-\frac{n_2}{n_1+n_2}log\left(\frac{\theta}{\tilde{\theta}}\right)-1+\left(\frac{\tilde{\lambda}}{\lambda}\right)^{\frac{1}{n_1+n_2}}\left(\frac{\tilde{\theta}}{\theta}\right)^{-\frac{n_2}{n_1+n_2}}+(\widetilde{\mu_1}-\mu_1)^2\tilde{\lambda}^{-\frac{1}{n_1+n_2}}\tilde{\theta}^{\frac{n_2}{n_1+n_2}}\right\}$$
$$+\frac{n_2}{2}\left\{\frac{1}{n_1+n_2}log\left(\frac{\lambda}{\tilde{\lambda}}\right)+\frac{n_1}{n_1+n_2}log\left(\frac{\theta}{\tilde{\theta}}\right)-1+\left(\frac{\tilde{\lambda}}{\lambda}\right)^{\frac{1}{n_1+n_2}}\left(\frac{\tilde{\theta}}{\theta}\right)^{\frac{n_1}{n_1+n_2}}+(\widetilde{\mu_2}-\mu_2)^2\lambda^{-\frac{1}{n_1+n_2}}\theta^{-\frac{n_1}{n_1+n_2}}\right\}. \quad (2.6)$$

As a function of $(\tilde{\lambda},\widetilde{\mu_1},\widetilde{\mu_2})$, the directed divergence is minimized when $(\tilde{\lambda},\widetilde{\mu_1},\widetilde{\mu_2})$ takes the value $\left(\lambda\left\{\frac{n_1}{n_1+n_2}\left(\frac{\tilde{\theta}}{\theta}\right)^{-\frac{n_2}{n_1+n_2}}+\frac{n_2}{n_1+n_2}\left(\frac{\tilde{\theta}}{\theta}\right)^{\frac{n_1}{n_1+n_2}}\right\}^{-(n_1+n_2)},\mu_1,\mu_2\right)$. Thus by substituting in (2.6), the minimum directed divergence is

$$inf_{\substack{\tilde{\lambda}>0\\ \tilde{\mu}_1\in R\\ \tilde{\mu}_2\in R}} k\{\theta,\lambda,\mu_1,\mu_2|\tilde{\theta},\tilde{\lambda},\tilde{\mu}_1,\tilde{\mu}_2\}=\frac{n_1+n_2}{2}log\left\{\frac{n_1}{n_1+n_2}\left(\frac{\tilde{\theta}}{\theta}\right)^{-\frac{n_2}{n_1+n_2}}+\frac{n_2}{n_1+n_2}\left(\frac{\tilde{\theta}}{\theta}\right)^{\frac{n_1}{n_1+n_2}}\right\}. \quad (2.7)$$

Hence, when $n_1 \ge n_2$, the intrinsic discrepancy loss from using $\tilde{\theta}$ as a proxy for θ is

$$\delta_x\{\tilde{\theta},(\theta,\lambda,\mu_1,\mu_2)\}=\begin{cases}\frac{n_1+n_2}{2}g(\phi) & \text{if} \quad \phi<1,\\ \frac{n_1+n_2}{2}g(\frac{1}{\phi}) & \text{if} \quad \phi\ge 1,\end{cases} \quad (2.8)$$

and, when $n_1 < n_2$, the intrinsic discrepancy loss is

$$\delta_x\{\tilde{\theta},(\theta,\lambda,\mu_1,\mu_2)\}=\begin{cases}\frac{n_1+n_2}{2}g(\frac{1}{\phi}) & \text{if} \quad \phi<1,\\ \frac{n_1+n_2}{2}g(\phi) & \text{if} \quad \phi\ge 1,\end{cases} \quad (2.9)$$

where $g(t)=\log\left\{\frac{n_1}{n_1+n_2}t^{-\frac{n_2}{n_1+n_2}}+\frac{n_2}{n_1+n_2}t^{\frac{n_1}{n_1+n_2}}\right\}$ and $\phi=\tilde{\theta}/\theta$.

2.2. The Bayesian Reference Criterion

The intrinsic discrepancy reference expected loss, or intrinsic expected loss, defined as

$$d(\tilde{\boldsymbol{\theta}}|\boldsymbol{x}) = \int_{\Theta}\int_{\Lambda} \delta_{\boldsymbol{x}}\{\tilde{\boldsymbol{\theta}},(\boldsymbol{\theta},\boldsymbol{\lambda})\}\pi(\boldsymbol{\theta},\boldsymbol{\lambda}|\boldsymbol{x})d\boldsymbol{\lambda}d\boldsymbol{\theta} \tag{2.10}$$

where $\pi(\boldsymbol{\theta},\boldsymbol{\lambda}|\boldsymbol{x}) \propto p(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\lambda})\pi(\boldsymbol{\theta},\boldsymbol{\lambda})$, and $\pi(\boldsymbol{\theta},\boldsymbol{\lambda})$ is the joint reference prior when $\boldsymbol{\theta}$ is the quantity of interest.

The intrinsic statistic $d(\boldsymbol{\theta}_0|\boldsymbol{x})$ is a measure of the evidence against the simplified model $p(\boldsymbol{x}|\boldsymbol{\theta}=\boldsymbol{\theta}_0,\boldsymbol{\lambda})$ provided by the data $\boldsymbol{x}$. The hypothesis $H_0 \equiv \{\boldsymbol{\theta}=\boldsymbol{\theta}_0\}$ should be rejected if(and only if) the posterior expected loss is sufficiently large. To decide whether or not the precise value $\boldsymbol{\theta}_0$ may be used as a proxy for the unknown value of $\boldsymbol{\theta}$, the Bayesian reference criterion(BRC) might be used as follows:

$$\text{Reject} \quad H_0 \quad \text{iff} \quad d(\boldsymbol{\theta}_0|\boldsymbol{x}) = \int_{\Theta}\int_{\Lambda} \delta_{\boldsymbol{x}}\{\boldsymbol{\theta}_0,(\boldsymbol{\theta},\boldsymbol{\lambda})\}\pi(\boldsymbol{\theta},\boldsymbol{\lambda}|\boldsymbol{x})d\boldsymbol{\lambda}d\boldsymbol{\theta} \;>\; d^* \tag{2.11}$$

The values of d^* around 2.5 would imply a ratio of $e^{2.5} \approx 12$, providing mild evidence against the null; while values around $5(e^5 \approx 150)$ can be regarded as strong evidence against H_0; values of $d^* \geq 7.5(e^{7.5} \approx 1800)$ can be safely used to reject the null.

In our problem, the reference prior when θ is the parameter of interest is given by $\pi(\theta,\lambda,\mu_1,\mu_2) \propto \theta^{-1}\lambda^{-1}$. The reference posterior distribution of $(\theta,\lambda,\mu_1,\mu_2)$,

$$\pi(\theta,\lambda,\mu_1,\mu_2|\boldsymbol{x}) \propto p(\boldsymbol{x}|\theta,\lambda,\mu_1,\mu_2)\pi(\theta,\lambda,\mu_1,\mu_2), \tag{2.12}$$

is proper if $n_1 > 1$ and $n_2 > 1$.

Integrating out the nuisance parameters $\{\lambda,\mu_1,\mu_2\}$, it leads to the marginal reference posterior of θ,

$$\pi(\theta|\boldsymbol{x}) \propto \theta^{\frac{n_1-3}{2}}(\theta s_1^2 + s_2^2)^{1-\frac{n_1+n_2}{2}}. \tag{2.13}$$

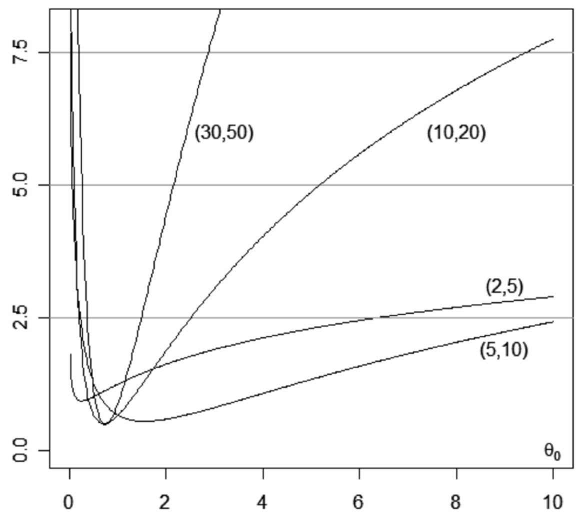
Therefore from the intrinsic discrepancy loss (2.8), (2.9) and the reference posterior (2.12), the intrinsic discrepancy reference expected loss is given as follow:

$$d(\tilde{\theta}|\boldsymbol{x}) = \int_0^\infty \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \delta_{\boldsymbol{x}}\{\tilde{\theta},(\theta,\lambda,\mu_1,\mu_2)\}\pi(\theta,\lambda,\mu_1,\mu_2|\boldsymbol{x})d\mu_2 d\mu_1 d\lambda d\theta \tag{2.14}$$
$$= \frac{\int_0^\infty \delta_{\boldsymbol{x}}(\tilde{\theta},\theta)\theta^{\frac{n_1-3}{2}}(\theta s_1^2+s_2^2)^{1-\frac{n_1+n_2}{2}}d\theta}{\int_0^\infty \theta^{\frac{n_1-3}{2}}(\theta s_1^2+s_2^2)^{1-\frac{n_1+n_2}{2}}d\theta}.$$

But the denominator is simplified by $B(\frac{n_1-1}{2},\frac{n_2-1}{2})/(s_1^2)^{(n_1-1)}(s_2^2)^{(n_2-1)}$ using the usual beta function.

The intrinsic statistic $d(\theta_0|\boldsymbol{x})$ is the reference posterior expectation of the amount of information $\delta_{\boldsymbol{x}}\{\tilde{\theta},(\theta,\lambda,\mu_1,\mu_2)\}$ which could be lost if the null model were used. Thus BRC is

$$\text{Reject} \quad \theta=\theta_0 \quad \text{iff} \quad d(\theta_0|\boldsymbol{x}) > d^*. \tag{2.15}$$



<Figure1> The intrinsic statistic $d(\theta_0|\boldsymbol{x})$ for the ratio of two normal variances θ_0, fixing $\theta=\sigma_2^2/\sigma_1^2=1$.

Figure 1 represents the intrinsic statistic calculated from simulated data with $\theta=1$ and several sample sizes of (n_1,n_2). We can see that for small (n_1,n_2) it is not possible to reject almost any value of the parameter and that the criterion becomes more discriminating as the sample size increases.

2.3. The Intrinsic Estimation

Bayes estimates are those which minimize the expected posterior loss. The intrinsic estimate is the Bayes estimate which corresponds to the intrinsic discrepancy loss and the reference posterior distribution. Introduced by Bernardo and Juárez (2003), this is a completely general objective Bayesian estimator which is invariant under reparametrization. The intrinsic expected loss $d(\tilde{\theta}|\boldsymbol{x})$ may be easily be computed by numerical integration. The intrinsic estimate of θ

$$\tilde{\theta}_{int}(\boldsymbol{x}) = \arg\min_{\tilde{\theta}>0} d(\tilde{\theta}|\boldsymbol{x}), \tag{2.16}$$

and the p-credible intrinsic region is the solution $R_p^{int}(\boldsymbol{x}) = (\theta_0, \theta_1)$ to the equation system

$$\left\{ d(\theta_0|\boldsymbol{x}) = d(\theta_1|\boldsymbol{x}),\ \int_{\theta_0}^{\theta_1} \pi(\theta|\boldsymbol{x}) d\theta = p \right\}. \tag{2.17}$$

3. Numerical Analysis

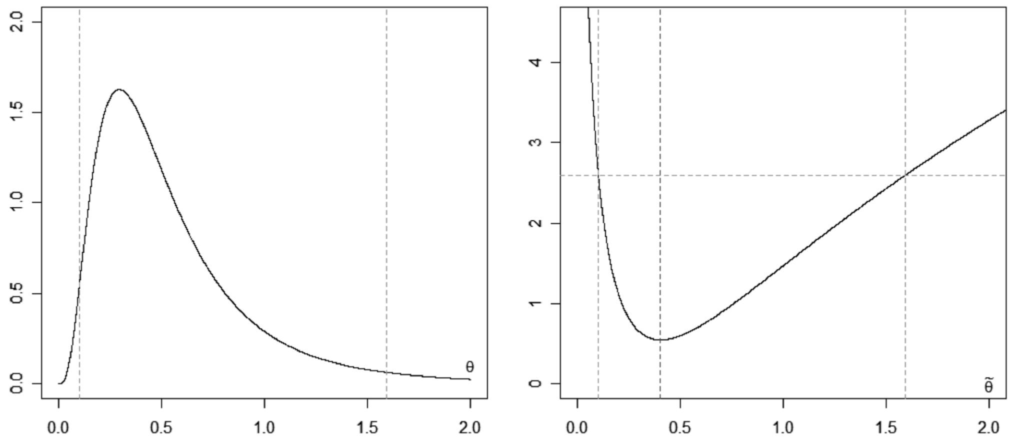
3.1. Example

The data "Calcium and Blood Pressure Story" in DASL(lib.stat.cmu.edu/DASL), which contains a subset of the data shown in Lyle et al.(1987), consist of blood pressure measurements on a subgroup of 21 African-American subjects, 10 who have taken calcium supplements and 11 who have taken placebo. The primary analysis variable is the blood pressure difference("begin" minus "End"). Summary statistics are as follows:

Group	n	mean	StdDev
Calcium	10	5.0000	8.7433
Placebo	11	-0.2727	5.9007

To test the difference between two means, the variance ratio, $\theta = \sigma_2^2/\sigma_1^2$, must be known. In the Frequentist test for $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 \neq \sigma_2^2$, we can reject H_0 with the significance level α when $F > F_{\alpha/2}(n_2-1, n_1-1)$ or $F < F_{1-\alpha/2}(n_2-1, n_1-1)$ where $F = s_2^2/s_1^2$. In our problem, the F-value is 0.4555 and $F_{0.025}(10,9) = 3.96$, $F_{0.975}(10,9) = 0.26$. Thus we cannot reject the null hypothesis H_0 with $\alpha = 0.05$. In BRC, the exact value of the expected intrinsic loss by numerical integration using (2.14) is 1.4692. According to the BRC using the threshold value $d^* = 2.5$, we cannot reject the null hypothesis $H_0 : \theta = 1$. Thus, there is insufficient evidence to indicate a difference in the population variances.

The frequentist 95% confidence interval for σ_2^2/σ_1^2 is (0.1150, 1.7218), the intrinsic 0.95-credible interval is $R_{0.95}^{int} = (0.1033, 1.5930)$ and the intrinsic estimator is $\tilde{\theta}_{int} = 0.4050$. The results from the reference analysis of this data set are encapsulated in Figure 2. The reference posterior distribution $\pi(\theta|\boldsymbol{x})$ is represented in the left panel, and the expected intrinsic loss $d(\tilde{\theta}|\boldsymbol{x})$ from using $\tilde{\theta}$ as proxy for θ is represented in the right panel.



<Figure 2> Reference posterior density $\pi(\theta|\boldsymbol{x})$ (left panel) and intrinsic expected loss $d(\tilde{\theta}|\boldsymbol{x})$ (right panel) for the variance ratio $\theta = \sigma_2^2/\sigma_1^2$. The intrinsic estimator is $\tilde{\theta}_{int} = 0.4050$ and the intrinsic 0.95-credible interval is $R_{0.95}^{int} = (0.1033, 1.5930)$.

As a result, the intrinsic credible interval is superior to the frequentist confidence interval in the sense of length of interval.

3.2. Simulation Study

In order to compare the performance under homogeneous repeated sampling of the BRC with their frequentist counterparts, ten thousand simulations were carried out for several sample sizes (n_1, n_2). Table 1 summarizes this comparison. The third column(BRC) reports the relative number of times when the hypothesis $H_0 : \theta = 1$ was rejected under each criterion, using the threshold values of $d^* = 2.4207$ for the BRC, and the fourth column(F-test) reports the relative number of times when the hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ was rejected α=0.05 for the F

-test.

<Table 1> Comparison of the behaviour under repeated sampling of the BRC and F-test.

(n_1,n_2)	σ_1^2	σ_2^2	BRC	F-test
	1	1	0.0419	0.0219
(2,5)	3	1	0.1601	0.0135
	5	1	0.2509	0.0110
	1	1	0.0219	0.0295
(5,10)	3	1	0.2419	0.0865
	5	1	0.4597	0.2171
	1	1	0.0377	0.0320
(10,20)	3	1	0.4518	0.3300
	5	1	0.7705	0.6686
	1	1	0.0456	0.0402
(20,30)	3	1	0.7457	0.6942
	5	1	0.9661	0.9518
	1	1	0.0412	0.0400
(30,50)	3	1	0.9024	0.8824
	5	1	0.9968	0.9953

From Table 1, we confirm that the powers of the BRC are larger than the powers of the corresponding frequentist test and the powers of both tests increase with sample size. Moreover, under H_0, the frequentist type-I error and the BRC type-I error are comparable.

4. Concluding Remarks

The reference-intrinsic approach described provides a powerful alternative to point and interval estimation and sharp hypothesis testing, with a clear interpretation in terms of information units. This study considers the reference-intrinsic approach for the ratio of two normal variances. In future we will derive the Bayesian reference criterion, the intrinsic estimator and the credible region which corresponds to the intrinsic discrepancy loss and the reference prior in bivariate normal distribution.

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