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NONUNIQUE COINCIDENCE POINT THEOREMS FOR ĆIRIĆ TYPE MAPPINGS

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Abstract. A few existence results of nonunique coincidence points for some kinds of Ćirić type mappings in metric and pseudocompact Tichonov spaces, respectively, are proved. The results presented in this paper extend some known results in the literature.

1. Introduction and preliminaries

The existence of nonunique fixed points, nonunique common fixed points and nonunique coincidence points for various nonlinear mappings in metric spaces and pseudocompact Tichonov spaces have been studied by a lot of investigators, for example, see [1]-[13] and the references therein.

In 1974, Ćirić [2] first proved the existence of nonunique fixed points for the following self mapping f in a metric space (X, d):

$$\begin{aligned} &\min\{d(fx, fy), d(fx, x), d(y, fy)\} \\ &\quad - \min\{d(x, fy), d(fx, y)\} \le rd(x, y) \end{aligned} \tag{1.1}$$

for all $x, y \in X$ and some $0 < r < 1$.

In 1980, Achari [1] extended the results of Ćirić [2] to a pair of nonlinear mappings:

$$\begin{aligned} &\min\{d(fx, gx), d(x, fx), d(y, gy)\} \\ &\quad - \min\{d(x, gy), d(y, fx)\} \le rd(x, y) \end{aligned} \tag{1.2}$$

for all $x, y \in X$ and some $0 < r < 1$.

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In 1993, Liu [6] extended and improved the results of Achari [1] and Ćirić [2] to the following three nonlinear mappings in metric and pseudocompact Tichonov spaces, respectively:

$$\begin{aligned} &\min\{d(fx,gy),d(fx,hx),d(hy,gy)\} \\ &\quad -\min\{d(hx,gy),d(fx,hy)\} \le rd(hx,hy) \end{aligned} \tag{1.3}$$

for all $x, y \in X$ and some $0 < r < 1$;

$$\begin{aligned} &\min\{F(fx,gy),F(fx,hx),F(hy,gy)\} \\ &\quad -\min\{F(hx,gy),F(fx,hy)\} < F(hx,hy) \end{aligned} \tag{1.4}$$

for all $x, y \in X$.

In 1994, Liu [8] generalized the results of Ćirić [2] from metric spaces to pseudocompact Tichonov spaces:

$$\begin{aligned} &\min\Big\{F(fx,fy),F(fx,gx),F(fy,gy),\frac{F^2(fx,fy)}{F(gx,gy)}, \\ &\quad \frac{F^2(fx,gx)}{F(gx,gy)},\frac{F^2(fy,gy)}{F(gx,gy)},\frac{F(fx,fy)F(fx,gx)}{F(gx,gy)}, \\ &\quad \frac{F(fx,fy)F(fy,gy)}{F(gx,gy)},\frac{F(fx,gx)F(fy,gy)}{F(gx,gy)}\Big\} \\ &-\min\big\{F(gx,fy),F(fx,gy)\big\} < F(gx,gy) \end{aligned} \tag{1.5}$$

for all $x, y \in X$ with $gx \ne gy$.

The purpose of the present paper is to establish some more general nonunique coincidence point theorems for several classes of Ćirić type mappings below on metric and pseudocompact Tichonov spaces, respectively:

$$\begin{aligned} &\min\Big\{d(fx,gy),d(fx,hx),d(hy,gy),\frac{d^2(fx,gy)}{d(hx,hy)}, \\ &\quad \frac{d(fx,gy)d(fx,hx)}{d(hx,hy)},\frac{d(fx,gy)d(hy,gy)}{d(hx,hy)}, \\ &\quad \frac{d(fx,hx)d(hy,gy)}{d(hx,hy)}\Big\} - \min\big\{d(hx,gy),d(fx,hy)\big\} \end{aligned} \tag{1.6}$$

$$\le r\max\Big\{d(hx,hy),\min\{d(fx,hx),d(hy,gy)\},\frac{d(hx,gy)d(fx,hy)}{d(hx,hy)}\Big\};$$

$$\begin{aligned}&\min\{d^2(fx,gy),d^2(fx,hx),d^2(hy,gy),\\&\quad d(fx,gy)d(hx,hy),d(fx,hx)d(hx,hy),\\&\quad d(hy,gy)d(hx,hy)\}-\min\{d^2(hx,gy),d^2(fx,hy)\}\\&\le r\max\{d^2(hx,hy),\min\{d^2(fx,hx),d^2(hy,gy)\},\\&\quad d(hx,gy)d(fx,hy)\};\end{aligned}\tag{1.7}$$

$$\begin{aligned}&\min\Big\{F(fx,gy),F(fx,hx),F(hy,gy),F(fy,hy),F(hx,gx),\\&\frac{F^2(fx,gy)}{F(hy,hx)},\frac{F^2(hy,gy)}{F(hy,hx)},\frac{F^2(hx,gx)}{F(hy,hx)},\frac{F(fx,hx)F(fx,gy)}{F(hy,hx)},\\&\frac{F(fx,hx)F(hy,gy)}{F(hy,hx)},\frac{F(fx,hx)F(fy,hy)}{F(hy,hx)},\\&\frac{F(fx,hx)F(hx,gx)}{F(hy,hx)},\frac{F(fy,hy)F(fx,gy)}{F(hy,hx)},\\&\frac{F(fy,hy)F(hy,gy)}{F(hy,hx)},\frac{F(fy,hy)F(hx,gx)}{F(hy,hx)}\Big\}\\&-\min\{F(hx,gy),F(fx,hy)\}<F(hy,hx);\end{aligned}\tag{1.8}$$

$$\begin{aligned}&\min\{F^2(fx,gy),F^2(fx,hx),F^2(hy,gy),F^2(fy,hx),\\&\quad F^2(hx,gx),F(fx,gy)F(hy,hx),F(fx,hx)F(hy,hx),\\&\quad F(hy,gy)F(hy,hx),F(fy,hx)F(hy,hx),\\&\quad F(hx,gx),F(hy,hx)\}\\&-\min\{F^2(hx,gy),F^2(fx,hy)\}<F^2(hy,hx).\end{aligned}\tag{1.9}$$

The results presented in this paper extend and unify some known results in [2] and [6].

Let f, g and h be self mappings of a metric space (X,d). For a point $x_0 \in X$, if there exists a sequence $\{x_n\}_{n\geq 0}$ in X such that $hx_{2n+1} = fx_{2n}, hx_{2n+2} = gx_{2n+1}$ for $n \geq 0$, then $O(f,g,h,x_0) = \{hx_n : n \geq 1\}$ is called an *orbit* of (f,g,h) at x_0. $\overline{O(f,g,h,x_0)}$ denotes the closure of $O(f,g,h,x_0)$ in X. X is said to be (f,g,h)-*orbitally complete* at x_0 if every Cauchy sequence in $O(f,g,h,x_0)$ converges in X. For $T \in \{f,g,h\}$, T is called to be *orbitally continuous* at x_0 if it is continuous on $O(f,g,h,x_0)$. Sessa [14] defined self mappings f and g on (X,d) to be *weakly commuting* if $d(fgx,gfx) \leq d(fx,gx)$ for all $x \in X$.

A topological space X is said to be *pseudocompact* if every real valued continuous function on X is bounded. It is clear that a compact space is pseudocompact. If X is an arbitrary Thchonov space, then X is pseudocompact if and only if every real valued continuous function on X is bounded and assumes its bounds.

2. Nonunique coincidence point theorems

Our main results are as follows:

THEOREM 2.1. *Let f, g and h be self mappings of a metric space (X,d) and X be (f,g,h)-orbitally complete at some $x_0 \in X$. Assume that*

(a) *either f is orbitally continuous at x_0, f and h are weakly commuting or g is orbitally continuous at x_0, g and h are weakly commuting;*

(b) *h is orbitally continuous at x_0;*

(c) *there exists $r \in (0,1)$ such that for any $x,y \in O(f,g,h,x_0)$ with $hx \neq hy$, at least one of* (1.6) *and* (1.7) *is satisfied.*

Then f and h or g and h have a coincidence point in $\overline{O(f,g,h,x_0)}$.

Proof. Suppose that $hx_n = hx_{n+1}$ for some $n \geq 0$. Obviously x_n is a coincidence point of f and h or g and h. Suppose that $hx_n \neq hx_{n+1}$ for each $n \geq 0$. Let $d_n = d(hx_n, hx_{n+1})$ for $n \geq 0$. Now we claim that

$$d_{n+1} \leq \sqrt{r} d_n \tag{2.1}$$

for all $n \geq 0$.

Let n be a nonnegative integer. Now we have to consider the following possible cases:

Case 1. Suppose that (1.6) holds for $x = x_{2n}$ and $y = x_{2n+1}$. It follows that

$$\begin{aligned}
\min\Big\{ & d(fx_{2n}, gx_{2n+1}), d(fx_{2n}, hx_{2n}), d(hx_{2n+1}, gx_{2n+1}), \\
& \frac{d^2(fx_{2n}, gx_{2n+1})}{d(hx_{2n}, hx_{2n+1})}, \frac{d(fx_{2n}, gx_{2n+1})d(fx_{2n}, hx_{2n})}{d(hx_{2n}, hx_{2n+1})}, \\
& \frac{d(fx_{2n}, gx_{2n+1})d(hx_{2n+1}, gx_{2n+1})}{d(hx_{2n}, hx_{2n+1})}, \\
& - \frac{d(fx_{2n}, hx_{2n})d(hx_{2n+1}, gx_{2n+1})}{d(hx_{2n}, hx_{2n+1})}\Big\} \\
& - \min\big\{d(hx_{2n}, gx_{2n+1}), d(fx_{2n}, hx_{2n+1})\big\} \\
\le r \max\Big\{ & d(hx_{2n}, hx_{2n+1}), \min\big\{d(fx_{2n}, hx_{2n}), d(hx_{2n+1}, gx_{2n+1})\big\}, \\
& \frac{d(hx_{2n}, gx_{2n+1})d(fx_{2n}, hx_{2n+1})}{d(hx_{2n}, hx_{2n+1})}\Big\},
\end{aligned}$$

which implies that

$$\begin{aligned}
& \min\Big\{ d_{2n+1}, d_{2n}, \frac{d_{2n+1}^2}{d_{2n}} \Big\} \\
& = \min\Big\{ d_{2n+1}, d_{2n}, d_{2n+1}, \frac{d_{2n+1}^2}{d_{2n}}, \frac{d_{2n+1}d_{2n}}{d_{2n}}, \frac{d_{2n+1}d_{2n+1}}{d_{2n}}, \frac{d_{2n}d_{2n+1}}{d_{2n}} \Big\} \\
& \quad - \min\big\{d(hx_{2n}, hx_{2n+2}), 0\big\} \\
& \le r \max\big\{d_{2n}, \min\{d_{2n}, d_{2n+1}\}, 0\big\} \\
& = r d_{2n},
\end{aligned}$$

which yields that $d_{2n+1} \le \sqrt{r} d_{2n}$.

Case 2. Suppose that (1.7) holds for $x = x_{2n}$ and $y = x_{2n+1}$. It follows from (1.7) that

$$\begin{aligned}
\min\big\{ & d^2(fx_{2n}, gx_{2n+1}), d^2(fx_{2n}, hx_{2n}), d^2(hx_{2n+1}, gx_{2n+1}), \\
& d(fx_{2n}, gx_{2n+1})d(hx_{2n}, hx_{2n+1}), d(fx_{2n}, hx_{2n})d(hx_{2n}, hx_{2n+1}), \\
& d(hx_{2n+1}, gx_{2n+1})d(hx_{2n}, hx_{2n+1})\big\} \\
& - \min\big\{d^2(hx_{2n}, gx_{2n+1}), d^2(fx_{2n}, hx_{2n+1})\big\}
\end{aligned}$$

$$\leq r\max\{d^2(hx_{2n},hx_{2n+1}),\min\{d^2(fx_{2n},hx_{2n}),\\ d^2(hx_{2n+1},gx_{2n+1})\},d(fx_{2n},hx_{2n+1})d(hx_{2n},gx_{2n+1})\},$$

that is,

$$\begin{aligned}&\min\{d_{2n+1}^2,d_{2n}^2,d_{2n+1}d_{2n}\}\\&=\min\{d_{2n+1}^2,d_{2n}^2,d_{2n+1}^2,d_{2n+1}d_{2n},d_{2n+1}d_{2n+1},d_{2n}d_{2n+1}\}\\&\quad-\min\{d(hx_{2n},hx_{2n+2}),0\}\\&\leq r\max\{d_{2n}^2,\min\{d_{2n}^2,d_{2n+1}^2\},0\}\\&=rd_{2n}^2,\end{aligned}$$

which means that $d_{2n+1}\leq\sqrt{r}d_{2n}$.

Anyhow, $d_{2n+1}\leq\sqrt{r}d_{2n}$ holds without any doubt. Similarly, $d_{2n}\leq\sqrt{r}d_{2n-1}$. Consequently, (2.1) holds. Note that

$$\begin{aligned}d(hx_n,hx_{n+p})&\leq\sum_{i=n}^{n+p-1}d_i\leq\left\{\sum_{i=n}^{n+p-1}(\sqrt{r})^i\right\}d_0\\&\leq\frac{(\sqrt{r})^n}{1-\sqrt{r}}d_0\end{aligned}\tag{2.2}$$

for all $n,p\geq 0$.

It follows from (2.2) that $\{hx_n\}_{n\geq 0}$ is a Cauchy sequence. Since X is (f,g,h)-orbitally complete and h is orbitally continuous at x_0, there exists a point $z\in\overline{O(f,g,h,x_0)}$ such that $hx_n\to z$ and $hhx_n\to hz$ as $n\to\infty$. If f is orbitally continuous at x_0 and f and h are weakly commuting, it follows that $fhx_{2n}\to fz$ as $n\to\infty$ and for any $n\geq 1$

$$\begin{aligned}d(fz,hz)&\leq d(fz,fhx_{2n})+d(fhx_{2n},hfx_{2n})+d(hfx_{2n},hz)\\&\leq d(fz,fhx_{2n})+d(fx_{2n},hx_{2n})+d(hhx_{2n+1},hz).\end{aligned}$$

Let n tend to ∞, we infer that $d(fz,hz)\leq 0$. Hence z is a coincidence point of f and h. If g and h are orbitally continuous at x_0 and weakly commuting, we can similarly attain that g and h have a coincidence point in $\overline{O(f,g,h,x_0)}$. This completes the proof. □

THEOREM 2.2. *Let f, g and h be self mappings of a metric space (X,d) and $x_0 \in X$. Assume that there exists an orbit $O(f,g,h,x_0) \subseteq X$ such that $hx_n \neq hx_{n+1}$ for each $n \geq 0$. Let X be (f,g,h)-orbitally complete at x_0, f, g and h be orbitally continuous at x_0, f and h, g and h be weakly commuting. If f, g and h satisfy* (1.6) *or* (1.7) *for all $x,y \in O(f,g,h,x_0)$ with $hx \neq hy$, then f, g and h have a coincidence point in $\overline{O(f,g,h,x_0)}$.*

Proof. As in the proof of Theorem 2.1, we infer that there exists a point $z \in \overline{O(f,g,h,x_0)}$ with $\lim_{n\to\infty} hx_n = z$ and $fz = hz = gz$. That is, z is a coincidence point of f, g and h. This completes the proof. □

REMARK 2.1. Theorems 2.1 and 2.2 extend Theorem 1 in [2] and Theorem 3.1 in [6], respectively.

THEOREM 2.3. *Let X be a pseudocompact Tichonov space and $F : X \times X \to [0,+\infty)$ satisfy that $F(x,y) = 0$ if and only if $x = y$. Let f, g and $h : X \to X$ satisfy $f(X) \cup g(X) \subseteq h(X)$. Assume that two functions a and b defined by $a(x) = F(fx,hx)$ and $b(x) = F(hx,gx)$ are continuous on X. If for all $x,y \in X$ with $hx \neq hy$, at least one of* (1.8) *and* (1.9) *is satisfied, then f and h or g and h have a coincidence point in X.*

Proof. Since X is a pseudocompact Tichonov space and a and b are continuous on X, it follows that there exist two points $u,v \in X$ satisfying

$$a(u) = \inf\{a(x) : x \in X\} \quad \text{and} \quad b(v) = \inf\{b(x) : x \in X\}.$$

Assume that, without loss of generality, $a(u) \leq b(v)$. Observe that $f(X) \subseteq h(X)$ implies that there exists a point $w \in X$ such that $fu = hw$. We now assert that u is a coincidence of f and h. Otherwise $fu \neq hu$, that is, $hu \neq hw$. Now we have to consider the following cases:

Case 1. Suppose that (1.8) is true for $x = u$ and $y = w$. Noting that

$b(w) \geq b(v) \geq a(u)$ and $a(w) \geq a(u)$, we deduce immediately that

$$\begin{aligned}
\min\Bigg\{ & F(fu,gw), F(fu,hu), F(hw,gw), F(fw,hw), F(hu,gu), \\
& \frac{F^2(fu,gw)}{F(hw,hu)}, \frac{F^2(hw,gw)}{F(hw,hu)}, \frac{F^2(hu,gu)}{F(hw,hu)}, \frac{F(fu,hu)F(fu,gw)}{F(hw,hu)}, \\
& \frac{F(fu,hu)F(hw,gw)}{F(hw,hu)}, \frac{F(fu,hu)F(fw,hw)}{F(hw,hu)}, \\
& \frac{F(fu,hu)F(hu,gu)}{F(hw,hu)}, \frac{F(fw,hw)F(fu,gw)}{F(hw,hu)}, \\
& \frac{F(fw,hw)F(hw,gw)}{F(hw,hu)}, \frac{F(fw,hw)F(hu,gu)}{F(hw,hu)}\Bigg\} \\
& - \min\big\{F(hu,gw), F(fu,hw)\big\} < F(hw,hu),
\end{aligned}$$

that is,

$$\begin{aligned}
& \min\left\{a(u), \frac{b^2(u)}{a(u)}, \frac{a(w)b(w)}{a(u)}, \frac{a(w)b(u)}{a(u)}\right\} \\
& = \min\Bigg\{b(w), a(u), b(w), a(w), b(u), \frac{b^2(w)}{a(u)}, \frac{b^2(w)}{a(u)}, \frac{b^2(u)}{a(u)}, \\
& \qquad \frac{a(u)b(w)}{a(u)}, \frac{a(u)b(w)}{a(u)}, \frac{a(u)a(w)}{a(u)}, \frac{a(u)b(u)}{a(u)}, \frac{a(w)b(w)}{a(u)}, \\
& \qquad \frac{a(w)b(w)}{a(u)}, \frac{a(w)b(u)}{a(u)}\Bigg\} - \min\big\{F(hu,gw), 0\big\} \\
& < a(u),
\end{aligned}$$

which implies that $a(u) < a(u)$, which is impossible.

Case 2. Suppose that (1.9) is valid for $x = u$ and $y = w$. It follows from (1.9) that

$$\begin{aligned}
\min\big\{ & F^2(fu,gw), F^2(fu,hu), F^2(hw,gw), F^2(fw,hw), F^2(hu,gu), \\
& F(fu,gw)F(hw,hu), F(fu,hu)F(hw,hu), F(hw,gw)F(hw,hu), \\
& F(fw,hw)F(hw,hu), F(hu,gu)F(hw,hu)\big\} \\
& - \min\big\{F^2(hu,gw), 0\big\} < F^2(hw,hu),
\end{aligned}$$

which yields that

$$\begin{aligned}&\min\left\{b^2(w),a^2(u),a^2(w),b^2(u)\right\}\\&=\min\left\{b^2(w),a^2(u),b^2(w),a^2(w),b^2(u),b(w)a(u),a^2(u),\right.\\&\qquad\left.b(w)a(u),a(w)a(u),b(u)a(u)\right\}-\min\left\{F^2(hu,gw),0\right\}\\&<a^2(u),\end{aligned}$$

which gives that $a^2(u) < a^2(u)$, a contradiction.

It follows from Case 1 and Case 2 that $fu = hu$. Hence u is a coincidence point of f and h. This completes the proof. □

Remark 2.2. Theorem 2.3 extends Theorem 3.3 in [6].

Corollary 2.1. *Let X be a pseudocompact Tichonov space and $F : X \times X \to [0, +\infty)$ be continuous and $F(x, y) = 0$ if and only if $x = y$. Let f, g and $h : X \to X$ be continuous and $f(X) \subseteq g(X) \subseteq h(X)$. If for any $x, y \in X$ with $hx \neq hy$, f, g and h satisfy at least one of* (1.8) *and* (1.9)*, then f and h or g and h have a coincidence point in X.*

Corollary 2.2. *Let (X, d) be a compact metric space. Let f, g and $h : X \to X$ be three continuous mappings and $f(X) \cup g(X) \subseteq h(X)$. If for any $x, y \in X$ with $hx \neq hy$, at least one of the following conditions*

$$\begin{aligned}&\min\Big\{d(fx,gy),d(fx,hx),d(hy,gy),d(fy,hy),d(hx,gx),\\&\quad\frac{d^2(fx,gy)}{d(hy,hx)},\frac{d^2(hy,gy)}{d(hy,hx)},\frac{d^2(hx,gx)}{d(hy,hx)},\frac{d(fx,hx)d(fx,gy)}{d(hy,hx)},\\&\quad\frac{d(fx,hx)d(hy,gy)}{d(hy,hx)},\frac{d(fx,hx)d(fy,hy)}{d(hy,hx)},\\&\quad\frac{d(fx,hx)d(hx,gx)}{d(hy,hx)},\frac{d(fy,hy)d(fx,gy)}{d(hy,hx)},\\&\quad\frac{d(fy,hy)d(hy,gy)}{d(hy,hx)},\frac{d(fy,hy)d(hx,gx)}{d(hy,hx)}\Big\}\\&-\min\left\{d(hx,gy),d(fx,hy)\right\}<d(hy,hx)\end{aligned}\tag{2.3}$$

and

$$
\begin{aligned}
(2.4)\qquad & \min\big\{d^2(fx,gy),d^2(fx,hx),d^2(hy,gy),d^2(fy,hx),\\
& \quad d^2(hx,gx),d(fx,gy)d(hy,hx),d(fx,hx)d(hy,hx),\\
& \quad d(hy,gy)d(hy,hx),d(fy,hx)d(hy,hx),\\
& \quad d(hx,gx),d(hy,hx)\big\}\\
& -\min\big\{d^2(hx,gy),d^2(fx,hy)\big\} < d^2(hy,hx)
\end{aligned}
$$

is satisfied, then f and h or g and h have a coincidence point in X.

Remark 2.3. Corollary 2.2 is a generalization of Theorem 4 in [2].

Remark 2.4. The following example reveals that the coincidence points in Corollary 2.2 may not be unique.

Example 2.1. Let $X = \{0,1,2\}$ with the usual metric. Define f, g and $h : X \to X$ by $f0 = g2 = h0 = 0$, $f1 = f2 = g0 = g1 = h1 = 1$ and $h2 = 2$. Obviously, (X,d) is a compact metric space, and f, g and h are continuous. It is easy to show that the conditions of Corollary 2.2 are satisfied. But f and h have two coincidence points 0 and 1, and g and h have no coincidence point.

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