

# The Small Angle Generator Based on a Laser Angle Interferometer

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KEYWORDS : Angle standard, Small angle generator, Self calibration, Laser angle interferometer, Index table

*To calibrate precision autocollimators, the Korean Research Institute of Standards and Science (KRIS) has built a small angle generator using a laser interferometer. The system is based on a sine bar mechanism in which the angle is determined from the ratio of two lengths. The rotational angle is measured by the angle interferometer and the heterodyne laser interferometer detects the relative displacement of two retro-reflectors attached to the rotating arm. The distance between the two retro-reflectors of the laser angle interferometer is self-calibrated by an index table positioned on the rotating arm. The resolution of the system is 0.002 seconds, and the accuracy is better than 0.04 seconds within a measuring range of  $\pm 1$  degree. The small angle generator can also be used with an index table that can divide one circle into 1440 angles. The combined system can generate any angle over 360 degrees to an accuracy of 0.11 seconds.*

Manuscript received: August 11, 2006 / Accepted: October 23, 2006

## 1. Introduction

Small angles are often measured to determine the rotational guide errors of moving stages as well as the angular deviations of angle standards such as polygons, angle gage blocks and index tables.<sup>1</sup> Photoelectric autocollimators are usually used for this purpose. However, despite their popularity and high resolution, these instruments suffer from low accuracy. For example, a state-of-the-art autocollimator with a resolution of 0.005" has an accuracy of  $\pm 0.03''$  over a measuring range of  $\pm 300''$ .<sup>2</sup> This means that the random errors of autocollimators are much smaller than their systematic errors. Therefore, periodical calibrations are required to determine and quantitatively compensate for the systematic errors. Many measurement laboratories have developed calibration systems for autocollimators.<sup>3,4</sup> Once such calibration device, called a small angle generator, must have very small uncertainties compared to those of the autocollimator to be calibrated. Generally, small angle generators used to calibrate autocollimators with high resolution must have a high degree of accuracy as well as precision. Some autocollimator manufacturers supply optical wedges to calibrate their autocollimators. Calibrations using an optical edge are very simple, but they are limited because the autocollimator can be calibrated at only one angle.

Several techniques for generating and measuring small angles up to a couple of degrees have been reported.<sup>3-7</sup> Most use the sine relationship which states that an angle can be determined from the ratio of the lengths of the opposite side and hypotenuse of a right triangle. By this means, an angle can be determined simply by measuring lengths. Traditionally, angle devices like the sine bar have been widely used to generate small angles. The hypotenuse is established in construction as the center distance of two hardened

precision gage rollers or balls, and the opposite side is established or calibrated by micrometer heads, gage blocks or linear transducers. However, an angle generator of this type is based on the length standard, so it is necessary to calibrate the system in order to convert the length unit into the angle unit. Furthermore, the mechanical accuracy, such as the roundness of the roller or radial run-out of the bearing, has an effect on the performance.

In this study, a very simple and accurate small angle generator using a laser angle interferometer was designed and built. The system had a rotating arm driven by a sine bar mechanism. The interferometer detected the relative motion of two retro-reflectors attached to the rotating arm. The rotating angle was determined by the distance between the two retro-reflectors and the laser reading. The system was self-calibrated and self-tested by an indexing table with accuracy of 0.08". The small angle generator provided a large measuring range of  $\pm 1^\circ$  and high degree of accuracy of 0.04".

The main application of our small angle generator is to calibrate a precision autocollimator, but it can also be used to calibrate angle gage blocks whose angles cannot be determined from an indexing table alone (such as, for example, angles of 10", 30", 1', 5', 10', and 20'). In order to generate any angle over a 360° full circle, we combine the small angle generator with an index table. With our angle generator, angles larger than 0.25° are generated by the index table and angles smaller than 0.25° are generated by the small angle generator. Using both systems, any angle within a 360° range can be generated.

## 2. System description

Figure 1 shows the main components of the small angle generator.

The system consisted of a main body, a rotating part and a laser angle interferometer. The main body, which was made of a cast-iron, was positioned on a granite surface plate with a vibration isolator. A laser interferometer head, the autocollimator to be calibrated and a precision bearing with a rotating arm were fixed to the main body. The rotating arm usually carried the precision mirror for the autocollimator and two retro-reflectors for the angle interferometer. The rotating arm had a massive and stable structure because it must carry the two indexing tables to calibrate the distance between the two retro-reflectors (referred to as the arm length hereafter). Its rotation induced a relative displacement between the two retro-reflectors, and this displacement was detected by the laser angle interferometer. As a result, the rotation angle  $\theta$  could be determined according to the simple formula

$$\theta = \arcsin(d/L) \quad (1)$$

Here,  $d$  is the relative displacement between the two retro-reflectors and  $L$  is the arm length. The optical source in Fig. 1 was a two-frequency Zeeman He-Ne laser with a wavelength of 632.8 nm.<sup>8</sup> Starting from the laser, a polarizing beam splitter divided the laser beam into two components with different frequencies, one vertically polarized and the other horizontally polarized, propagating in the separate arms of the interferometer. The two beams were reflected at the two retro-reflectors and recombined at the beam splitter. The beams interfered to generate a heterodyne signal at a detector. A heterodyne interferometer was used as the angle interferometer because it was insensitive to the alignment of the interference beams compared to a homodyne interferometer. It also had a high resolution over a wide range. The positions of the two beams at the detector changed horizontally as the arm of the angle interferometer rotated, which may cause the intensity of the interference signal to fluctuate and the nonlinearity of the laser reading to increase. Equation 1 is not valid for a solid retro-reflector; it must be modified to compensate for the refractive index of glass relative to air.<sup>9</sup> Thus, we used a hollow retro-reflector in the angle interferometer, even though a solid retro-reflector is thermally and mechanically more stable. Optical components, such as beam splitters and mirrors, were cemented to the compact tilting stages. These stages, which were rigidly fixed to the main body, were used to accurately align the optical components. In order to reduce the effect of air turbulence, the optical components and laser beam path were covered with an aluminum box. A motorized micrometer and a piezo-transducer (PZT) were used to rotate the arm. Usually they were controlled manually; however a computer could control the motorized micrometer and the PZT to rotate the arm to a predefined angle. The motorized micrometer covered the full rotating range of the system, whereas the PZT covered only 5".

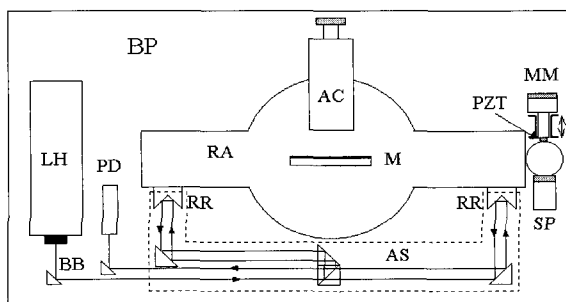


Fig. 1 Schematic diagram of the small-angle generator (LH: laser, BB: beam bender, PD: detector for interference signal, RR: retro-reflector, RA: rotating arm, AS: shield to protect the generator from the airflow, SP: spring, M: mirror, AC: autocollimator, MM: motorized micrometer, PZT: piezo-transducer, and BP: base plate)

### 3. Self-calibration of the small angle generator

The accuracy of a small angle generator mainly depends on how accurately the arm length is determined. A simple method to determine the arm length is to use an optical microscope and a linear laser interferometer, which can measure the distance between corner edges of the two retro-reflectors. The accuracy of the result depends on the sharpness of the corner edges. We used two index tables to calibrate the arm length. Index table A, which was positioned on the rotating arm, allowed 1440 angles (every 1/4°) within one circle (360°) to be determined to an accuracy better than 0.08". It was used as a reference to calibrate the arm length. Index table B, which was positioned on index table A, had a 1° rotational resolution and 0.3" accuracy. In this experiment, the use of index table B made it easier to calibrate the arm length, although we could have measured the arm length without it. The mirror, which was mounted on index table B, was shielded from temperature variations and air turbulence by a protective cover. It was used in conjunction with the automatic autocollimator with a resolution of 0.01". The rotating arm, index tables A and B and mirror were concentrically aligned and rigidly fixed to one another. The procedure for calibrating the arm length was as follows.

- 1) Set the small angle generator at the reference position and reset the autocollimator and laser readings to zero.
- 2) Rotate index table A to 2°. Rotate the small angle generator in the opposite direction so that autocollimator reading is zero. Then,  $d$  measured by the interferometer is converted to the arm length  $L$  using the relationship  $L = d / \sin \theta$  (here  $\theta$  is 2°). This relationship ensures that we can determine the arm length with an uncertainty related to the accuracy of the interferometer and index table A and the repeatability of the autocollimator. For simplification, we suppose that the uncertainty of the interferometer is much smaller than that of index table A and the short-term noise of the autocollimator is zero.<sup>9</sup> Then, the arm length is determined relative to the accuracy of index table A (0.1" accuracy). This means that a small angle generator cannot have an angular accuracy better than 0.1" over a measuring range of 2°. To improve the system accuracy, the arm length must be measured with higher accuracy. For this purpose, we adopted the partial closure method often used in precision angle measurements.
- 3) Return the small angle generator to the reference position and rotate index table B 2° in the opposite direction so that the autocollimator can view the mirror. Then, reset the autocollimator and laser readings to zero.

Table 1 Measured results of the distance between two retro-reflectors. The distance was self-calibrated by an index table using the partial closure method

Interval of index table A (degrees)	Distance between the two retro-reflectors (mm)		
	Average of fifteen measurements	Deviation (peak to peak)	Standard deviation
0 – 30	400.1283	0.0074	0.0021
30 – 60	400.1270	0.0080	0.0024
60 – 90	400.1286	0.0065	0.0015
180 – 210	400.1275	0.0075	0.0018
210 – 240	400.1284	0.0070	0.0019
240 – 270	400.1272	0.0064	0.0025
Average	400.1278	0.0016	0.0006

- 4) Repeat (2) and (3) fifteen times, giving fifteen results for the arm length  $L$  over a  $30^\circ$  interval of index table A. The accuracy of index table A over this  $30^\circ$  is also below  $0.1''$ , so the average of fifteen results will be fifteen times more accurate than a single result. Measurements over six different intervals of index table A were performed. The average arm length determined by the six measurements was  $400.1278 \pm 0.0018$  mm ( $k = 2$ ). The results are summarized in Table 1.

**4. Performance and results**

Table 2 shows a roughly evaluated uncertainty budget. This was treated in accordance with ISO guidelines for the expression of uncertainties.<sup>10</sup> The expanded uncertainty of the system at a 95 % confidence level was  $0.04''$  over a measuring range of  $\pm 1^\circ$ . The most significant sources of uncertainty in the system were the arm length error and thermal stability.

Table 2 Estimated uncertainty budget of the small angle generator over a measuring range of  $2^\circ$

Sources of uncertainty	Uncertainty (")
Arm length determined by self-calibration (0.0009 mm)	0.016
Thermal expansion correction of the arm length (1.5 ppm)	0.010
Short-term repeatability	0.003
Long-term drift over 60 minutes	0.005
Flatness of the retro-reflectors (0.006 $\mu\text{m}$ ) <sup>a</sup>	0.003
Wavelength of the laser (0.4 ppm)	0.003
Combined standard uncertainty $u_c$	0.020
Expanded uncertainty ( $k = 2$ )	0.040

<sup>a</sup> The retro-reflector specifications state a flatness and aperture of  $\lambda/20$  and 25 mm, respectively. The laser beams moved across a small area (about 0.3 mm in the horizontal direction) and the retro-reflectors rotated over an angle of  $2^\circ$ . We assumed that 1/5 of the flatness had an effect on the measurements.

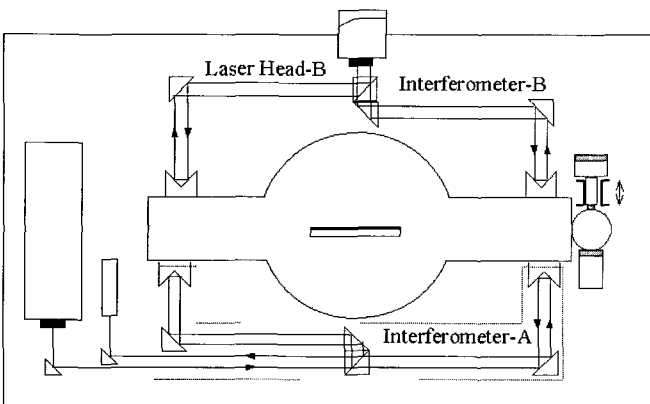


Fig.2 Experimental setup for checking the performance of the system. Interferometer B was temporarily installed on the small angle generator. In this setup, the distance between the two retro-reflectors of interferometer B was also self-calibrated by the partial closure method using an index table. The results are shown in Fig. 3

To evaluate the accuracy of the system and confirm the estimated uncertainty of the measurements, another interferometer, angle

interferometer B, was temporarily installed in the system, as shown Fig. 2. The same optical components found in interferometer A were used. The arm length of interferometer B was also measured by the self-calibration method. Its measured value was 382.425 mm, which was determined using only one index table interval ( $0 - 30^\circ$ ). The arm length of interferometer B was intentionally chosen to be different from that of interferometer A so that any difference between the values obtained by the two interferometers would contain systematic errors caused by the two arm lengths, nonlinearity of the laser, compensation of the refractive index of air, etc. A plot of the difference between the two values as a function of the rotating angle is shown in Fig. 3. The abscissa in Fig. 3 is the rotating angle measured by interferometer A. Six measurements were performed over one hour. The difference was proportional to the rotation angle. As shown in the figure, the estimated uncertainty of the measurements was in good agreement with the data. The maximum difference was  $0.05''$  at a rotation angle of  $1^\circ$ , which was less than the sum of the uncertainties of the two interferometers.

The long-term stability of the system is shown in Fig. 4, which gives 6000 individual observations made over thirteen hours. Usually, long-term stability is not required for a small angle generator because it takes less than ten minutes to calibrate an autocollimator. However, the peak-to-peak range of Fig. 4 was about  $0.05''$ . The figure shows that this variation was mainly due to long-term drift. During the thirteen hours of operation, variations in the temperature of the rotating arm and the refractive index of air were  $0.2^\circ\text{C}$  and  $4.2 \times 10^{-6}$  respectively. The system can compensate for the thermal expansion of the arm length automatically. But thermal effects on angle measurements are more complicated and cannot be estimated accurately from linear measurements. (Our system had a symmetric optical configuration, which was not very sensitive to changes in the temperature and refractive index of air. Nevertheless, thermal distortion of the optical components and the mechanical parts by temperature drift was inevitable, which caused the optical path to change without any rotation of the arm.) Therefore, to obtain a high degree of accuracy over long period, the mechanical parts should be made of low thermal expansion material.

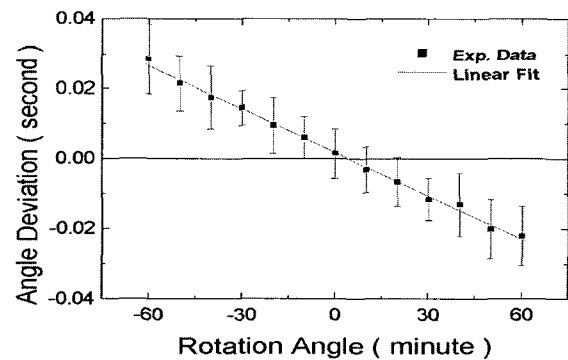


Fig.3 Angle deviation between the two angle interferometers as a function of rotation angle. The maximum angle deviation was below 0.04 seconds at a rotation angle of  $1^\circ$ , which is smaller than the sum of uncertainties for the two angle interferometers

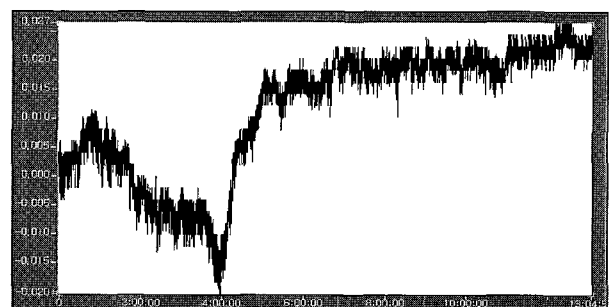


Fig. 4 Long-term stability of the system

Figure 5 shows calibration curves for three commercial precision photoelectric autocollimators. The autocollimators had device-specific errors that were greater than their resolution. This implies that the autocollimator must be calibrated for precise measurements. Since the systematic errors of the three devices were greater than the accuracy of the small angle generator, the generator could be used to calibrate them.

Continuous measurements over a full-circle range of  $360^\circ$  are required for general angle measurements. A goniometer system using a rotary encoder and an automatic stack of three index tables is generally used to achieve this.<sup>11,12</sup> Most of these systems are very large and complicated, but they are accurate. At our institute, a system in which a small angle generator is combined with an index table is used to generate angles over a full circle. In this angle generator, angles greater than  $0.25^\circ$  are generated by the index table and angles smaller than  $0.25^\circ$  are generated by the small angle generator. Using both systems, any angle over a full circle can be generated. Fig. 6 shows an angle generator combining a small angle generator with an index table. Its accuracy over a full circle is under  $0.11''$  without calibration.

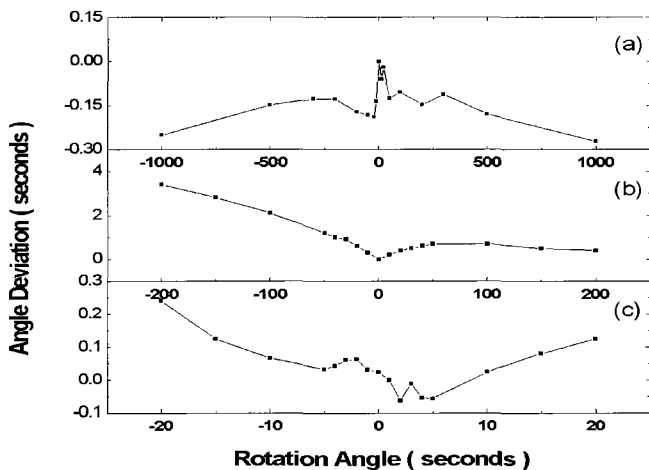


Fig. 5 Calibration curves for three different types of photoelectric autocollimators. The resolutions of the autocollimators are  $0.05''$  for (a),  $0.1''$  for (b) and  $0.01''$  for (c)

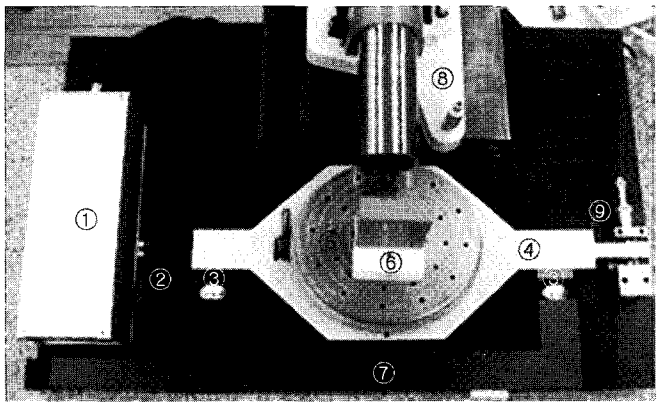


Fig. 6 Full-circle angle generator using a small angle generator and an index table. ① He-Ne laser head, ② photodiode, ③ retro-reflector, ④ rotating arm, ⑤ index table, ⑥ mirror, ⑦ angle interferometer optics, ⑧ autocollimator, and ⑨ micrometer head & PZT

## 5. Conclusion

We demonstrated a small angle generator for calibrating autocollimators. The rotational angle was measured by a laser angle interferometer that detected the relative motion of two retro-reflectors

attached to the rotating arm. The system was based on a sine bar mechanism in which the angle was determined from the ratio of two lengths of the opposite side and hypotenuse of a right triangle. The distance between the two retro-reflectors was self-calibrated by the partial closure method using an index table. We are convinced that this method is a simple and very accurate technique for calibrating the arm length in an angle interferometer. Our preliminary test showed that the system was fully satisfactory for calibrating commercial autocollimators with a high degree of resolution and accuracy. The system had an accuracy that was better than  $0.04''$  over a range of  $\pm 1^\circ$ . However, the long-term stability of the system was worse than we expected, although this is not a concern when calibrating autocollimators. There are possibilities for improving the long-term stability using a low thermal expansion material such as an Invar alloy or Zerodur.

The small angle generator can also be used as an angle standard by combining it with an index table. It can generate any angle in the full-circle range of  $360^\circ$ . Using this system, we can calibrate an autocollimator as well as angle gage blocks and polygons with angles that do not match the subdivision of the index table. Its accuracy over a full circle is better than  $0.11''$  without calibration.

## ACKNOWLEDGEMENT

We acknowledge the financial support of the Korean Ministry of Science and Technology.

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