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第30卷(1), 2007. 3, pp. 55~83

Welfare Effects of Publicly Provided Self-Insurance Against Unemployment

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This paper examines the welfare aspects of loan-based self-insurance against unemployment, and discusses the scope of government intervention in its provision. This paper deals with these issues in a model where the individuals may experience unemployment shocks frequently to leave little savings for retirement, so that the government may have to provide them with unemployment and retirement insurance benefits during their unemployment and retirement, respectively. We identify the two interesting features in the model: the externality that the self-insurance exerts upon other social insurances, and the incentive of private sector to provide loans that exerts the externality upon other social insurances. In particular, this paper shows that, although the inefficiency associated with private loan warrants the government provision of loans to unemployed workers, the over-incentive of the private sector to offer loans may reduce the scope of the government intervention. This paper also shows that, unless the inefficiency associated with private loans is high, the private incentive for loans would reduce welfare because of the externality generated by private loans.

Key Words : self-insurance, loan, unemployment insurance, externality

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I. Introduction

It is well-known that unemployment insurance is not so effective as a means of income support for the unemployed due to the moral hazard associated with the system.[1] More often than not, as is shown by Stiglitz and Yun (2005), self-insurance through borrowing could enhance welfare of workers by providing them with intertemporal income smoothing.[2] One may ask, however, how much the loan-based self-insurance can improve welfare when there is some chance of default on the part of borrowers. The possibility of default raises some important issues on the welfare effects of the loan-based self-insurance and the role of government in its provision, which this paper explores extensively.

When a worker may get unemployed for a considerably long period of his lifetime, his savings may not be enough to cover what he has borrowed *and* minimum consumption during his unemployment and during his retirement. If a worker fails to repay what he has borrowed at the time of retirement or to secure the minimum consumption for retirement after the repayment, there is no other way for the government but to bail him out. That is, the government not only has to relieve the worker of his repayment burden, but also has to provide him with the minimum consumption during unemployment and retirement.

To see how the default possibility associated with borrowings and the government bail-out affect the welfare role of self-insurance against unemployment and the need for government intervention, this paper considers a system of social insurances that includes unemployment insurance and retirement insurance as well as the loan-based self-insurance. In a model of the social insurance system this paper highlights the three important points. First, the possibility of default associated with borrowings will inevitably limit its function of income-smoothing and aggravate the incentive problem,

1) See Flemming (1978), Hopenhayen and Nicolini (1991).

2) See also Altman and Feldstein (1998), Costain (1999)

which will negatively affect the welfare role of the loan-based self-insurance.

Second, the possibility of default and the resulting government bail-out generate the externalities among self-insurance and other social insurances. Since the costs for the loan-based self-insurance and for the retirement insurance are commonly related to the probability of default the externality between the two insurances is expected to occur. In other words, the moral hazard associated with one insurance aggravates the incentive problem of the other insurance. The existence of this externality obviously affects the optimal provision of the self-insurances against unemployment: the optimal level of self-insurance would have to be set lower than would be the case when there is no externality. In particular, the optimal self-insurance level could be zero, contrasted with the literature on insurance and moral hazard.[3]

Thirdly, the government bail-out may induce private sector to provide loan-based self-insurance to unemployed workers. This is because the government does in fact subsidize private loan by bailing out those who default. The private loan, however, exerts an externality upon retirement insurance by increasing the probability of default, which may lead the private sector to offer excessive loans for unemployed workers. Also, the private loan is in general more costly than the public mandatory loan due to the various reasons, including informational problems (such as moral hazard and adverse selection) and imperfect competition in the capital market.[4]

Even when it is made clear that self-insurance increases welfare, we still need to justify the government intervention for the provision of self-insurance and determine the level of self-insurance the government has to provide. The externality and the cost disadvantage associated with private loan would lead to the necessity of government intervention in providing self-insurance against unemployment. Since the government cannot in fact effectively control private loans through taxes, however, it can indirectly dis-

3) It is generally believed that optimal level of insurance is always positive even in the context of moral hazard, because, at the zero level of insurance, marginal benefit of providing insurance is always positive while the marginal incentive cost of it is negligible.

4) A loan package with the bailout possibility involves insurance. Thus, the underlying reasons for the cost disadvantage associated with private loans are what we frequently refer to as one of the most important reasons for the public provision of social insurances.

courage private loans from being offered by providing its own loans to unemployed workers. Despite the provision of public loans and the cost disadvantage associated with private loan, however, the private sector still has some incentives to offer loans for unemployed workers because the private cost of loans for the private sector is lower than its social cost. The private incentives for loans would thus reduce welfare compared to the case with no such incentives, as was pointed out by Arnott and Stiglitz (1986, 1991) and Greenwald and Stiglitz (1986). This paper also explores how the government provision of loan-based self-insurance would be affected by the private incentives.

In general the public loan is an imperfect substitute for the private loan: the unit increase in the former is accompanied by the less-than-unit reduction in the latter. Thus, the government intervention has two conflicting welfare effects: it can lower the cost of borrowings by (partially) replacing relatively inefficient private loan, while it creates the additional moral hazard cost caused by the increase in the total amount of loans. The net effect of government provision of loan-based self-insurance would then critically depend upon the cost disadvantage associated with the private loan.

When the cost disadvantage is small, the government may not provide its loans because the reduction in cost of borrowings that it can expect from offering loans will be small whereas the moral hazard cost of public loan is high in the presence of private loan. As the cost disadvantage goes higher beyond a certain level, however, the public loan will increase from zero while private loan decreases. At a certain level of cost disadvantage the amount of private loan will become zero and the public loan will reach its maximum, which involves some extra amount of loans (in excess of the socially optimal level) aiming to discourage private loan from being offered.[5] When the cost disadvantage is high enough to make the extra public loan unnecessary, the amount of public loan would not be affected by the possibility of private loan.

We can see from these arguments that the private incentive for loans may keep the government from intervening in the provision of loan-based self-insurance when the cost disadvantage is small. Once the government intervenes, however, it may provide more

5) The extra amount of public mandatory loan for that purpose would decrease, however, as the cost disadvantage goes up further.

loan-based self-insurance under certain circumstances than it would have in the absence of the private incentive. We also show in this paper that the total borrowings for the unemployed workers in the presence of private incentives for loans is always greater than that in the case of no private loan, suggesting that the private incentives for loans would reduce welfare.

The rest of this paper is organized as follows. The next section describes the basic model that can address the issues on the loan-based self-insurance. Section 3 examines properties of the optimal mix between UI benefit and self-insurance in the absence or presence of private incentives for loans, and discusses the role of government in the provision of loan-based self-insurance. Finally, Section 4 collects the main results of this paper with some concluding remarks.

II. The Model[6)]

Consider a 3-period model in which an individual may work for period 1 and 2 at the wage w per period, and then retires in period 3. The model assumes away the inter-temporal wage change and discounting just for simplicity. The worker may be confronted with unemployment shock in each of the two periods. The probability of an unemployment shock's occurring to an individual in period 1 is q, while that in period 2 depends upon whether or not he gets unemployed in period 1. The probability of a shock in period 2 for a worker who was previously unemployed is p_U, while that for a worker who was not unemployed is p_N.

Upon an unemployment shock a worker may choose to search or not to search for a job. If he searches, he has to incur search cost e_1 (or e_2) in period 1 (or in period 2)

6) The basic model used in this paper is identical to the one formulated by Kim, Stiglitz and Yun (2006), except that this model incorporates the private incentives for self-insurance which play critical roles in this paper. Furthermore, this paper deals with various theoretical issues on the self-insurance, whereas Kim-Stiglitz-Yun(2006) conducts empirical simulations based upon panel data sets to confirm welfare implications of the loan-based self-insurances.

while he immediately gets reemployed in the period.[7] If he chooses not to search, however, he will be unemployed in the period. The search cost e_1, e_2 for an individual are i.i.d. random variables with distribution functions F(.) and G(.), respectively. When the realized search cost for an individual in a period is lower (higher) than a threshold level, which is determined optimally for the period as will be discussed later, he will choose to search (not to search). Let $H_1 \equiv \frac{f(e_1)}{1-F(e_1)}$, $H_2 \equiv \frac{g(e_2)}{1-G(e_2)}$, and assume that H_1, H_2 are assumed to be constant over e_1, e_2, respectively. Note that H_1, H_2 indicate the search elasticity of reemployment, i.e., the degree of sensitiveness of reemployment with respect to search activity, in period 1, 2, respectively.

The government implements a program supporting the unemployed workers in period 1 (or in period 2), which provides them with unemployment insurance benefit r_1 (or r_2) and with loans R_1 (or R_2) against their future retirement incomes. The borrowers may not be able to repay what they have borrowed at the time of retirement, in which case the government bails them out. The government also implements a retirement insurance program, which provides a basic subsidy to those ending up with zero retirement income. In particular, the retirement insurance program provides a fixed amount of subsidy S to those unemployed in both of the two periods.[8] The cost incurred by these programs, including expected UI benefits, expected cost of bail-outs and expected subsidy for those with no retirement income, is covered by the taxes imposed upon the employed workers.[9]

The model introduces a possibility of private provision of loans for the unemployed workers. The private loan, however, is subject to higher probability of default compared

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7) The search activity in the model takes no time and guarantees a job for the worker with probability 1. Thus, a worker choosing to search will not be unemployed in the period, like the one with no unemployment shock.

8) This paper maintains that an individual is guaranteed the minimum consumption $(\frac{S}{2})$ by the government, i.e., that lender are not allowed to claim against the basic subsidy S.

9) Since the retirement insurance is financed by the taxes imposed upon employees, it can be viewed as a pay-as-you-go system that is strongly redistributive and that provides insurances against retirement risk to individuals.

to public mandatory loan due to the following reasons. First, private lenders are, in general, not in a position to be able to secure collaterals as much as the government, inducing opportunistic behaviors on the part of borrowers. Second, the private loans are subject to adverse selection in the case of privately-informed heterogeneity among workers, whereas the public loan is free of this problem because it is mandatory for all individuals. Instead of formulating these types of moral hazard and adverse selection in detail, the model assumes that private lenders incur higher unit cost P than the government in providing loans to unemployed workers, as will be shown below.

Given the government programs characterized by (r_1, R_1, r_2, R_2, S) and private loan R, let $V_{i,j}$ (i,j=N, U, where N, U indicate the state of employment and unemployment, respectively) denote the lifetime utility expected at the beginning of period 2 for a worker whose employment status is i in period 1 and j in period 2. Normalizing the constant wage to 1 for simplicity, we have

$$V_{NN} = Max_{s_2} \ \{U(1-s_2)+U(s_1+s_2)\} = 2U(\frac{1+s_1}{2})$$
$$V_{NU} = U(r_2+R_2)+U(s_1-R_2)$$
$$V_{UN} = Max_{s_2'} \ \{U(1-s_2')+U(s_2'-R_1-R)\} = 2U(\frac{1-R_1-R}{2})$$
$$V_{UU} = 2U(\frac{S}{2}) \quad ,$$

where S_1 is the savings that an individual chooses in period 1 to maximize his lifetime expected utility. Note that individual workers set their savings S_2, S_2' so as to equalize their consumptions between periods 2 and 3. Note also that the government is assumed to provide those unemployed in both of the two periods with the equal amount of subsidy, $\frac{S}{2}$, in period 2 and 3.

The total lifetime expected utility will then be

$$V = Max_{e_1, s_1} (1-\bar{q})[U(1-s_1-T_1) + Max_{e_2} \{\bar{p}_N V_{NU} + (1-\bar{p}_N)V_{NN} - p_N \int_0^{\bar{e}_2'} edG\}$$
$$+\bar{q}\{U(r_1 + R_1 - T_2 + R(1-P)) + Max_{e_2} \{\bar{p}_U V_{UU} + (1-\bar{p}_U)V_{UN} - p_U \int_0^{\bar{e}_2} edG\}]$$
$$-q \int_0^{\bar{e}_1} edF \tag{1}$$

where $\bar{e}_1, \bar{e}_2, \bar{e}_2'$ are the threshold levels of search cost so that

$$\bar{q} \equiv q(1-F(\bar{e}))$$
$$\bar{p}_U \equiv p_U(1-G(\bar{e}_2))$$
$$\bar{p}_N \equiv p_N(1-G(\bar{e}_2'))$$

and

$$(1-\bar{q})T_1 = \bar{q}r_1 + (1-\bar{q})\bar{p}_N r_2.$$
$$T_2 = \bar{p}_U(S+R_1)$$
$$P = \bar{p}_U(1+t)$$

Note that the tax T_1 for UI benefit r_1 and the tax T_2 for the cost of default of public mandatory loan R_1 are set to satisfy government budget constraint. Note also that a competitive private lender charges the expected cost of default, $P(=\bar{p}_U(1+t))$, to the borrowers, which is higher than that $\bar{p}_U$ charged by the government for the public loan. The difference in the costs between private and public loans, or the cost disadvantage associated with private loans, is represented by t.

Since private loan is more costly than public mandatory loan, it will be always optimal for any loan to be offered by the government, not by the private sector. As will be explained later, however, the private sector may have some incentives to offer loans despite the cost disadvantage, crowding out the public loans and reducing individual welfare.10)

The threshold levels of search cost in period 2, $\bar{e}_2, \bar{e}_2'$, will be set as follows:

$$\bar{e}_2 = V_{UN} - V_{UU} \qquad (2)$$
$$\bar{e}_2' = V_{NN} - V_{NU}$$

Similarly, the threshold level of search cost in period 1, $\bar{e}_1$, will be determined as follows:

$$\bar{e}_1 = Max_{s_1}\{U(1-s_1-T_1)+V_N\} - Max_{R_1}\{U(r_1+R_1-T_2+R(1-P))+V_U\}, \qquad (3)$$

where

$$V_N \equiv Max_{e_2}\{\bar{p}_N V_{NU} + (1-\bar{p}_N)V_{NN} - p_N \int_0^{\bar{e}_2'} edG\}$$
$$V_U \equiv Max_{e_2}\{\bar{p}_U V_{UU} + (1-\bar{p}_U)V_{UN} - p_U \int_0^{\bar{e}_2} edG\}]$$

Let us examine how an individual makes his decision on savings S_1 in period 1. To the extent that the savings S_1 affects the work incentive in period 2, the socially optimal savings would be different from the individual choice. Since this would impose additional complication upon the design of optimal social insurance system, however, this paper assumes that $p_N \approx 0$, implying that there is little chance for those who have not been unemployed in period 1 to get unemployment shock in period 2. More specifically, this assumption allows the optimal savings in period 1 to be set so as to smooth out consumption across the periods just like the way the individual choice of savings in period 1 is made. That is,

10) This is consistent with the proposition that the competitive equilibrium under moral hazard is not constrained efficient, argued by Arnott-Stiglitz (1986, 1991) and Greenwald-Stiglitz (1986).

$$s_1 = \frac{1-2T_1}{3}, \tag{4}$$

so that

$$U(1-s_1-T_1) = U(\frac{1+s_1}{2}) = U(\frac{2-T_1}{3}).$$

Based on this model we will examine for the rest of the paper the properties of optimal government program for unemployed workers. In particular, we will first characterize the baseline optimum in which no borrowings are offered by the private sector, and then will explore how the private provision of borrowings affects the government program.

III. Optimal Government Program

1. The Baseline Optimum

Let us denote by $(r_1^*, R_1^*, r_2^*, R_2^*, T_1^*, T_2^*)$ the baseline optimum that maximizes the expected utility of an individual given the government budget constraint and given that no loan is privately offered. The baseline government program will then satisfy the following conditions, (5), (6), (7) and (8) given the retirement income support S provided by the government to those who have been unemployed in both of the two periods.

$$r_1: \quad A(r_1, R_1) \equiv \{1 - \frac{U'(\frac{2-T_1}{3})}{U'(r_1+R_1-T_2)}\} - \frac{H_1}{1-\bar{q}} r_1 U'(\frac{2-T_1}{3}) = 0 \tag{5}$$

$$R_1: \ B(r_1, R_1) \equiv \{1 - \frac{U'(\frac{1-R_1}{2})}{U'(r_1 + R_1 - T_2)}\} - \frac{\overline{p}_U}{1-\overline{p}_U} H_2 \{R_1 U'(\frac{1-R_1}{2}) + S' U'(\frac{S}{2})\}$$
$$= 0 \qquad (6)$$

$$r_2: \ \{1 - \frac{U'(\frac{2-T_1}{3})}{U'(r_2 + R_2)}\} - H_2 r_2 U'(\frac{2-T_1}{3}) = 0 \qquad (7)$$

$$R_2: \ R_2 = \frac{s_1 - r_2}{2}. \qquad (8)$$

Note that the government will set its loans R_2 in period 2 just to smooth out consumptions of individuals over time. Each of the optimal government programs (r_1^*, R_1^*, r_2^*) is determined so as to balance its benefit of consumption smoothing across states or periods with the moral hazard costs associated with them. Note also that the solutions for (r_1^*, R_1^*) are separated from the others in the sense that we can solve for (r_1^*, R_1^*) from (5) and (6) independently of the other solutions. Once we get (r_1^*, R_1^*), we will solve for s_1^* by (4), and then solve for (r_2^*, R_2^*) from (7) and (8). The second-order conditions for (r_1^*, R_1^*) are satisfied for an interior solution, which is proved in the Appendix.

To examine the characteristics of the baseline government program (r_1^*, R_1^*) we state the following results:

Proposition 1

The correlation between the two unemployment shocks (indicated by p_U) increases UI benefit but reduces loan-based self-insurance. The moral hazard cost in period 1 (indicated by H_1) reduces UI benefit and increases loan-based self-insurance, while the reverse is true for the moral hazard cost in period 2 (indicated by H_2). And, the total amount of support for an unemployed worker is decreasing in the correlation and the moral hazard costs.

The proof is delegated to the Appendix. The results are intuitive. First, as the correlation between the two unemployment shocks increases, the need for income smoothing across the two periods will be reduced while the expected moral hazard cost associated with default in period 2 increases, leading to lower level of loan-based self-insurance. Also, although the reduction in the borrowings for an unemployed worker necessitates more of UI benefit r_1, the total amount of his consumption would be lowered. Second, as incentive problem in period 1 or 2 (indicated by H_1 or H_2) gets more serious, the optimal UI benefit or loan-based self-insurance would reduce, respectively. Similarly, this would lead to more of loan-based self-insurance or to more of UI benefit, respectively, but the total amount of consumption for an unemployed worker will be lowered.

We can state the following result on the possibility that the baseline optimum involves zero borrowings R_1^*, with the proof being delegated to the Appendix.

Proposition 2

The baseline optimum may entail zero loan-based self-insurance R_1^*.

The possibility of zero loan-based self-insurance stems from the externality inherent in the model. That is, the amount R_1 of loan-based self-insurance increases the expected expenditure of income support $\overline{p}_U S$ through its adverse incentive effect of increasing $\overline{p}_U$. Thus, even at the zero level of borrowings, its moral hazard cost would not become trivial, leading to the possibility of zero loan-based self-insurance. This result can be contrasted with the conventional argument that optimal level of insurance is always positive despite the moral hazard effect.

We can also examine how the optimal mix between UI benefit and self-insurance changes over time, i.e., can compare the optimal mix in period 1 with that in period 2. To identify the effect of unemployment timing upon the optimal mix, we will confine ourselves to the case where $p_U = 0$. In other words, we will examine how the optimal mix changes over time when a worker gets unemployed only once in his lifetime. We can then establish the following result:

Proposition 3

Suppose that $p_N > 0$ and $p_U = 0$, and that $H_1 = H_2$. Then,

$$r_1^* < r_2^* \text{ for any } p_N.$$

The above proposition suggests that the government would rely more upon UI benefit as unemployment support for the old compared to the young. The intuition behind this is the following. When unemployed in their young ages, the workers can rely upon self-insurance relatively easily because they can make sufficient amount of savings in their later careers in response to the unemployment shock. If unemployed in their later ages, however, they would have few periods to make savings in response to the shock, leading them to rely more upon UI than upon self-insurance.

2. Private Provision of Loans and Optimal Government Program

When the private sector as well as the government offers loans, however, the government has to take into consideration the private incentive for loans in designing its program against unemployment. The amount of private loan to be provided will be determined so as to maximize the expected utility V of an individual given the provision R_1 of public loan, that is,

$$R: \ \{1 - \frac{U'(\frac{1-R_1-R}{2})}{U'(r_1 + R_1 - T_2 + R(1-\bar{p}_U(1+t))}\} - \frac{\bar{p}_U}{1-\bar{p}_U}\{t + H_2 R(1+t)U'(\frac{1-R_1-R}{2})\} = 0 \qquad (9)$$

As we can see from (9), the amount of private loan is determined so as to balance the marginal benefit of income-smoothing (the first term) with the marginal (private) cost represented by the cost disadvantage and moral hazard cost (the second term). To the extent that private loan increases the probability $\overline{p}_U$ of default, it also increases moral hazard costs associated with public loan and retirement insurance. In other words, private loan exerts externality upon public loan and retirement insurance, which are not taken into account in the determination of its amount. This suggests that some positive amount of private loan may be offered, despite its cost disadvantage compared to public mandatory loan, which will in turn affect the government unemployment program.

The private incentive for loans is closely related to its cost disadvantage indicated by t. We can see from (6) and (9) that borrowings will not be offered by the private sector, i.e., $R=0$, if

$$t \geq H_2\{R_1^* U'(\frac{1-R_1^*}{2}) + SU'(\frac{S}{2})\} \equiv t^o, \tag{10}$$

where R_1^* is the optimal amount of self-insurance. This leads to the following proposition:

Proposition 4

There exists the parameter value $H_2(t)$ or $t(H_2)$ for a given value of the other parameter t or H_2, respectively, such that, for $H_2 \leq H_2(t)$ or for $t > t(H_2)$, no loan is offered by the private sector and the baseline optimum (r_1^*, R_1^*) will be chosen by the government.

The proof is delegated to the Appendix. The above proposition confirms the intuition that as the moral hazard problem for the borrowings becomes less serious, i.e., as H_2 decreases, the externality that the private loan exerts will become smaller, reducing the room for the private loan to undercut public loan. In particular, it can be noted that the private loan would not be offered when there is no moral hazard problem in period 2, i.e., $H_2 = 0$. Also, when the cost disadvantage associated with private loans t is greater than a certain level set by the moral hazard cost H_2 or by the amount of externality the private loan generates, the private sector would not offer loans for unemployed workers.

We will now examine in this subsection how the government unemployment program would be affected by the private incentive for the loans. Noting that the government program $(r_2(t), R_2(t))$ is influenced only through the tax T_1, we will focus on the government program $(r_1(t), R_1(t))$.

In choosing its program for unemployed workers the government should take into account the response of private loan R to the government program. We can collect the following results on the private responses.

Corollary 1

$$\frac{\partial R}{\partial r_1} < 0, \quad -1 < \frac{\partial R}{\partial R_1} < 0 .$$

That is, less of the private loan would be offered to workers as more of UI benefit or of the public loan is provided to them. This makes sense, because both government program and private loan are substitutes for each other.

The optimal government program $(r_1(t), R_1(t))$ and the level of private loan $R(t)$ will critically depend upon the cost disadvantage t. When the private loan is positive, i.e., $R(t) > 0$, the optimal government program $(r_1(t), R_1(t))$ will satisfy

$$\begin{aligned} r_1 : \quad & \{1 - \frac{U'(\frac{2-T_1}{3})}{U'(r_1 + R_1 - T_2 + R(1 - \bar{p}_U - t))}\} \\ & - \frac{1}{1-\bar{q}}\{H_1 r_1 U'(\frac{2-T_1}{3}) + \frac{\partial R}{\partial r_1}\bar{p}_U H_2 \{R_1 U'(\frac{1-R_1-R}{2}) + SU'(\frac{S}{2})\}\} \\ & = 0 \end{aligned} \tag{11}$$

$$R_1: \quad \{1-\frac{U'(\frac{1-R_1-R}{2})}{U'(r_1+R_1-T_2+R(1-\bar{p}_U(1+t))}\} \\ -\frac{\bar{p}_U}{1-\bar{p}_U}H_2\{R(1+t)U'(\frac{1-R_1-R}{2})+(1+\frac{\partial R}{\partial R_1})\{R_1U'(\frac{1-R_1-R}{2})+SU'(\frac{S}{2})\}\} \,, \\ =0 \tag{12}$$

where the private loan $R(t)$ is determined by (9). The condition (12) can alternatively be rewritten by (9) as follows:

$$R_1: \quad t-(1+\frac{\partial R}{\partial R_1})\{R_1U'(\frac{1-R_1-R}{2})+SU'(\frac{S}{2})\}=0 \tag{12'}$$

As we compare (11) and (12) with (5) and (6), respectively, the introduction of private loan will affect the government program (r_1, R_1) for unemployment, especially the public loan R_1, in several ways. First, the public loan will be partly crowded out by the private loan, as was pointed out by Arnott and Stiglitz (1991) in other context. Second, the public loan will increase the level of externality (in terms of moral hazard costs) that the private loan exerts. These two considerations would act toward reducing public loan in the presence of private loan. Being a substitute for private loan, on the other hand, the government may increase its loans to discourage private incentive for its relatively costly loans, which is captured by the term $\frac{\partial R}{\partial R_1}$.

Taking these effects into account, we will examine how the presence of private loan affects the loans that the government would like to offer for a given cost disadvantage t. What we can demonstrate first is that, for a certain interval of t, the government may increase its loan-based self-insurance above its baseline optimum level so as to keep the private loan to zero. To see this let us first denote by (r_1^o, R_1^o) the solution of (11) and (12) when $R=0$, and let t^1 be as follows:

$$t^1 = \frac{1-\bar{p}_U}{\bar{p}_U}\{1-\frac{U'(\frac{1-R_1^o}{2})}{U'(r_1^o+R_1^o-T_2)}\}$$
$$=(1+\frac{\partial R}{\partial R_1})H_2\{R_1^o U'(\frac{1-R_1^o}{2})+SU'(\frac{S}{2})\} \quad (13)$$

Note that $t^1 < t^o$ as $\frac{\partial R}{\partial R_1} < 0$. For $t \in [t^1, t^o]$ the government program $(r_1(t), R_1(t))$ will then satisfy the following two conditions:

$$1-\frac{U'(\frac{2-T_1}{3})}{U'(r_1(t)+R_1(t)-T_2))}-\frac{1}{1-\bar{q}}H_1 r_1(t)U'(\frac{2-T_1}{3})=0 \quad (14)$$

$$\frac{1-\bar{p}_U}{\bar{p}_U}\{1-\frac{U'(\frac{1-R_1(t)}{2})}{U'(r_1(t)+R_1(t)-T_2)}\}=t \quad (15)$$

In particular, we can establish the following proposition.

Proposition 5

For $t \in [t^1, t^o]$, $R(t)=0$, $R_1(t) > R_1^*$, $r_1(t) < r_1^*$.

The above proposition highlights the aspect of public loan that discourages private sector from providing loans for unemployed workers. If the cost disadvantage t is high the benefit of replacing private loan with public loan will become large. Since the private incentive for loans will be small under high t, on the other hand, the additional amount of public loans in excess of the baseline optimum that is required to keep private loan to zero will be small. it would be worthwhile for the government to the degree of externality exerted by the public loan upon private loan will decrease. These two factors will contribute to the case where, for a certain range of cost disadvantage t, the amount of public loan is set to be greater than the baseline optimum, even though private loans are not provided. To the extent that $R_1(t) > R_1^*$ in this case, therefore, the welfare will be lower than in the baseline optimum, whereas the welfare of the baseline optimum can be achieved when the cost disadvantage is high enough (i.e., greater than t^o) to kill private incentive for loans by itself.

When the cost disadvantage t is lower than t^1, however, the government would find it better to let some private loan be offered than to prevent it. The set of UI benefit and public mandatory loan, $(r_1(t), R_1(t))$, the government would like to provide will satisfy (11) and the following condition:

$$R_1 : \quad t-(1+\frac{\partial R}{\partial R_1})H_2\{R_1 U'(\frac{1-R_1-R}{2})+SU'(\frac{S}{2})\}=0 \tag{16}$$

while the amount of private loan offered is determined by (9).

Let

$$t^2=(1+\frac{\partial R}{\partial R_1})H_2 SU'(\frac{S}{2})$$

We can then establish the following proposition.

Proposition 6

(i) There exists $t'(<t^1)$ such that $R_1(t)> R_1^*$ for $t \in (t', t^o)$.

(ii) For $t \in [0, t^2]$, $R_1(t)=0$.

(iii) $R_1(t)+R(t) \geq R_1^*$.

The proof is delegated to the Appendix. Lower cost disadvantage t would reduce the benefit of replacing private loan with public one while the increased private loans will make the moral hazard costs of public loan higher. This argument and Proposition 5 lead us to say that, if the cost disadvantage t associated with private loan is higher than a certain level t', the government would want to provide more loans to unemployed workers than the baseline optimum. On the other hand, when the cost disadvantage is lower than a certain level t^2, it is better for the government not to provide any public loan to the unemployed workers. That is, the public loan will be completely crowded out by private loans. Note that the threshold level of cost disadvantage t^2 is determined by the basic retirement benefit S. If $S=0$, for example, the private loans would exert no externality so that $t^2=0$, i.e., that private sector would have no incentive to offer loans even when the cost disadvantage is zero.

Finally, the total amount of loans - both government and private - is always greater than the baseline optimal level of public loan if the cost disadvantage t is not greater than t^o. There are two factors underlying this result: the one is that the private loan does not take into account the moral hazard cost it generates upon the public loan, and the other is that the government has additional incentive for loans to discourage private loan which is relatively more costly than public mandatory loan. This suggests that the private incentives for loan-based self-insurance reduces welfare compared to that of the baseline optimum unless the cost disadvantage associated with private loans is high.

IV. Conclusion

This paper analyzes a social insurance system that involves the loan-based self-insurance as well as unemployment and retirement insurances for the individuals who may experience unemployment shocks and may end up with no savings left for retirement. Although the self-insurance could provide unemployed workers with a chance for intertemporal income smoothing over time, however, the possibility of default and the resulting government bail-out may limit the effectiveness of self-insurance because it may create rooms for moral hazard behaviors and for the externalities between the self-insurance and other social insurance programs.

The externalities created by the social insurance programs may also induce the private sector to provide some loans to the unemployed workers. In general, the private loan is more costly than the public mandatory loan because of various informational problems and market imperfections. Despite its cost disadvantage, however, positive amount of private loan may be offered because private lenders would not take into account the externality they exert upon other social insurances. The response of private loan would affect the government provision of loans, because they are substitutes for each other and the government has to care about the externality the private loan exerts, and because the government may want to discourage private loan by increasing its own loans. This paper shows that the amount of public loan offered for the unemployed workers may be zero for small amount of the cost disadvantage, and that the total amount of loans offered for the unemployed workers is always greater than the level of loan-based self-insurance in the baseline optimum. This paper also shows that, unless the cost disadvantage associated with private loans is high, the private incentive for loans would reduce welfare compared to the baseline optimum because of the externality generated by private loans.

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Appendix <Proofs of Propositions>

$$U_0 \equiv U(\frac{1-T_1}{3}), \quad U_1 \equiv U(r_1 + R_1 - T_2 + R(1-\bar{p}_U - t)),$$
$$U_2 \equiv U(\frac{1-R_1-R+S}{2}), \quad U_3 \equiv U(\frac{S+S'}{2})$$

<Second-order Condition>

Second-order condition: $D \equiv A_1 B_2 - A_2 B_1 > 0$

* (A) $\frac{\partial(1-\bar{p}_U)}{1-\bar{p}_U}\frac{R_1}{\partial R_1} < 1$: a sufficient condition

$$A_1 = -\frac{H_1}{1-\bar{q}}U_0'(1+\frac{\bar{q}}{1-\bar{q}}H_1 U_0' r_1) + \frac{\bar{q}}{3}U_0''(1+H_1 U_1')(\frac{1}{U_1'}+\frac{H_1}{1-\bar{q}}r_1)$$
$$+\frac{U_0'}{(U_1')^2}U_1'' \quad < 0$$

$$A_2 = \frac{U_0'}{(U_1')^2}U_1''(1-\bar{p}_U(1+\frac{H_2}{2}R_1)U_2')) \ < 0 \ \text{ by (A)}$$

$$B_1 = \frac{1-\bar{p}_U}{\bar{p}_U}\frac{U_2'}{(U_1')^2}U_1'' < 0$$

$$B_2 = -\frac{H_2}{2\overline{p}_U}U_2'(1-\frac{U_2'}{U_1'}) + \frac{1-\overline{p}_U}{\overline{p}_U}\frac{1}{(U_1')^2}(\frac{U_2''U_1'}{2} + U_2'U_1''(1-\overline{p}_U(1+\frac{H_2}{2}U_2'(R_1+S)))$$
$$-\frac{H_2}{2}(U_2' - R_1\frac{U_2''}{2}) \; < \; 0$$

by (A)

(i) $A_1 < 0,\ B_2 < 0$

(ii) $\frac{B_2}{A_2} > \frac{1-\overline{p}_U}{\overline{p}_U}\frac{U_2'}{U_0'} > \frac{B_1}{A_1}$, implying that $D > 0$.

<Proposition 1>

$A_{p_U} > 0$ because $\frac{\partial \overline{q}}{\partial p_U} < 0$ and $\frac{\partial T_2}{\partial p_U} > 0$.

Also, $B_{p_U} < 0$ because $\frac{p_U}{U_1'}$ is increasing in p_U. We then have

$$\frac{\partial r_1}{\partial p_U} = \frac{1}{D}(-A_{p_U}B_2 + B_{p_U}A_2) > 0 \qquad \frac{\partial R_1}{\partial p_U} = \frac{1}{D}(-A_1B_{p_U} + B_1A_{p_U}) < 0,$$

proving the first part of the proposition.

Note also that $A_{H_1} = -\frac{r_1U_0'}{1-\overline{q}}$, $B_{H_1} = 0$, $A_{H_2} = 0$, $B_{H_2} = -\frac{R_1U_2' + SU_3'}{2}$

$$\frac{\partial r_1}{\partial H_1} = \frac{1}{D}(-A_{H_1}B_2) < 0 \qquad \frac{\partial R_1}{\partial H_1} = \frac{1}{D}(A_{H_1}B_1) > 0$$

$$\frac{\partial r_1}{\partial H_2} = \frac{1}{D}(B_{H_2}A_2) > 0 \qquad \frac{\partial R_1}{\partial H_2} = \frac{1}{D}(-A_1B_2) < 0.$$

<Proposition 2>

Let r_1^o be the solution of r_1 for (6) when $R_1 = 0$, i.e.,

$$\frac{H_1 r_1^o U_0'}{1-\bar{q}} = 1 - \frac{U_2'}{U_1'}.$$

Then, $R_1^* = 0$ if

$$\frac{1-\bar{p}_U}{\bar{p}_U}\frac{H_1 r_1^o U_0'}{1-\bar{q}} - \frac{H_2 S U_3'}{2} \le 0,$$

which may hold for low H_1, q, S or for high H_2, p_U.

<Proposition 3>

Since $p_U = 0$, $T_2 = 0$, and R_1 is set so as to equalize $(r_1 + R_1)$ and $(\frac{1-R_1}{2})$. That is, $R_1 = \frac{1-2r_1}{3}$, so that $r_1 + R_1 = \frac{1-R_1}{2} = \frac{1+r_1}{3}$. Also, by (1), we have $r_2 + R_2 = \frac{s_1 + r_2}{2}$. Thus, from (8) and (1), we have

$$H(\frac{r_1}{1-\bar{q}} - r_2) = \frac{1}{U'(\frac{s_1+r_2}{2})} - \frac{1}{U'(\frac{1+r_1}{3})}.$$

If $r_1 \ge r_2$, the above condition will not hold because $\frac{s_1 + r_2}{2} < \frac{1+r_1}{3}$ since $s_1 < \frac{1}{3}$.

<Proposition 4>

Substituting (7) into (9), we have at $R = 0$

$$H_2\{R_1^* U'(\frac{1-R_1^*}{2}) + SU'(\frac{S}{2})\} - t = 0.$$

Thus, if $H_2 < \dfrac{t}{R_1^* U'(\frac{1-R_1^*}{2}) + SU'(\frac{S}{2})} \equiv H_2(t)$ or if

$$t > H_2\{R_1^* U'(\frac{1-R_1^*}{2}) + SU'(\frac{S}{2})\} \equiv t(H_2), \quad R^* = 0.$$

<Proposition 5>

It is clear from (9) and (15) that $R(t) = 0$ for $t \geq t^o$. Regarding the solutions for $R_1(t)$ and $r_1(t)$ of (14) and (15), we can first prove the second-order condition to hold.

<Second-order Conditions for (14) and (15)>

* (A) $\dfrac{\partial(1-\overline{p}_U)}{1-\overline{p}_U}\dfrac{R_1}{\partial R_1} < 1$: a sufficient condition

$$A_1' = -\frac{H_1}{1-\overline{q}}U_0'(1 + \frac{\overline{q}}{1-\overline{q}}H_1 U_0'(r_1 + S)) + \frac{\overline{q}}{3}U_0''(1 + H_1 U_1')(\frac{1}{U_1'} + \frac{H_1}{1-\overline{q}}(r_1 + S))$$
$$+ \frac{U_0'}{(U_1')^2}U_1'' < 0$$

$$A_2' = \frac{U_0'}{(U_1')^2}U_1''(1-\bar{p}_U(1+\frac{H_2}{2}(R_1+S)U_2')) < 0 \text{ by (A)}$$

$$B_1' = \frac{1-\bar{p}_U}{\bar{p}_U}\frac{U_2'}{(U_1')^2}U_1'' < 0$$

$$B_2' = -\frac{H_2}{2\bar{p}_U}U_2'(1-\frac{U_2'}{U_1'}) + \frac{1-\bar{p}_U}{\bar{p}_U}\frac{1}{(U_1')^2}(\frac{U_2''U_1'}{2} + U_2'U_1''(1-\bar{p}_U(1+\frac{H_2}{2}U_2'(R_1+S')))$$
$$< 0$$

by (A)

(i) $A_1' < 0,\ B_2' < 0$

(ii) $\frac{B_2'}{A_2'} > \frac{1-\bar{p}_U}{\bar{p}_U}\frac{U_2'}{U_0'} > \frac{B_1'}{A_1'}$, implying that $D' \equiv A_1'B_2' - A_2'B_1' > 0$

Applying the Cramer's rule to (14) and (15), we have

$$\frac{dr_1}{dt} = \frac{1}{D'}(-A_2') > 0, \qquad \frac{dR_1}{dt} = \frac{1}{D'}(A_1') < 0.$$

Since $R_1(t) = R_1^*$, $r_1(t) = r_1^*$ when $t = t^o$, we have the desired results.

<Proposition 6>

(i) Since $R_1(t) > R_1^*$ for $t \in (t^1, t^o)$ by Proposition 5, the desired results obtain.

(ii) $t - (1+\frac{\partial R}{\partial R_1})H_2SU'(\frac{S}{2}) < 0$ for $t \le t^2$ by the definition of t^2, implying the desired result by (16).

(iii) Let $TR \equiv R + R_1$. Then, we can see from (12) that the choice of TR, TR^*, satisfies

$$\Omega(TR^*) \equiv \frac{1-\overline{p}_U(TR^*)}{\overline{p}_U(TR^*)}\{1 - \frac{U'(\frac{1-TR^*}{2})}{U'(r_1 + TR^*(1-\overline{p}_U) - Rt))}\})$$

$$- H_2\{(TR^*)U'(\frac{1-TR^*}{2}) + SU'(\frac{S}{2})\}$$

$$- \frac{\partial R}{\partial R_1} H_2\{R_1 U'(\frac{1-TR^*}{2}) + SU'(\frac{S}{2})\}$$

$$= 0$$

Note that $\overline{p}_U$ depends on R and R_1 only through TR. Comparing this with (6), we can see

$$\Omega(R^*) \geq - \frac{\partial R}{\partial R_1} H_2\{R_1 U'(\frac{1-TR^*}{2}) + SU'(\frac{S}{2})\} > 0 ,$$

because $U'(r_1 + TR(1-\overline{p}_U) - Rt) \geq U'(r_1 + TR(1-\overline{p}_U))$.

This implies that $TR^* > R_1^*$.

Corollary 1

The response can be derived from (6) as follows:

$$\frac{\partial R}{\partial r_1} = -\frac{B}{D}, \quad \frac{\partial R}{\partial R_1} = -\frac{C}{D}$$

where

$$B \equiv \{1 - \overline{p}_U - \overline{p}_U(1+t)(1 + H_2 R U_2')\}U_1''$$

$$C \equiv (1-\bar{p}_U-\bar{p}_U(1+t)(1+H_2RU_2^{'}))U_1^{''}(1-\bar{p}_U(1+H_2(R_1+R(1+t))U_2^{'}))$$
$$+(1-\bar{p}_U+\bar{p}_U H_2 R(1+t)U_1^{'})\frac{U_2^{''}}{2}$$
$$-\bar{p}_U H_2 U_2^{'}\{U_1^{'}-U_2^{'}+(1+t)(1+H_2RU_2^{'})U_1^{'}\}$$

$$D \equiv C - t\bar{p}_U(1-\bar{p}_U-\bar{p}_U(1+t)(1+H_2RU_2^{'}))U_1^{''} - H_2\bar{p}_U(1+t)U_1^{'}U_2^{'}$$

and

$$U_1 \equiv U(r_1+R_1-T_2+R(1-\bar{p}_U(1+t))$$
$$U_2 \equiv U(\frac{1-R_1-R+S}{2})$$

Since $B<0$, $C<0$, and $0>C>D$, the desired result obtains.

논문초록

실직대비 공적 자가보험의 후생효과

윤 정 열

본 논문은 실직에 대비한 대출자가보험의 후생효과와 정부개입 필요성에 관해 분석하고 있고, 이 문제를 다루는 데 있어서 본 논문에서는 근로자가 오랜 실직기간으로 퇴직후 연금소득이 부족하여 대출상환을 못하고 정부로부터 최저소득 지원을 받게 되는 가능성을 전제하고 있다. 본 논문에서는 대출자가보험이 동태적 소비균등화 및 근로유인 강화 측면에서 긍정적 후생효과를 갖지만 대출상환 불능 가능성이 그 효과를 제약할 수 있음을 보이고 있다. 또 대출상환 불능자에 대한 정부 지원이 민간부문의 대출자가보험을 유인할 수 있음을 보이고, 이에 따라 정부의 대출자가보험 공급 여지가 감소할 수는 있지만 일정 부분 정부의 역할이 필요하게 됨을 보여주고 있다.

주제어 : 자가보험, 대출, 실업보험, 외부효과