

# Modified Probabilistic Neural Network of Heterogeneous Probabilistic Density Functions for the Estimation of Concrete Strength

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**Abstract:** Recently, probabilistic neural network (PNN) has been proposed to predict the compressive strength of concrete for the known effect of improvement on PNN by the iteration method. However, an empirical method has been incorporated in the PNN technique to specify its smoothing parameter, which causes significant uncertainty in predicting the compressive strength of concrete. In this study, a modified probabilistic neural network (MPNN) approach is hence proposed. The global probability density function (PDF) of variables is reflected by summing the heterogeneous local PDFs which are automatically determined by the individual standard deviation of each variable. The proposed MPNN is applied to predict the compressive strength of concrete using actual test data from a concrete company. The estimated results of MPNN are compared with those of the conventional PNN. MPNN showed better results than the conventional PNN in predicting the compressive strength of concrete and provided promising results for the probabilistic approach to predict the concrete strength by using the individual standard deviation of a variable.

**Keywords:** concrete compressive strength, strength prediction, multivariate Gaussian distribution, classification, modified probabilistic neural network (MPNN)

## 1. Introduction

A large of methods for predicting concrete strength researchers has been proposed by researchers over many years. Traditional methods for predicting 28-day compressive strength of concrete are basically based upon statistical analyses, by which many linear and nonlinear regression equations have been constructed to model such prediction problems.<sup>1,2</sup> Recently standard multi-layer feed-forward neural networks with a back propagation algorithm have been used to predict the strength of concrete.<sup>3-5</sup> Back propagation neural networks (BPNN) have the advantage of being able to effectively consider various inputs without using complicated equations, in contrast to conventional regression analyses. In addition, they can easily adapt to new data through a re-training process. However, BPNN needs more effort to determine the architecture of networks and more computational time in training the networks. Moreover, the estimated results from BPNN are not probabilistic but deterministic, even though the test results for the compressive strength of concrete specimens under the same conditions, such as mix proportions, curing conditions, methods of transporting, placing, testing, etc.,

show distributed characteristics in nature. Probabilistic neural network (PNN), therefore, is effective alternative because PNN needs less time to determine the networks' architecture and to train the networks. Moreover, PNN provides probabilistic viewpoints as well as deterministic classification results.

Probabilistic neural network has been widely used for pattern recognition problems such as texture recognition,<sup>6,7</sup> image recognition,<sup>8,9</sup> medical/biochemical field,<sup>10,11</sup> signal processing,<sup>12</sup> finance,<sup>13</sup> civil/geotechnical engineering,<sup>14-16</sup> etc.

Kim et al.<sup>17</sup> proposed PNN to predict the estimation of the compressive strength of concrete on the basis of concrete mix proportions and verified performance of PNN through comparison with the results of the actual compression tests; the estimation performance of PNN is improved by the iteration method. It is that the prediction of the concrete strength (i.e. classification) was performed iteratively. However, an empirical method has been incorporated to specify  $\sigma$  (smoothing parameter) in the PNN technique, causing significant uncertainty in the estimation results. In addition, the probability density function (PDF) is the sum of homogeneous multivariate Gaussian distribution because only one global smoothing parameter is used. In this paper, the modified probabilistic neural network (MPNN) was proposed to reflect the global probability density function by summing the heterogeneous local probability density function. The heterogeneous local probability density function of MPNN is automatically determined to use the individual standard deviation of variables. Training and test patterns for MPNN are prepared using the data sets on the mix proportions of concrete company. The proposed methods are verified using the actual test

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data from concrete company. The estimated results of MPNN are compared with those of PNN.

## 2. Modified probabilistic neural network

### 2.1 Probabilistic neural network

PNN is basically a pattern classifier that combines the well-known Bayesian decision strategy with the Parzen non-parametric estimator of the probability density functions of different classes.<sup>18</sup> PNN has gained interest because it offers a way to interpret the network's structure in the form of a probability density function and it is easy to implement.

An accepted norm for decision rules or strategies used to classify patterns is that they do so in a way that minimizes the "expected risk." Such strategies are called "Bayesian strategies" and can be applied to problems containing any number of classes.

Consider the two-category situation in which the state of nature  $\theta$  is known to be either  $\theta_A$  or  $\theta_B$ . To decide whether  $\theta = \theta_A$  or  $\theta = \theta_B$  based on a set of measurements represented by the  $P$ -dimensional vector  $X = [X_1 \wedge X_j \wedge X_p]^T$ , the Bayesian decision rule becomes:

$$d(X) = \theta_A \text{ if } h_A l_A f_A(X) > h_B l_B f_B(X) \quad (1a)$$

$$d(X) = \theta_B \text{ if } h_A l_A f_A(X) < h_B l_B f_B(X) \quad (1b)$$

where  $f_A(X)$  and  $f_B(X)$  are the probability density functions (PDFs) for categories A and B, respectively;  $l_A$  is the loss function associated with the decision  $d(X) = \theta_B$  when  $\theta = \theta_A$ ;  $l_B$  is the loss associated with the decision  $d(X) = \theta_A$  when  $\theta = \theta_B$  (the losses associated with correct decisions are taken to be equal to zero);  $h_A$  is the priori probability of occurrence of patterns from category A; and  $h_B = 1 - h_A$  is the priori probability that  $\theta = \theta_B$ . In the simplified case that assumes both loss function and a priori probability are equal to each other, the Bayesian rule classifies an input pattern to the class that has its PDF greater than the PDF of the other class. Therefore, the accuracy of the decision boundaries depends on the accuracy with which the underlying PDFs are estimated. Parzen showed how one may construct a family of estimates of  $f(X)$ ,<sup>19</sup> and Cacoullos has also extended Parzen's results to estimates in the special case that the multivariate kernel is a product of univariate kernels.<sup>20</sup> In the particular case of the Gaussian kernel, the multivariate estimates can be expressed as:

$$f_A(X) = \frac{1}{(2\pi)^{p/2} \sigma^p} \frac{1}{m} \sum_{i=1}^m \exp \left[ -\frac{(X - X_{Ai})^T (X - X_{Ai})}{2\sigma^2} \right] \quad (2)$$

where  $X$  is the test vector to be classified;  $f_A(X)$  is the value of the PDF of category A at point  $X$ ;  $m$  is the number of training vectors in category A;  $P$  is the dimensionality of the training vectors;  $X_{Ai}$  is the  $i^{\text{th}}$  training vector for category A; and  $\sigma$  is the smoothing parameter. Note that  $f_A(X)$  is the simple sum of small multivariate Gaussian distributions centered at each training sample because only one global smoothing parameter is used.

### 2.2 Modified probabilistic neural network

Each variable, such as slump, water/cement ratio, unit water,

fine aggregate, unit cement content, natural sand ( $S_1$ ), crushed sand ( $S_2$ ), unit coarse aggregate content, and admixture, has an individual standard deviation and different probabilistic property. However, the probability density function did not consider the individual probabilistic property of variables in PNN because only one global smoothing parameter was used. Therefore, in this paper, the modified probabilistic neural network (MPNN) was proposed to reflect the global probability density function by summing the heterogeneous local probability density function automatically determined to use the individual standard deviation of variables. The basic idea is to individually use the heterogeneous local probability density function in a variable because the probabilistic property of variables is not homogenous but heterogeneous. The individual probability density function was derived from the standard deviation of variables. The probability density function for  $i^{\text{th}}$  sample is determined to sum different standard deviations of the  $P$ -dimensionality of the training vector with  $j^{\text{th}}$  variables. The probability density function of MPNN can be expressed as:

$$f(X) = \frac{1}{(2\pi)^{p/2} (\sigma_j)^p} \exp \left( -\frac{(X - \mu_{i,j})^T (X - \mu_{i,j})}{2(\sigma_j)^2} \right) \quad (3)$$

where  $f(X)$  is the probability density function;  $i$  is the number of training pattern (1~155);  $j$  is the number of variables (1~9);  $X$  is the input data;  $P$  denotes the dimensionality of the training vectors;  $\mu_{i,j}$  is the  $i^{\text{th}}$  training vector with the  $j^{\text{th}}$  variables;  $\sigma_j$  is the standard deviation with the  $j^{\text{th}}$  variables.

## 3. Estimation of concrete compressive strength using the MPNN

The concrete used at construction sites is mostly produced in a ready-mixed concrete company according to specified concrete mix proportions. In general, slump tests are performed before the placing of concrete, but the compression tests of specimens are carried out 28 days after placing. Therefore, it is difficult to estimate the compressive strength at construction sites. Ready-mixed concrete company uses their own mix proportions based on codes, previous experience, and experiments. In this study, the MPNN for estimating the concrete compressive strength was incorporated using the actual mix proportion data provided by company. The material properties of concrete from the company are shown in Table 1. Normal Portland cement was used. The maximum size of aggregate was 25 mm, the range of compressive strengths was from 9.8 to 39.2 MPa, and slump values were 5, 8, 10, 12, 15, 18, and 21 cm.

**Table 1** Material properties of concrete.

| Properties of material |                           | Experiment data              |
|------------------------|---------------------------|------------------------------|
|                        |                           | Company                      |
| Specific gravity       | Cement                    | 3.15                         |
|                        | Natural sand              | 2.58                         |
|                        | Coarse aggregate          | 2.63                         |
| Fineness modulus       | Natural sand              | 2.70                         |
|                        | Coarse aggregate          | 6.60                         |
| Admixtures             | Air-entraining admixtures | AE water-reducing (Standard) |

Verifications on the applicability of the MPNN to the problem of strength estimation were performed using actual mix proportion data provided by company. At first, eight variables including water-cement ratio, fine aggregate percentage, unit water content, unit cement content, unit fine aggregate content, unit coarse aggregate content, admixtures, and slump were used as the input set for the models, while the specified compressive strength was defined as the output (class) to be estimated.

All the input data are normalized to 0.1~0.9 to give an equal weighting factor before implementing the data to the network. All the classes are defined using a compressive strength of 9.8~39.2 MPa with a step size of 0.98 MPa. Consequently, the compressive strength to be estimated is divided into 31 classes in total.

There are 7 independent training patterns with slump values of 5, 8, 10, 12, 15, 18, and 21cm in each class. Five samples which are randomly selected among 7 samples in a class are used for training patterns and the others are used for testing; that is, the numbers of training and test patterns are 155 and 62; respectively. Examples of specified concrete mix proportions of the company for training are shown in Table 2. The global probability density function of MPNN was derived from eq. (3). Table 3 shows the standard deviation values and means of the normalized variables. Table 4 shows estimation results for 14 randomly-selected test patterns from company.

#### 4. A comparison between MPNN with PNN

In this section, we compared the estimation capability of the MPNN and conventional PNN. Fig. 1 shows the estimation errors for 62 test patterns of company using eight parameters as input to MPNN. The estimation errors are defined by the root mean square (RMS) errors are as follows:

$$e = \sqrt{\frac{1}{N} \sum_{i=1}^N (f - \bar{f})^2} \quad (4)$$

where  $N$  is the number of test patterns and  $f$  and  $\bar{f}$  denote the actual and predicted concrete strength, respectively.

Table 5 shows the estimation results of the concrete compressive strength using conventional PNN with empirical smoothing parameters ( $\sigma=1.0, 0.5, 0.1$ ). Additionally, Table 6 shows the estimation results of PNN improved by the iteration method.<sup>17</sup> From these results, it was found that the proposed method is very effective and accurate to predict the compressive strength of concrete.

#### 5. Comparison of predictions and test results

The predicted strengths were compared with the results of the actual compression tests of concrete company for verification.

**Table 2** Samples of specified concrete mix proportions of company.

| Specified strength (MPa) | Slump (cm) | W/C* ratio (%) | Fine aggregate percentage (%) | Unit water content (kN/m <sup>3</sup> ) | Unit cement content (kN/m <sup>3</sup> ) | Natural sand (kN/m <sup>3</sup> ) | Unit coarse aggregate content (kN/m <sup>3</sup> ) | Admixture (%) |
|--------------------------|------------|----------------|-------------------------------|---|--|-----------------------------------|--|---------------|
| 9.8                      | 8          | 82.0           | 54.8                          | 1.69                                    | 2.07                                     | 10.19                             | 8.58   | 1.06          |
| 11.76                    | 10         | 73.8           | 53.1                          | 1.71                                    | 2.33                                     | 9.75                              | 8.77   | 1.19          |
| 13.72                    | 12         | 66.3           | 51.4                          | 1.72                                    | 2.58                                     | 9.32                              | 8.97   | 1.32          |
| 13.72                    | 21         | 66.9           | 50.6                          | 1.86                                    | 2.80                                     | 8.89                              | 8.84   | 1.43          |
| 15.68                    | 10         | 63.0           | 50.9                          | 1.68                                    | 2.66                                     | 9.23                              | 9.08   | 1.36          |
| 15.68                    | 15         | 63.0           | 50.4                          | 1.76                                    | 2.79                                     | 8.98                              | 9.01   | 1.43          |
| 17.64                    | 5          | 59.0           | 50.5                          | 1.59                                    | 2.71                                     | 9.25                              | 9.25   | 1.39          |
| 17.64                    | 12         | 59.0           | 49.8                          | 1.71                                    | 2.91                                     | 8.90                              | 9.14   | 1.49          |
| 17.64                    | 18         | 58.0           | 49.2                          | 1.80                                    | 3.09                                     | 8.59                              | 9.05   | 1.58          |
| 20.58                    | 12         | 53.0           | 48.6                          | 1.70                                    | 3.22                                     | 8.57                              | 9.24   | 1.65          |
| 20.58                    | 18         | 53.0           | 48.0                          | 1.78                                    | 3.39                                     | 8.29                              | 9.15   | 1.73          |
| 23.52                    | 8          | 49.0           | 48.2                          | 1.62                                    | 3.32                                     | 8.56                              | 9.37   | 1.7           |
| 23.52                    | 12         | 49.0           | 47.8                          | 1.69                                    | 3.46                                     | 8.35                              | 9.29   | 1.77          |
| 26.46                    | 10         | 45.0           | 47.3                          | 1.65                                    | 3.65                                     | 8.23                              | 9.35   | 1.86          |
| 26.46                    | 18         | 45.0           | 46.5                          | 1.77                                    | 3.92                                     | 7.84                              | 9.19   | 2.00          |
| 29.4                     | 10         | 42.0           | 46.6                          | 1.64                                    | 3.92                                     | 8.02                              | 9.37   | 2.00          |
| 29.4                     | 15         | 42.0           | 46.1                          | 1.72                                    | 4.12                                     | 7.76                              | 9.25   | 2.10          |
| 34.3                     | 10         | 37.0           | 45.7                          | 1.63                                    | 4.40                                     | 7.69                              | 9.32   | 2.25          |
| 34.3                     | 18         | 37.0           | 44.9                          | 1.75                                    | 4.74                                     | 7.29                              | 9.11   | 2.42          |
| 37.24                    | 18         | 34.7           | 44.4                          | 1.75                                    | 5.08                                     | 7.09                              | 9.05   | 2.59          |
| 39.2                     | 15         | 33.0           | 44.4                          | 1.71                                    | 5.14                                     | 7.11                              | 9.08   | 2.62          |

\*Water and cement

**Table 3** Standard deviation and mean of normalized variables.

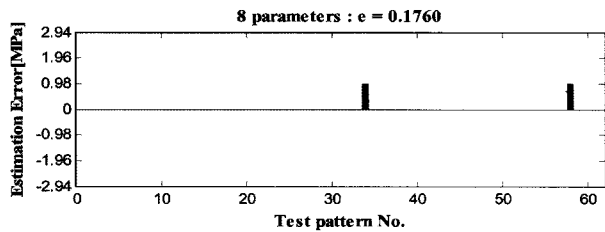
|            | Slump  | W/C ratio | Fine aggregate | Unit water | Unit cement | Natural sand | Unit coarse aggregate | Admixture |
|------------|--------|-----------|----------------|------------|-------------|--------------|-----------------------|-----------|
| $\sigma^*$ | 0.2842 | 0.0027    | 0.0401         | 0.2066     | 0.0461      | 0.0650       | 0.0889                | 0.0459    |

\*Standard deviation

**Table 4** Examples of estimation results for MPNN.

| Specified strength (MPa) | Slump (cm) | W/C ratio (%) | Fine aggregate percentage (%) | Unit water content (kN/m <sup>3</sup> ) | Unit cement content (kN/m <sup>3</sup> ) | Natural sand (kN/m <sup>3</sup> ) | Unit coarse aggregate content (kN/m <sup>3</sup> ) | Admixture (%) | Output Class |
|--------------------------|------------|---------------|-------------------------------|---|--|-----------------------------------|--|---------------|--------------|
| 10.78                    | 12         | 77.8          | 53.7                          | 1.74                                    | 2.24                                     | 10.04                             | 8.64   | 1.15          | 10.78        |
| 14.7                     | 18         | 64.0          | 50.3                          | 1.81                                    | 2.84                                     | 9.05                              | 8.94   | 1.45          | 14.7         |
| 17.64                    | 12         | 59.0          | 49.8                          | 1.71                                    | 2.91                                     | 9.08                              | 9.14   | 1.49          | 17.64        |
| 18.62                    | 8          | 56.2          | 49.8                          | 1.62                                    | 2.90                                     | 9.23                              | 9.25   | 1.48          | 18.62        |
| 20.58                    | 12         | 53.0          | 48.6                          | 1.70                                    | 3.22                                     | 8.74                              | 9.24   | 1.65          | 20.58        |
| 21.56                    | 18         | 51.6          | 47.8                          | 1.79                                    | 3.46                                     | 8.38                              | 9.15   | 1.77          | 21.56        |
| 23.52                    | 12         | 49.0          | 47.8                          | 1.69                                    | 3.46                                     | 8.52                              | 9.29   | 1.77          | 23.52        |
| 25.48*                   | 12         | 45.8          | 47.2                          | 1.69                                    | 3.62                                     | 8.16                              | 9.34   | 1.85          | 26.46        |
| 28.42                    | 8          | 42.6          | 47.0                          | 1.61                                    | 3.77                                     | 8.36                              | 9.41   | 1.93          | 28.42        |
| 29.4                     | 12         | 42.0          | 46.4                          | 1.68                                    | 4.00                                     | 8.08                              | 9.33   | 2.04          | 29.4         |
| 32.34                    | 18         | 39.0          | 45.3                          | 1.75                                    | 4.53                                     | 7.59                              | 9.14   | 2.31          | 32.34        |
| 34.3                     | 8          | 37.0          | 45.9                          | 1.60                                    | 4.32                                     | 7.95                              | 9.37   | 2.21          | 34.3         |
| 37.24*                   | 8          | 34.1          | 44.6                          | 1.70                                    | 4.91                                     | 7.24                              | 9.17   | 2.51          | 38.22        |
| 39.2                     | 8          | 33.0          | 45.1                          | 1.69                                    | 4.78                                     | 7.65                              | 9.30   | 2.44          | 39.2         |

\*Misclassification



**Fig. 1** Estimation results of MPNN.

**Table 5** Estimation results of PNN.

|                | PNN             |
|----------------|-----------------|
| $\sigma = 1.0$ | RMS :1.5141<br> |
| $\sigma = 0.5$ | RMS :1.4621<br> |
| $\sigma = 0.1$ | RMS :1.1407<br> |

The results of the compression tests of concrete may be affected by the type of test specimens, specimen size, the type of moulds, curing conditions, the preparation of end surfaces, the rigidity of a testing machine, and the rate of application of stress. Additional significant changes which have an influence on the compressive strength of concrete include those in the type of Portland cement, admixtures, source of aggregates, mix proportions, batching,

**Table 6** Estimation results of iteration PNN ( $\sigma = 0.1$ ).

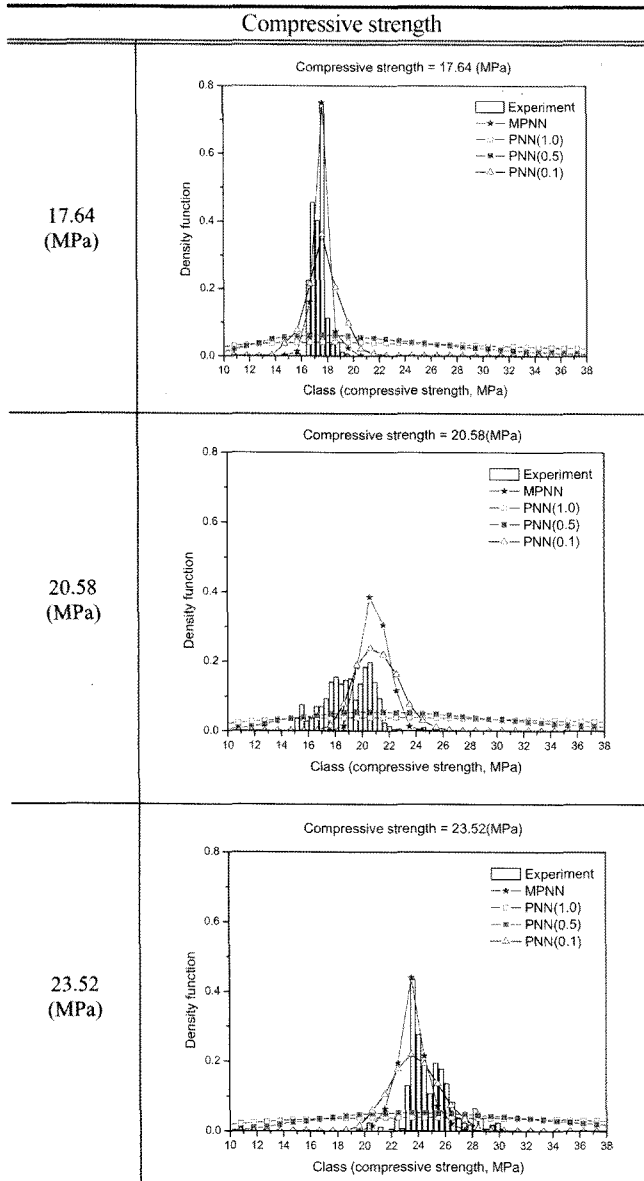
|          | PNN                            |
|----------|--------------------------------|
| Iter = 1 | 1st Iteration $e = 1.1407$<br> |
| Iter = 2 | 2nd Iteration $e = 0.7672$<br> |
| Iter = 3 | 3rd Iteration $e = 0.3936$<br> |

mixing, and delivery. In these tests, the specified strengths of concrete are 17.64, 20.58, and 23.52 MPa, and the slump is 12 cm. Cylindrical specimens with a size of  $\phi 100 \times 200$  mm, which are made according to the mix proportions shown in Table 2, are used. The tests follow the requirements of KS F2405<sup>21</sup> and ASTM C39-93a.<sup>22</sup> The results of the tests are shown in Table 7. Comparing the specified strengths in the table, the maximum percentage of the differences between the specified strength by tests ( $f'_{ci}$ ) and the specified strength in concrete mix proportions ( $f'_c$ ) is 1.7 percent. This indicates that the results of tests show good agreement with the specified strengths.

Table 8 shows the estimated results obtained by PNN and MPNN compared with the test results of concrete company. The training patterns are the same as in the previous section (217 training patterns) and the test patterns for specified compressive

**Table 7** Results of compression tests.

| Mix proportion                     |            | Test            |                                      |                                    |
|------------------------------------|------------|-----------------|--------------------------------------|------------------------------------|
| Specified strength ( $f'_c$ , MPa) | Slump (cm) | Number of tests | Standard deviation ( $\sigma$ , MPa) | Specified strength ( $f'_c$ , MPa) |
| 17.64                              | 12         | 345             | 1.11                                 | 17.64(0.0)                         |
| 20.58                              | 12         | 435             | 1.59                                 | 20.58(0.0)                         |
| 23.52                              | 12         | 363             | 1.62                                 | 23.91(1.7)                         |

**Table 8** Comparison of estimated results with test results.

strengths of 17.64, 20.58, 23.52 MPa with a slump of 12 cm are the same as shown in Table 2. The estimated results can be represented from the viewpoint of probability. To see more detailed results, for the case of the specified compressive strength of 23.52 MPa in Table 8, the number of specimens is 363 and the test result shows normal distribution, wherein the mean value of the specified strength is 23.91 MPa and the standard deviation is 1.62 MPa.

The distribution of concrete strength was estimated by the Parzen non-parametric estimator of the probability density functions used in the MPNN algorithm. Density estimation was

carried out for the empirical smoothing parameters used in the conventional PNN ( $\sigma = 1.0, 0.5, 0.1$ ) and the standard deviation determined in MPNN, as shown in Table 3. The estimated density function using the standard deviation of MPNN showed the most reasonable agreement with the distribution of the test results.

In the case of MPNN, general trends for the cases of specified strengths in company are almost the same as the aforementioned results. These results, which bring a probability point of view in addition to classification/prediction capability, are practically attractive, while the conventional methods, such as statistical analyses, neural network, etc., can give a deterministic value since the tests of concrete strength for specimens under the same conditions show distributed characteristics in nature.

## 6. Conclusions

Modified (advanced) probability neural network (MPNN) is proposed and incorporated to predict the compressive strength of concrete. The concrete mix proportions and the slump values from a ready mixed concrete company are used as inputs for the MPNN, and the compressive strength of concrete is set as classes (targets) to be predicted by the networks. The conclusions drawn in this study are as follows.

1) It has been found that the estimation performance of the proposed method is more effective than that of conventional PNN based on the results of RMS errors.

2) Although the estimation capability of PNN was improved by the iteration method, the proposed MPNN showed a better estimation capability even without the iteration method, decreasing the computational time.

3) The validity of the proposed method was proven by comparing the predicted strength with the test results of the concrete. The estimation results of MPNN using the individual standard deviation of input variables showed best match with the distribution of the test results compared with user defined smoothing parameters ( $\sigma = 1.0, 0.5, 0.1$ ).

The key point in this paper is that MPNN used a heterogeneous local PDF for training patterns, but the conventional PNN used homogenous local PDF. In addition, MPNN automatically calculates the global PDF by using the standard deviation in a respective variable. However, PNN calculates the global PDF by using one smoothing parameter defined by the user. The proposed MPNN technique, therefore, can be presented as a newly-developed application of the toolkit to predict the concrete compressive strength.

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