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Fuzzy strongly (r, s)-semicontinuous, fuzzy strongly (r, s)-semiopen and fuzzy strongly (r, s)-semiclosed mappings

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Abstract

In this paper, we investigate some of characteristic properties of fuzzy strongly (r, s)-semicontinuous, fuzzy strongly (r, s)-semiopen and fuzzy strongly (r, s)-semiclosed mappings on the intuitionistic fuzzy topological space in Šostak's sense.

Key words : fuzzy strongly (r, s)-semicontinuous mappings

1. Introduction

After the introduction of fuzzy sets by Zadeh [11], Chang [2] was the first to introduce the concept of a fuzzy topology on a set X by axiomatizing a collection T of fuzzy subsets of X, where he referred to each member of T as an open set. In his definition of fuzzy topology, fuzziness in the concept of openness of a fuzzy subset was absent. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [10], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [9].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker and his colleagues [4, 6] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

In this paper, we investigate some of characteristic properties of fuzzy strongly (r, s)-semicontinuous, fuzzy strongly (r, s)-semiopen and fuzzy strongly (r, s)-semiclosed mappings on the intuitionistic fuzzy topological space in Šostak's sense.

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2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A of X is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1. [1] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on X. Then

(1) $A \subseteq B$ if $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.

(2) $A = B$ if $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = (\gamma_A, \mu_A)$.

(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.

(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.

(6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* on X is a map $T : I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.

(3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair (X, T) is called a *smooth fuzzy topological space.*

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in T$.

(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.

(3) If $A_i \in T$ for all i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2. [5] Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* (SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a map $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(0_\sim) = \mathcal{T}_1(1_\sim) = 1$ and $\mathcal{T}_2(0_\sim) = \mathcal{T}_2(1_\sim) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense* (SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Let $(X, \mathcal{T})$ be an intuitionistic fuzzy topological space in Šostak's sense. Then it is easy to see that for each $(r, s) \in I \otimes I$, the family $\mathcal{T}_{(r,s)}$ defined by

$$\mathcal{T}_{(r,s)} = \{A \in I(X) \mid \mathcal{T}_1(A) \geq r \text{ and } \mathcal{T}_2(A) \leq s\}$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $(r, s) \in I \otimes I$. Then the map $T^{(r,s)} : I(X) \to I \otimes I$ defined by

$$T^{(r,s)}(A) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim \\ (r, s) & \text{if } A \in T - \{0_\sim, 1_\sim\} \\ (0, 1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Šostak's sense on X.

Definition 2.3. [7] Let A be an intuitionistic fuzzy set of a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-open* if $\mathcal{T}_1(A) \geq r$ and $\mathcal{T}_2(A) \leq s$,

(2) *fuzzy (r, s)-closed* if $\mathcal{T}_1(A^c) \geq r$ and $\mathcal{T}_2(A^c) \leq s$.

Definition 2.4. [7] Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$. For each $A \in I(X)$, the *fuzzy (r, s)-closure* is defined by

$$\text{cl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B,\ B \text{ is fuzzy } (r, s)\text{-closed}\}$$

and the *fuzzy (r, s)-interior* is defined by

$$\text{int}(A, r, s) = \bigcup\{B \in I(X) \mid A \supseteq B,\ B \text{ is fuzzy } (r, s)\text{-open}\}.$$

Lemma 2.5. [7] For an intuitionistic fuzzy set A of a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$,

(1) $\text{int}(A, r, s)^c = \text{cl}(A^c, r, s)$.

(2) $\text{cl}(A, r, s)^c = \text{int}(A^c, r, s)$.

Definition 2.6. [8] Let A be an intuitionistic fuzzy set of a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) a *fuzzy strongly* (r, s)*-semiopen* set if there is a fuzzy (r, s)-open set B in X such that $B \subseteq A \subseteq \text{int}(\text{cl}(B, r, s), r, s)$,

(2) a *fuzzy strongly* (r, s)*-semiclosed* set if there is a fuzzy (r, s)-closed set B in X such that $\text{cl}(\text{int}(B, r, s), r, s) \subseteq A \subseteq B$.

3. Fuzzy strongly (r, s)-semicontinuous, fuzzy strongly (r, s)-semiopen and fuzzy strongly (r, s)-semiclosed mappings

Definition 3.1. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy strongly* (r, s)*-semiclosure* is defined by

$$\text{sscl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B, B \text{ is fuzzy strongly } (r, s)\text{-semiclosed}\}$$

and the *fuzzy strongly* (r, s)*-semiinterior* is defined by

$$\text{ssint}(A, r, s) = \bigcup\{B \in I(X) \mid A \supseteq B, B \text{ is fuzzy strongly } (r, s)\text{-semiopen}\}.$$

Obviously, $\text{sscl}(A, r, s)$ is the smallest fuzzy strongly (r, s)-semiclosed set which contains A and $\text{sscl}(A, r, s) = A$ for any fuzzy strongly (r, s)-semiclosed set A. Also, $\text{ssint}(A, r, s)$ is the greatest fuzzy strongly (r, s)-semiopen set which is contained in A and $\text{ssint}(A, r, s) = A$ for any fuzzy strongly (r, s)-semiopen set A. Moreover, we have

$$\text{int}(A, r, s) \subseteq \text{ssint}(A, r, s) \subseteq A$$

and

$$A \subseteq \text{sscl}(A, r, s) \subseteq \text{cl}(A, r, s).$$

Also, we have the following results:

(1) $\text{ssint}(0_\sim, r, s) = 0_\sim$, $\text{ssint}(1_\sim, r, s) = 1_\sim$.

(2) $\text{ssint}(A, r, s) \subseteq A$.

(3) $\text{ssint}(A \cap B, r, s) \subseteq \text{ssint}(A, r, s) \cap \text{ssint}(B, r, s)$.

(4) $\text{ssint}(\text{ssint}(A, r, s)) = \text{ssint}(A, r, s)$.

(5) $\text{sscl}(0_\sim, r, s) = 0_\sim$, $\text{sscl}(1_\sim, r, s) = 1_\sim$.

(6) $\text{sscl}(A, r, s) \supseteq A$.

(7) $\text{sscl}(A \cup B, r, s) \subseteq \text{sscl}(A, r, s) \cup \text{sscl}(B, r, s)$.

(8) $\text{sscl}(\text{sscl}(A, r, s), r, s) = \text{sscl}(A, r, s)$.

Theorem 3.2. For an intuitionistic fuzzy set A of a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$, we have

(1) $\text{ssint}(A, r, s)^c = \text{sscl}(A^c, r, s)$.

(2) $\text{sscl}(A, r, s)^c = \text{ssint}(A^c, r, s)$.

Proof. (1) Since $A^c \subseteq \text{sscl}(A^c, r, s)$ and $\text{sscl}(A^c, r, s)$ is fuzzy strongly (r, s)-semiclosed in X, $\text{sscl}(A^c, r, s)^c \subseteq A$ and $\text{sscl}(A^c, r, s)^c$ is fuzzy strongly (r, s)-semiopen in X. Thus

$$\begin{aligned}\text{sscl}(A^c, r, s)^c &= \text{ssint}(\text{sscl}(A^c, r, s)^c, r, s) \\ &\subseteq \text{ssint}(A, r, s)\end{aligned}$$

and hence $\text{ssint}(A, r, s)^c \subseteq \text{sscl}(A^c, r, s)$. Conversely, since $\text{ssint}(A, r, s) \subseteq A$ and $\text{ssint}(A, r, s)$ is fuzzy strongly (r, s)-semiopen in X, $A^c \subseteq \text{ssint}(A, r, s)^c$ and $\text{ssint}(A, r, s)^c$ is fuzzy strongly (r, s)-semiclosed in X. Thus

$$\begin{aligned}\text{ssint}(A, r, s)^c &= \text{sscl}(\text{ssint}(A, r, s)^c, r, s) \\ &\supseteq \text{sscl}(A^c, r, s).\end{aligned}$$

(2) Similar to (1). □

Definition 3.3. [7] Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be

(1) a *fuzzy* (r, s)*-continuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-open set of X for each fuzzy (r, s)-open set B of Y,

(2) a *fuzzy* (r, s)*-open* mapping if $f(A)$ is a fuzzy (r, s)-open set of Y for each fuzzy (r, s)-open set A of X,

(3) a *fuzzy* (r,s)*-closed* mapping if $f(A)$ is a fuzzy (r,s)-closed set of Y for each fuzzy (r,s)-closed set A of X.

Definition 3.4. Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is said to be

(1) a *fuzzy strongly* (r,s)*-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy strongly (r,s)-semiopen set of X for each fuzzy (r,s)-open set B of Y,

(2) a *fuzzy strongly* (r,s)*-semiopen* mapping if $f(A)$ is a fuzzy strongly (r,s)-semiopen set of Y for each fuzzy (r,s)-open set A of X,

(3) a *fuzzy strongly* (r,s)*-semiclosed* mapping if $f(A)$ is a fuzzy strongly (r,s)-semiclosed set of Y for each fuzzy (r,s)-closed set A of X.

It is obvious that every fuzzy (r,s)-continuous mapping is also a fuzzy strongly (r,s)-semicontinuous mapping and every fuzzy (r,s)-open ((r,s)-closed) mapping is also a fuzzy strongly (r,s)-semiopen ((r,s)-semiclosed) mapping. However, the following examples show that all of the converses need not be true.

Example 3.5. Let $X = \{x,y\}$ and let A_1 and A_2 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.1,0.7), \quad A_1(y) = (0.4,0.3)$$

and

$$A_2(x) = (0.3,0.5), \quad A_2(y) = (0.8,0.2).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A),\mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A),\mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $(\mathcal{T}_1,\mathcal{T}_2)$ and $(\mathcal{U}_1,\mathcal{U}_2)$ are SoIFTs on X. Consider the identity mapping $1_X : (X,\mathcal{T}_1,\mathcal{T}_2) \to (X,\mathcal{U}_1,\mathcal{U}_2)$. Then it is a fuzzy strongly $(\frac{1}{2},\frac{1}{3})$-semicontinuous mapping which is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-continuous mapping.

Example 3.6. Let $X = \{x,y\}$ and let A_1 and A_2 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.5,0.1), \quad A_1(y) = (0.7,0.2)$$

and

$$A_2(x) = (0.2,0.4), \quad A_2(y) = (0.5,0.4).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A),\mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A),\mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $(\mathcal{T}_1,\mathcal{T}_2)$ and $(\mathcal{U}_1,\mathcal{U}_2)$ are SoIFTs on X. Consider the identity mapping $1_X : (X,\mathcal{T}_1,\mathcal{T}_2) \to (X,\mathcal{U}_1,\mathcal{U}_2)$. Then it is a fuzzy strongly $(\frac{1}{2},\frac{1}{3})$-semiopen mapping which is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-open mapping.

Example 3.7. Let $X = \{x,y\}$ and let A_1 and A_2 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.8,0.1), \quad A_1(y) = (0.6,0.2)$$

and

$$A_2(x) = (0.1,0.6), \quad A_2(y) = (0.5,0.3).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A),\mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A),\mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2},\frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $(\mathcal{T}_1,\mathcal{T}_2)$ and $(\mathcal{U}_1,\mathcal{U}_2)$ are SoIFTs on X. Consider the identity mapping $1_X : (X,\mathcal{T}_1,\mathcal{T}_2) \to (X,\mathcal{U}_1,\mathcal{U}_2)$. Then it is a fuzzy strongly $(\frac{1}{2},\frac{1}{3})$-semiclosed mapping which is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-closed mapping.

Theorem 3.8. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy strongly (r, s)-semicontinuous mapping.

(2) $f^{-1}(B)$ is a fuzzy strongly (r, s)-semiclosed set of X for each fuzzy (r, s)-closed set B of Y.

(3) $f(\text{sscl}(A, r, s)) \subseteq \text{cl}(f(A), r, s)$ for each intuitionistic fuzzy set A of X.

(4) $\text{sscl}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{cl}(B, r, s))$ for each intuitionistic fuzzy set B of Y.

(5) $f^{-1}(\text{int}(B, r, s)) \subseteq \text{ssint}(f^{-1}(B), r, s)$ for each intuitionistic fuzzy set B of Y.

Proof. (1) $\Rightarrow$ (2) Let B be any fuzzy (r, s)-closed set of Y. Then B^c is a fuzzy (r, s)-opens set of Y. Since f is a fuzzy strongly (r, s)-semicontinuous mapping, $f^{-1}(B^c)$ is a fuzzy strongly (r, s)-semiopen set of X. Thus $f^{-1}(B)$ is a fuzzy strongly (r, s)-semiclosed set of X.

(2) $\Rightarrow$ (3) Let A be any intuitionistic fuzzy set of X. Then $\text{cl}(f(A), r, s)$ is a fuzzy (r, s)-closed set of Y. By (2), $f^{-1}(\text{cl}(f(A), r, s))$ is a fuzzy strongly (r, s)-semiclosed set of X. Since $f(A) \subseteq \text{cl}(f(A), r, s)$, we have

$$\begin{aligned}\text{sscl}(A, r, s) &\subseteq \text{sscl}(f^{-1}f(A), r, s)\\ &\subseteq \text{sscl}(f^{-1}(\text{cl}(f(A), r, s)), r, s)\\ &= f^{-1}(\text{cl}(f(A), r, s)).\end{aligned}$$

Hence

$$f(\text{sscl}(A, r, s)) \subseteq ff^{-1}(\text{cl}(f(A), r, s)) \subseteq \text{cl}(f(A), r, s).$$

(3) $\Rightarrow$ (4) Let B be any intuitionistic fuzzy set of Y. By (3),

$$f(\text{sscl}(f^{-1}(B), r, s)) \subseteq \text{cl}(ff^{-1}(B), r, s) \subseteq \text{cl}(B, r, s).$$

Thus

$$\begin{aligned}\text{sscl}(f^{-1}(B), r, s) &\subseteq f^{-1}f(\text{sscl}(f^{-1}(B), r, s))\\ &\subseteq f^{-1}(\text{cl}(B, r, s)).\end{aligned}$$

(4) $\Rightarrow$ (5) Let B be any intuitionistic fuzzy set of Y. Then B^c is an intuitionistic fuzzy set of Y. By (4),

$$\begin{aligned}\text{sscl}(f^{-1}(B)^c, r, s) &= \text{sscl}(f^{-1}(B^c), r, s)\\ &\subseteq f^{-1}(\text{cl}(B^c, r, s)).\end{aligned}$$

By Lemma 2.5 and Theorem 3.2,

$$\begin{aligned}f^{-1}(\text{int}(B, r, s)) &= f^{-1}(\text{cl}(B^c, r, s))^c\\ &\subseteq \text{sscl}(f^{-1}(B^c), r, s)^c\\ &= \text{ssint}(f^{-1}(B), r, s).\end{aligned}$$

(5) $\Rightarrow$ (1) Let B be any fuzzy (r, s)-open set of Y. Then $\text{int}(B, r, s) = B$. By (5),

$$\begin{aligned}f^{-1}(B) &= f^{-1}(\text{int}(B, r, s))\\ &\subseteq \text{ssint}(f^{-1}(B), r, s)\\ &\subseteq f^{-1}(B).\end{aligned}$$

So $f^{-1}(B) = \text{ssint}(f^{-1}(B), r, s)$ and hence $f^{-1}(B)$ is a strongly (r, s)-semiopen set of X. Thus f is a fuzzy strongly (r, s)-semicontinuous mapping. □

Theorem 3.9. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a bijection and $(r, s) \in I \otimes I$. Then f is a fuzzy strongly (r, s)-semicontinuous mapping if and only if $\text{int}(f(A), r, s) \subseteq f(\text{ssint}(A, r, s))$ for each intuitionistic fuzzy set A of X.

Proof. Let f be a fuzzy strongly (r, s)-semicontinuous mapping and A any intuitionistic fuzzy set of X. Since $\text{int}(f(A), r, s)$ is a fuzzy (r, s)-open set of Y, we have $f^{-1}(\text{int}(f(A), r, s))$ is a fuzzy strongly (r, s)-semiopen set of X. Since f is fuzzy strongly (r, s)-semicontinuous and one-to-one, we have

$$\begin{aligned}f^{-1}(\text{int}(f(A), r, s)) &\subseteq \text{ssint}(f^{-1}f(A), r, s)\\ &= \text{ssint}(A, r, s).\end{aligned}$$

Since f is onto,

$$\begin{aligned}\text{int}(f(A), r, s) &= ff^{-1}(\text{int}(f(A), r, s))\\ &\subseteq f(\text{ssint}(A, r, s)).\end{aligned}$$

Conversely, let B be any fuzzy (r, s)-open set of Y. Then $\text{int}(B, r, s) = B$. Since f is onto,

$$\begin{aligned}f(\text{ssint}(f^{-1}(B), r, s)) &\supseteq \text{int}(ff^{-1}(B), r, s)\\ &= \text{int}(B, r, s) = B.\end{aligned}$$

Since f is one-to-one, we have

$$\begin{aligned}f^{-1}(B) &\subseteq f^{-1}f(\text{ssint}(f^{-1}(B), r, s))\\ &= \text{ssint}(f^{-1}(B), r, s)\\ &\subseteq f^{-1}(B).\end{aligned}$$

Thus $f^{-1}(B) = \text{ssint}(f^{-1}(B), r, s)$ and hence $f^{-1}(B)$ is a fuzzy strongly (r, s)-semiopen set of X. Therefore f is a fuzzy strongly (r, s)-semicontinuous mapping. □

Theorem 3.10. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy strongly (r, s)-semiopen mapping.

(2) $f(\text{int}(A, r, s)) \subseteq \text{ssint}(f(A), r, s)$ for each intuitionistic fuzzy set A of X.

(3) $\text{int}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{ssint}(B, r, s))$ for each intuitionistic fuzzy set B of Y.

Proof. (1) $\Rightarrow$ (2) Let A be any intuitionistic fuzzy set of X. Clearly $\text{int}(A, r, s)$ is a fuzzy (r, s)-open set of X. Since f is a fuzzy strongly (r, s)-semiopen mapping, $f(\text{int}(A, r, s))$ is a fuzzy strongly (r, s)-semiopen set of Y. Thus

$$\begin{aligned} f(\text{int}(A, r, s) &= \text{ssint}(f(\text{int}(A, r, s))r, s) \\ &\subseteq \text{ssint}(f(A), r, s). \end{aligned}$$

(2) $\Rightarrow$ (3) Let B be any intuitionistic fuzzy set of Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set of X. By (2),

$$\begin{aligned} f(\text{int}(f^{-1}(B), r, s)) &\subseteq \text{ssint}(ff^{-1}(B), r, s) \\ &\subseteq \text{ssint}(B, r, s). \end{aligned}$$

Thus we have

$$\begin{aligned} \text{int}(f^{-1}(B), r, s) &\subseteq f^{-1}f(\text{int}(f^{-1}(B), r, s)) \\ &\subseteq f^{-1}(\text{ssint}(B, r, s)). \end{aligned}$$

(3) $\Rightarrow$ (1) Let A be any fuzzy (r, s)-open set of X. Then $\text{int}(A, r, s) = A$ and $f(A)$ is an intuitionistic fuzzy set of Y. By (3),

$$\begin{aligned} A = \text{int}(A, r, s) &\subseteq \text{int}(f^{-1}f(A), r, s) \\ &\subseteq f^{-1}(\text{ssint}(f(A), r, s)). \end{aligned}$$

Hence we have

$$\begin{aligned} f(A) &\subseteq ff^{-1}(\text{ssint}(f(A), r, s)) \\ &\subseteq \text{ssint}(f(A), r, s) \\ &\subseteq f(A). \end{aligned}$$

Thus $f(A) = \text{ssint}(f(A), r, s)$ and hence $f(A)$ is a fuzzy strongly (r, s)-semiopen set of Y. Therefore f is a fuzzy strongly (r, s)-semiopen mapping. □

Theorem 3.11. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy strongly (r, s)-semiclosed mapping.

(2) $\text{sscl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s))$ for each intuitionistic fuzzy set A of X.

Proof. (1) $\Rightarrow$ (2) Let A be any intuitionistic fuzzy set of X. Clearly $\text{cl}(A, r, s)$ is a fuzzy (r, s)-closed set of X. Since f is a fuzzy strongly (r, s)-semiclosed mapping, $f(\text{cl}(A, r, s))$ is a fuzzy strongly (r, s)-semiclosed set of Y. Thus we have

$$\begin{aligned} \text{sscl}(f(A), r, s) &\subseteq \text{sscl}(f(\text{cl}(A, r, s)), r, s) \\ &= f(\text{cl}(A, r, s)). \end{aligned}$$

(2) $\Rightarrow$ (1) Let A be any fuzzy (r, s)-closed set of X. Then $\text{cl}(A, r, s) = A$. By (2),

$$\begin{aligned} \text{sscl}(f(A), r, s) &\subseteq f(\text{cl}(A, r, s) \\ &= f(A) \\ &\subseteq \text{sscl}(f(A), r, s). \end{aligned}$$

Thus $f(A) = \text{sscl}(f(A), r, s)$ and hence $f(A)$ is a fuzzy strongly (r, s)-semiclosed set of Y. Therefore f is a fuzzy strongly (r, s)-semiclosed mapping. □

Theorem 3.12. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a bijection and $(r, s) \in I \otimes I$. Then f is a fuzzy strongly (r, s)-semiclosed mapping if and only if $f^{-1}(\text{sscl}(B, r, s)) \subseteq \text{cl}(f^{-1}(B), r, s)$ for each intuitionistic fuzzy set B of Y.

Proof. Let f be a fuzzy strongly (r, s)-semiclosed mapping and B any intuitionistic fuzzy set of Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set of X. Since f is fuzzy strongly (r, s)-semiclosed and onto,

$$\begin{aligned} \text{sscl}(B, r, s) &= \text{sscl}(ff^{-1}(B), r, s) \\ &\subseteq f(\text{cl}(f^{-1}(B), r, s)). \end{aligned}$$

Since f is one-to-one, we have

$$\begin{aligned} f^{-1}(\text{sscl}(B, r, s)) &\subseteq f^{-1}f(\text{cl}(f^{-1}(B), r, s)) \\ &= \text{cl}(f^{-1}(B), r, s). \end{aligned}$$

Conversely, let A be any fuzzy (r, s)-closed set of X. Then $\mathrm{cl}(A, r, s) = A$. Since f is one-to-one,

$$f^{-1}(\mathrm{sscl}(f(A), r, s)) \subseteq \mathrm{cl}(f^{-1}f(A), r, s) = \mathrm{cl}(A, r, s) = A.$$

Since f is onto, we have

$$\mathrm{sscl}(f(A), r, s) = ff^{-1}(\mathrm{sscl}(f(A), r, s)) \subseteq f(A) \subseteq \mathrm{sscl}(f(A), r, s).$$

Thus $f(A) = \mathrm{sscl}(f(A), r, s)$ and hence $f(A)$ is a fuzzy strongly (r, s)-semiclosed set of Y. Therefore f is a fuzzy strongly (r, s)-semiclosed mapping. □

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