Honam Mathematical J. **29** (2007), No. 4, pp. 723–732

UNIQUENESS RESULTS FOR THE NONLINEAR HYPERBOLIC SYSTEM WITH JUMPING NONLINEARITY

TACKSUN JUNG AND Q-HEUNG CHOI

Abstract. We investigate the existence of solutions $u(x,t)$ for a perturbation $b[(\xi+\eta+1)^+-1]$ of the hyperbolic system with Dirichlet boundary condition

$$L\xi = \mu[(\xi+\eta+1)^+ - 1] + f \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R},$$
$$L\eta = \nu[(\xi+\eta+1)^+ - 1] + f \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \tag{0.1}$$

where $u^+ = \max\{u, 0\}$, μ, ν are nonzero constants. Here ξ, η are periodic functions.

1. Introduction

Let L be the wave operator in $\mathbb{R}^2$, i.e., $Lu = u_{tt} - u_{xx}$. The eigenvalue problem

$$Lu = \lambda u \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R},$$
$$u(\pm\frac{\pi}{2}, t) = 0, \quad u(x, t+\pi) = u(x,t) = u(-x,t), \tag{1.1}$$

has infinitely many eigenvalues $\lambda_{mn} = (2n+1)^2 - 4m^2$ $(m, n = 0, 1, 2, ...)$ and corresponding normalized eigenfunctions $\phi_{mn}, \psi_{mn} (m, n \geq 0)$.

Let n be fixed and define

$$\lambda_n^+ = \inf_m\{\lambda_{mn} : \lambda_{mn} > 0\} = 4n + 1,$$
$$\lambda_n^- = \sup_m\{\lambda_{mn} : \lambda_{mn} < 0\} = -4n - 3.$$

Received November 22, 2007. Accepted November 28, 2007.
2000 Mathematics Subject Classification: 35B10, 35L05, 35L20.
Key words and phrases: Hyperbolic system, eigenvalue problem, Dirichlet boundary condition, uniqueness.

Then we obtain that $\lim_{n\to\infty}\lambda_n^+ = +\infty$, $\lim_{n\to-\infty}\lambda_n^- = -\infty$. Thus it is easy to check that the only eigenvalues in the interval (-15, 9) are given by

$$\lambda_{32} = -11 < \lambda_{21} = -7 < \lambda_{10} = -3 < \lambda_{00} = 1 < \lambda_{11} = 5.$$

In this paper we investigate the existence of solutions $u(x,t)$ for a perturbation $b[(\xi+\eta+1)^+ - 1]$ of the hyperbolic system with Dirichlet boundary condition

$$\begin{aligned} &L\xi = \mu[(\xi+\eta+1)^+ - 1] + f \quad \text{in} \quad (-\frac{\pi}{2},\frac{\pi}{2})\times\mathbb{R},\\ &L\eta = \nu[(\xi+\eta+1)^+ - 1] + f \quad \text{in} \quad (-\frac{\pi}{2},\frac{\pi}{2})\times\mathbb{R},\\ &\xi(\pm\frac{\pi}{2},t) = 0, \quad \xi(x,t+\pi) = \xi(x,t) = \xi(-x,t),\\ &\eta(\pm\frac{\pi}{2},t) = 0, \quad \eta(x,t+\pi) = \eta(x,t) = \eta(-x,t), \end{aligned} \tag{1.2}$$

where $u^+ = \max\{u, 0\}$, μ, ν are nonzero constants.

The following type nonlinear equation with Dirichlet boundary condition was studied by many authors.

$$\begin{aligned} &u_{tt} - u_{xx} = b[(u+1)^+ - 1] \quad \text{in} \quad (-\frac{\pi}{2},\frac{\pi}{2})\times\mathbb{R},\\ &u(\pm\frac{\pi}{2},t) = 0, \quad u(x,t+\pi) = u(x,t) = u(-x,t). \end{aligned} \tag{1.3}$$

In [6] Lazer and McKenna point out that this kind of nonlinearity $b[(u+1)^+ - 1]$ can furnish a model to study traveling waves in suspension bridges. So the nonlinear equation with jumping nonlinearity have been extensively studied by many authors. For fourth elliptic equation Tarantello [11] , Micheletti and Pistoia [8][9] proved the existence of nontrivial solutions used degree theory and critical points theory separately. For one-dimensional case Lazer and McKenna [7] proved the existence of nontrivial solution by the global bifurcation method. For this jumping nonlinearity we are interest in the multiple nontrivial solutions of the equation. Here we used variational reduction method to find the nontrivial solutions of problem (1.1).

The organization of this paper is as following. In section 2, we investigate some properties of the Hilbert space spanned by eigenfunctions of the wave operator. We show that only the trivial solution exists for the steady state problem of (1.3) when $b < 0$. We also show that problem (1.3) has only trivial solution for $-3 < \mu + \nu < 1$ and $f = 0$. In section 3, we investigate the existence of solutions (ξ, η) for a perturbation $b[(\xi + \eta + 1)^+ - 1]$ of the hyperbolic system with Dirichlet boundary condition for $-3 < \mu + \nu < 1$ and nonconstant f.

2. Unique Trivial Solution

Let L be the wave operator in $\mathbb{R}^2$, i.e., $Lu = u_{tt} - u_{xx}$. The eigenvalue problem

$$
\begin{aligned}
&Lu = \lambda u \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\
&u(\pm\frac{\pi}{2}, t) = 0, \quad u(x, t + \pi) = u(x, t) = u(-x, t),
\end{aligned} \tag{2.1}
$$

has infinitely many eigenvalues $\lambda_{mn} = (2n+1)^2 - 4m^2$ $(m, n = 0, 1, 2, ...)$ and corresponding normalized eigenfunctions $\phi_{mn}, \psi_{mn} (m, n \geq 0)$ given by

$$
\begin{aligned}
\phi_{0n} &= \frac{\sqrt{2}}{\pi} \cos(2n+1)x \quad \text{for} \quad n \geq 0, \\
\phi_{mn} &= \frac{2}{\pi} \cos 2mt \cdot \cos(2n+1)x \quad \text{for} \quad m > 0, \quad n \geq 0, \\
\psi_{mn} &= \frac{2}{\pi} \sin 2mt \cdot \cos(2n+1)x \quad \text{for} \quad m > 0, \quad n \geq 0.
\end{aligned}
$$

Let n be fixed and define

$$
\begin{aligned}
\lambda_n^+ &= \inf_m \{\lambda_{mn} : \lambda_{mn} > 0\} = 4n + 1, \\
\lambda_n^- &= \sup_m \{\lambda_{mn} : \lambda_{mn} < 0\} = -4n - 3.
\end{aligned}
$$

Then we obtain that $\lim_{n \to \infty} \lambda_n^+ = +\infty$, $\lim_{n \to -\infty} \lambda_n^- = -\infty$. Thus it is easy to check that the only eigenvalues in the interval (-15, 9) are given by

$$
\lambda_{32} = -11 < \lambda_{21} = -7 < \lambda_{10} = -3 < \lambda_{00} = 1 < \lambda_{11} = 5.
$$

Let Ω be the square $(-\pi/2, \pi/2) \times (-\pi/2, \pi/2)$ and H_0 the Hilbert space defined by

$$H_0 = \{u \in L^2(\Omega) : u \text{ is even in } x\}.$$

The set of functions $\{\phi_{mn}, \psi_{mn}\}$ is an orthonormal basis in H_0. Let us denote an element u in H_0 as

$$u = \sum (h_{mn}\phi_{mn} + k_{mn}\psi_{mn}),$$

and we define a subspace H of H_0 as

$$H = \{u \in H_0 : \sum |\lambda_{mn}|(h_{mn}^2 + k_{mn}^2) < \infty\}.$$

Then this is a complete normed space with a norm

$$\|u\|_H = [\sum |\lambda_{mn}|(h_{mn}^2 + k_{mn}^2)]^{\frac{1}{2}}.$$

Since $|\lambda_{mn}| \geq 1$ for all m, n, we have that

(i) $\|u\|_H \geq \|u\|$, where $\|u\|$ denotes the L^2 norm of u,

(ii) $\|u\| = 0$ if and only if $\|u\|_H = 0$.

Define $L_\beta u = Lu + \beta u$. Then we have the following lemma.

Lemma 2.1. *Let $\beta \in \mathbb{R}$, $\beta \neq -\lambda_{mn}$ $(m, n \geq 0)$. Then we have:*

L_β^{-1} is a bounded linear operator from H_0 into H.

Proof. Suppose that $\beta \neq -\lambda_{mn}$. Since $\lim_{n\to\infty} \lambda_n^+ = +\infty$, $\lim_{n\to-\infty} \lambda_n^- = -\infty$, we know that the number of elements in the set $\{\lambda_{mn} : |\lambda_{mn}| < |\beta|\}$ is finite, where λ_{mn} is an eigenvalue of L. Let

$$u = \sum (h_{mn}\phi_{mn} + k_{mn}\psi_{mn}).$$

Then

$$L_\beta^{-1} u = \sum \left(\frac{1}{\lambda_{mn} + \beta} h_{mn}\phi_{mn} + \frac{1}{\lambda_{mn} + \beta} k_{mn}\psi_{mn}\right).$$

Hence we have the inequality

$$\|L_\beta^{-1} u\|_H = \sum \frac{|\lambda_{mn}|}{|\lambda_{mn} + \beta|^2}(h_{mn}^2 + k_{mn}^2) \leq C \sum (h_{mn}^2 + k_{mn}^2)$$

for some $C > 0$, which means that

$$\|L_\beta^{-1}u\|_H \leq C_1\|u\|, C_1 = \sqrt{C}.$$

So L_β^{-1} is a bounded linear operator from H_0 to H. □

Lemma 2.2. *Let $-3 < b < 1$. Then the equation*

$$Lu = b[(u+1)^+ - 1]$$

has only the trivial solution in H_0.

Proof. Since $\lambda_{10} = -3$ and $\lambda_{00} = 1$, let $\beta = -\frac{1}{2}(\lambda_{00}+\lambda_{10}) = -\frac{1}{2}(-3+1) = 1$. The equation is equivalent to

$$u = (L+\beta)^{-1}[(b+\beta)(u+1)^+ - \beta(u+1)^- - (b+\beta)], \tag{2.2}$$

where we use the equality $u = u^+ - u^-$.

By lemma 2.1 $(L+\beta)^{-1}$ is a compact linear map from H_0 into H_0. Therefore its L^2 norm $\frac{1}{2}$. We note that

$$\begin{aligned}
& \|(b+\beta)[(u_1+1)^+ - (u_2+1)^+] - \beta[(u_1+1)^- - (u_2+1)^-]\| \\
\leq \; & \max\{|b+\beta|, |\beta|\}\|u_1 - u_2\| \\
< \; & \frac{1}{2}(\lambda_{00} - \lambda_{10})\|u_1 - u_2\| \\
= \; & 2\|u_1 - u_2\|
\end{aligned}$$

where we used the inequality $|s_1^+ - s_2^+| + |s_1^- - s_2^-| \leq |s_1 + s_2|$.

So the right hand side of (2.2) defines a Lipschitz mapping of H_0 into H_0 with Lipschitz constant $\gamma < 1$. Therefore, by the contraction mapping principle, there exists a unique solution $u \in H_0$. Since $u \equiv 0$ is a solution of equation (2.2), $u \equiv 0$ is the unique solution. □

Theorem 2.1. *Let μ, ν be nonzero constants and $1+\frac{\mu}{\nu} \neq 0$. Assume that $-3 < \mu + \nu < 1$. Then the hyperbolic system*

$$(2.3) \quad \begin{aligned} & L\xi = \mu[(\xi+\eta+1)^+ - 1] \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\ & L\eta = \nu[(\xi+\eta+1)^+ - 1] \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\ & \xi(\pm\frac{\pi}{2}, t) = 0, \quad \xi(x, t+\pi) = \xi(x,t) = \xi(-x,t), \\ & \eta(\pm\frac{\pi}{2}, t) = 0, \quad \eta(x, t+\pi) = \eta(x,t) = \eta(-x,t), \end{aligned}$$

has only the trivial solution.

Proof. From problem (2.3) we get that $L\xi = \frac{\mu}{\nu} L\eta$. By Lemma 2.3, the problem

$$(2.4) \quad \begin{aligned} & Lu = 0 \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\ & u(\pm\frac{\pi}{2}, t) = 0, \quad u(x, t+\pi) = u(x,t) = u(-x,t), \end{aligned}$$

has only the trivial solution. So the solution (ξ, η) of problem (2.5) satisfies $\xi = \frac{\mu}{\nu}\eta$. On the other hand, from problem (2.5) we get the equation

$$\begin{cases} L(\xi+\eta) = (\mu+\nu)[(\xi+\eta+1)^+ - 1] \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\ \xi(\pm\frac{\pi}{2}, t) = 0, \quad \xi(x, t+\pi) = \xi(x,t) = \xi(-x,t), \\ \eta(\pm\frac{\pi}{2}, t) = 0, \quad \eta(x, t+\pi) = \eta(x,t) = \eta(-x,t). \end{cases}$$

Let $w = \xi + \eta$. Then the above equation is equivalent to

$$\begin{cases} Lw = (\mu+\nu)[(w+1)^+ - 1] \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\ w(\pm\frac{\pi}{2}, t) = 0, \quad w(x, t+\pi) = w(x,t) = w(-x,t). \end{cases}$$

When $-3 < \mu + \nu < 1$, the above equation has only the trivial solution. Hence the solution (ξ, η) of problem (2.3) satisfies

$$\begin{cases} \xi + \eta = 0 \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\ \xi = \frac{\mu}{\nu}\eta \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\ \xi(\pm\frac{\pi}{2}, t) = 0, \quad \xi(x, t+\pi) = \xi(x,t) = \xi(-x,t), \\ \eta(\pm\frac{\pi}{2}, t) = 0, \quad \eta(x, t+\pi) = \eta(x,t) = \eta(-x,t). \end{cases}$$

Therefore system(2.3)has only the trivial solution $(\xi, \eta) = (0, 0)$. □

By using the similar method as in the proof of Theorem 2.1, we have the following corollary.

Corollary 1. *Let μ, ν be nonzero constants and $1 - \frac{\mu}{\nu} \neq 0$. Assume that $-3 < \mu + \nu < 1$. Then the hyperbolic system*

$$
\begin{aligned}
&L\xi = \mu[(\xi - \eta + 1)^+ - 1] \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R},\\
&L\eta = \nu[(\xi - \eta + 1)^+ - 1] \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R},\\
&\xi(\pm\frac{\pi}{2}, t) = 0, \quad \xi(x, t + \pi) = \xi(x, t) = \xi(-x, t),\\
&\eta(\pm\frac{\pi}{2}, t) = 0, \quad \eta(x, t + \pi) = \eta(x, t) = \eta(-x, t),
\end{aligned}
\tag{2.5}
$$

has only the trivial solution.

3. Nonconstant Load

In this section we investigate the existence of solutions (ξ, η) for a perturbation $b[(\xi + \eta + 1)^+ - 1]$ of the hyperbolic system with Dirichlet boundary condition

$$
\begin{aligned}
&L\xi = \mu[(\xi + \eta + 1)^+ - 1] + f \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R},\\
&L\eta = \nu[(\xi + \eta + 1)^+ - 1] + f \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R},\\
&\xi(\pm\frac{\pi}{2}, t) = 0, \quad \xi(x, t + \pi) = \xi(x, t) = \xi(-x, t),\\
&\eta(\pm\frac{\pi}{2}, t) = 0, \quad \eta(x, t + \pi) = \eta(x, t) = \eta(-x, t),
\end{aligned}
\tag{3.1}
$$

where $u^+ = \max\{u, 0\}$, μ, ν are nonzero constants. Here we assume that $-3 < \mu + \nu < 1$.

Theorem 3.1. *Let μ, ν be nonzero constants and $1 + \frac{\mu}{\nu} \neq 0$, $\lambda_{00} - (\mu + \nu) \neq 0$. Assume that $-3 < \mu + \nu < 1$. Then the hyperbolic system*

$$
\begin{aligned}
&L\xi = \mu[(\xi + \eta + 1)^+ - 1] + \phi_{00} \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R},\\
&L\eta = \nu[(\xi + \eta + 1)^+ - 1] + \phi_{00} \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R},\\
&\xi(\pm\frac{\pi}{2}, t) = 0, \quad \xi(x, t + \pi) = \xi(x, t) = \xi(-x, t),\\
&\eta(\pm\frac{\pi}{2}, t) = 0, \quad \eta(x, t + \pi) = \eta(x, t) = \eta(-x, t),
\end{aligned}
\tag{3.2}
$$

has a unique solution.

Proof. From problem (3.2) we get that $L\xi - \phi_{00} = \frac{\mu}{\nu}(L\eta - \phi_{00})$. By Lemma 2.3, the problem

$$
\begin{aligned}
&Lu = (1 - \frac{\mu}{\nu})\phi_{00} \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\
&u(\pm\frac{\pi}{2}, t) = 0, \quad u(x, t+\pi) = u(x,t) = u(-x,t)
\end{aligned} \tag{3.3}
$$

has a unique solution $u = \frac{1}{\lambda_{00}}(1 - \frac{\mu}{\nu})\phi_{00}$. So the solution (ξ, η) of problem (3.2) satisfies $\xi = \frac{\mu}{\nu}\eta + \frac{1}{\lambda_{00}}(1 - \frac{\mu}{\nu})\phi_{00}$. On the other hand, from problem (3.2) we get the equation

$$
\begin{cases}
L(\xi + \eta) = (\mu + \nu)[(\xi + \eta + 1)^+ - 1] + 2\phi_{00} \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\
\xi(\pm\frac{\pi}{2}, t) = 0, \quad \xi(x, t+\pi) = \xi(x,t) = \xi(-x,t), \\
\eta(\pm\frac{\pi}{2}, t) = 0, \quad \eta(x, t+\pi) = \eta(x,t) = \eta(-x,t).
\end{cases}
$$

Let $w = \xi + \eta$. Then the above equation is equivalent to

$$
\begin{cases}
Lw = (\mu + \nu)[(w + 1)^+ - 1] + 2\phi_{00} \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\
w(\pm\frac{\pi}{2}, t) = 0, \quad w(x, t+\pi) = w(x,t) = w(-x,t).
\end{cases}
$$

When $-3 < \mu + \nu < 1$, the above equation has a unique solution $w = \frac{1}{\lambda_{00} - (\mu+\nu)}\phi_{00}$. Hence the solution (ξ, η) of problem (3.2) satisfies

$$
\begin{cases}
\xi + \eta = \frac{1}{\lambda_{00} - (\mu+\nu)}\phi_{00} \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\
\xi = \frac{\mu}{\nu}\eta + \frac{1}{\lambda_{00}}(1 - \frac{\mu}{\nu})\phi_{00} \quad \text{in} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\
\xi(\pm\frac{\pi}{2}, t) = 0, \quad \xi(x, t+\pi) = \xi(x,t) = \xi(-x,t), \\
\eta(\pm\frac{\pi}{2}, t) = 0, \quad \eta(x, t+\pi) = \eta(x,t) = \eta(-x,t).
\end{cases}
$$

Therefore system(3.2)has a unique solution (ξ, η) as follows

$$
\xi = \frac{1}{1 + \frac{\nu}{\mu}}(\frac{\nu}{\mu}\frac{1}{\lambda_{00}}(1 - \frac{\mu}{\nu}) + \frac{1}{\lambda_{00} - (\mu+\nu)})\phi_{00},
$$

$$
\eta = \frac{1}{1 + \frac{\mu}{\nu}}(\frac{1}{\lambda_{00} - (\mu+\nu)} - \frac{\nu}{\mu}\frac{1}{\lambda_{00}}(1 - \frac{\mu}{\nu}))\phi_{00}.
$$

□

By using the similar method as in the proof of Theorem 3.1, we have the following corollary.

Corollary 2. *Let μ, ν be nonzero constants and $\lambda_{00} + (\mu + \nu) \neq 0$. Assume that $-3 < \mu + \nu < 1$. Then the hyperbolic system*

$$(3.4)\quad \begin{aligned} &L\xi = \mu[(\xi - \eta + 1)^+ - 1] + \phi_{00} \quad in \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\ &L\eta = \nu[(\xi - \eta + 1)^+ - 1] + \phi_{00} \quad in \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}, \\ &\xi(\pm\frac{\pi}{2}, t) = 0, \quad \xi(x, t + \pi) = \xi(x, t) = \xi(-x, t), \\ &\eta(\pm\frac{\pi}{2}, t) = 0, \quad \eta(x, t + \pi) = \eta(x, t) = \eta(-x, t), \end{aligned}$$

has a unique solution.

References

[1] H.Amann, *Saddle points and multiple solutions of differential equation*, Math.Z.(1979), 27-166.

[2] A. Ambrosetti and P. H. Rabinowitz, *Dual variation methods in critical point theory and applications*, J. Functional analysis, 14(1973), 349-381.

[3] Q. H. Choi, T. Jung and P. J. McK. na, *The study of a nonlinear suspension bridge equation by a variational reduction mehtod*, Appl. Anal., 50(1993),73-92.

[4] Q. H. Choi and T. Jung, *An application of a variational reduction method to a nonlinear wave equation*, J. Differential equations, 117(1995), 390-410.

[5] A. C. Lazer and P. J. McKenna, *A symmetry theorem and applications to nonlinear partial differential equations*,J. Differential Equations 72(1988), 95-106.

[6] A. C. Lazer and P. J. McKenna, *Large-amplitude periodic oscillations in suspension bridges: some new connections with nonlinear analysis*, SIAM Review, 32(1990), 537-578.

[7] A. C. Lazer and P. J. McKenna, *Global bifurcation and a theorem of Tarantello*, J. Math. Anal. Appl, 181(1994), 648-655.

[8] A.M. Micheletti and A.Pistoia, *Multiplicity results for a fourth-order semilinear elliptic problem*, Nonlinear Analysis TMA, 31(1998), 895-908.

[9] A. M. Micheletti, A.Pistoia, *Nontrivial solutions for some fourth order semilinear elliptic problems*, Nonlinear Analysis, 34(1998), 509-523.

[10] Paul H. Rabinobitz, *Minimax methods in critical point theory with applications to differential equations*,Mathematical Science regional conference series, No. 65, AMS (1984).

[11] G. Tarantello, *A note on a semilinear elliptic problem*, Diff. Integ. Equations. 5(3)(1992), 561-565.

TACKSUN JUNG
Department of Mathematics,
Kunsan National University,
Kunsan 573-701, Korea
E-mail: tsjung@kunsan.ac.kr

Q-HEUNG CHOI
Department of Mathematics Education,
Inha University,
Incheon 402-751, Korea
E-mail: qheung@inha.ac.kr