International Journal of Automotive Technology, Vol. 8, No. 2, pp. 211−217 (2007)

1229−9138/2007/033−10

MAXIMUM BRAKING FORCE CONTROL UTILIZING THE ESTIMATED BRAKING FORCE

D. HONG[1)], I. HWANG[2)], M. SUNWOO[1)] and K. HUH[1)*]

[1)]Department of Automotive Engineering, Hanyang University, Seoul 133-791, Korea
[2)]Central R&D Center, Mando Corporation, 413-5 Gomae-dong, Giheung-gu, Yongin-si, Gyeonggi 449-901, Korea

(Received 12 July 2006; Revised 23 October 2006)

ABSTRACT−The wheel slip control systems are able to control the braking force more accurately and can be adapted to different vehicles more easily than conventional ABS (Anti-lock Brake System) systems. In realizing the wheel slip control systems, real-time information such as the tire braking force at each wheel is required. In addition, the optimal target slip values need to be determined depending on the braking objectives such as minimum braking distance and stability enhancement. In this paper, a robust wheel slip controller is developed based on the adaptive sliding mode control method and an optimal target slip assignment algorithm is proposed for maximizing the braking force. An adaptive law is formulated to estimate the braking force in real-time. The wheel slip controller is designed based on the Lyapunov stability theory considering the error bounds in estimating the braking force and the brake disk-pad friction coefficient. The target slip assignment algorithm searches for the optimal target slip value based on the estimated braking force. The performance of the proposed wheel slip control system is verified in HILS (Hardware-In-the-Loop Simulator) experiments and demonstrates the effectiveness of the wheel slip control in various road conditions.

KEY WORDS : Electro-hydraulic brake, Maximum braking force, Wheel slip control, Adaptive sliding mode control, Optimal target slip

1. INTRODUCTION

The wheel slip control systems using the brake-by-wire actuator have been studied widely in the vehicle brake research. They are able to control the braking force more accurately and can be adapted to different vehicles more easily than conventional ABS (Anti-lock Brake System) systems. With the wheel slip control system, the braking force can be accurately adjusted to various operating conditions and road conditions. By maintaining a specified tire slip for each wheel, the braking force can be maximized for the shortest stopping distance or can be optimally distributed for the vehicle stability.

In order to achieve the superior braking performance through the wheel slip control, the optimal target slip values need to be determined and real-time information such as the tire braking force at each wheel is required. The tire braking force can be calculated by the wheel dynamics equation if the braking torque can be measured or known. In the case of EHB (Electro-Hydraulic Brake), the tire braking torque cannot be measured directly and can be approximated based on the characteristics of the brake disk-pad friction. However, the friction characteristics can change significantly depending on aging of the brake, moisture on the contact area, heat, etc. Müller *et al.* (2001) demonstrate that the brake disk-pad friction can change more than 50% under various braking conditions. Yoon *et al.* (2004) developed a monitoring system to estimate the longitudinal tire force and disk-pad friction coefficient in real-time.

Various control methods have been reported for the wheel slip control. In the case of EMB (Electro-Mechanical Brake), the braking force is calculated from the measured braking torque and the wheel slip controller is designed based on the feedback linearization method (Semmler *et al.*, 2002). By assuming that the braking force is known (Buckholtz, 2002; Chun *et al.*, 2004) or the upper bound of the force is known (Kawabe *et al.*, 1996), sliding mode controllers have been designed to control the wheel slip. Several control methods were also developed to adjust the brake torque as the control input for the wheel slip control systems (Johansen *et al.*, 2003; Schinkel and Hunt, 2002; Ünsal and Kachroo, 1999). In the case of EHB, the control input should be formulated in terms of braking pressure, not braking torque. An adaptive wheel slip control system for EHB has been proposed (Yi *et al.*, 2002) based on the estimated road friction coefficient, but it requires an accurate tire model. Kageyama *et al.* (1996)

**Corresponding author.* e-mail: khuh2@hanyang.ac.kr

obtain the m-slip curve based on the estimated parameters of a tire model and determine the slip value for the zero slope of the curve. Ono *et al.* (2003) estimate the slope of the m-slip curve and control the braking torque so that the slope becomes zero.

In this paper, a robust wheel slip controller is developed for EHB based on the adaptive sliding mode control method and an optimal target slip assignment algorithm. The dynamics of the wheel slip ratio is utilized to represent the dynamic relation between the braking pressure and the slip ratio. The dynamic relation includes not only the longitudinal tire force, but also the brake disk-pad friction coefficient. An adaptive law is formulated to estimate the longitudinal braking force in real-time. The wheel slip controller is designed using the Lyapunov stability theory for the robust control performance and considers the error bounds in estimating the braking force and the brake disk-pad friction coefficient. The target slip assignment algorithm is developed for the maximum braking force and searches for the optimal target slip value based on the estimated braking force. The controller determines the required pressure in the EHB master cylinder in order to maintain the target wheel slip ratio during braking. The performance of the proposed wheel-slip control system is verified in HILS (Hardware-In-the-Loop Simulator) experiments and demonstrates the effectiveness of the wheel slip control in various road conditions.

2. OVERALL STRUCTURE

Figure 1 shows the wheel slip control system proposed in this paper. The proposed system consists of the optimal target slip assignment algorithm, adaptive law, sliding mode controller and EHB actuator. The sliding mode controller calculates the pressure change as the control input to compensate the error between the target slip and the current slip ratio. The calculation of the control input requires the on-line information of the tire braking force which is estimated based on the adaptive law. The target slip ratio for maximizing the braking force is determined from the optimal target slip assignment algorithm utilizing the estimated braking force. The EHB unit includes a pressure servo type actuator and applies the brake pressure to a brake caliper.

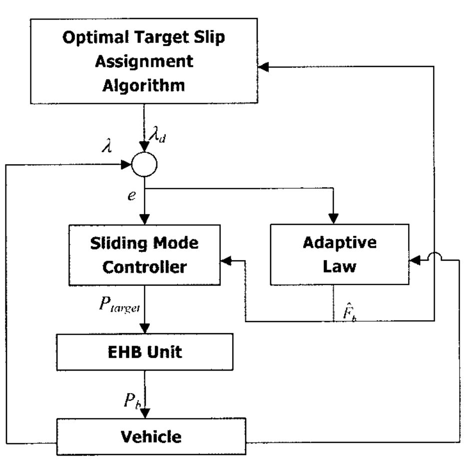

Figure 1. Block diagram of the proposed system.

3. WHEEL SLIP CONTROLLER DESIGN

3.1. Slip Dynamics

When a brake is engaged in the wheel, the braking force is generated due to friction between a tire and road surface as illustrated in Figure 2. In the case of the hydraulic disk brake as shown in Figure 3, the braking torque is determined from the brake parameters and brake pressure.

$$T_b=2\cdot P_b A_p R_b \mu_b \tag{1}$$

where T_b : braking torque
P_b : hydraulic braking pressure
A_p : piston area
R_b : effective radius between the center of disk rotor and pad
μ_b : brake disk-pad friction coefficient

The wheel dynamics equation can be expressed as follows by substituting the above relation.

$$J_w\dot{\omega} = -T_b + r_w F_b = -2\cdot P_b A_p R_b \mu_b + r_w F_b \tag{2}$$

where ω denotes the rotational wheel velocity, J_w is the moment of inertia of wheel, r_w is the wheel radius, and F_b is the braking force, respectively.

The wheel slip generated between a tire and road

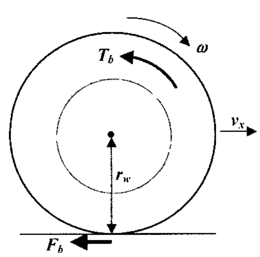

Figure 2. Force diagram of the wheel during braking.

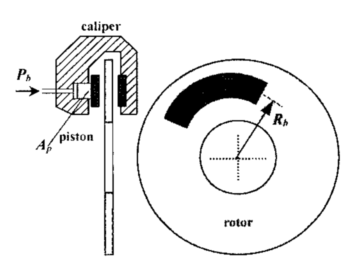

Figure 3. Hydraulic disk brake.

surface during braking is defined as the relative difference between the vehicle longitudinal velocity and the wheel velocity.

$$\lambda=\frac{v_x - r_w\omega}{v_x} \tag{3}$$

where λ is the slip ratio and v_x is the vehicle longitudinal velocity.

By differentiating Equation (3) and substituting Equation (2), the slip dynamics equation can be obtained.

$$\dot{\lambda} = -\frac{\dot{v}_x}{v_x}(\lambda-1) - \frac{r_w}{J_w v_x}\left(r_w F_b - 2A_p R_b \mu_b P_b\right) \tag{4}$$

3.2. Adaptive Sliding Mode Control

In this study, the adaptive sliding mode control technique based on Lyapunov stability theory is applied in designing the wheel slip controller. The adaptive law is derived to estimate the tire braking force and the control law is formulated such that its performance is robust to the estimation error in braking force and disk-pad friction coefficient. The control objective is to drive the wheel slip, λ, to the target wheel slip, λ_d, by applying the EHB actuator pressure as the control input.

The sliding surface, s, can be defined as the error between the current slip and target slip.

$$s=\lambda-\lambda_d \tag{5}$$

The quadratic function of the sliding surface and of the braking force estimation error is chosen as a Lyapunov function in order to determine the control and adaptive laws.

$$V(s,\tilde{F}_b) = \frac{1}{2}s^2 + \frac{1}{2\gamma}\tilde{F}_b^2, \quad \tilde{F}_b = F_b - \hat{F}_b \tag{6}$$

where $\hat{F}_b$ denotes the estimated braking force. γ is the adaptation gain and has a positive value.

By differentiating Equation (6) and substituting Equation (4), the time derivative of Lyapunov function is calculated as follows:

$$\begin{aligned}\dot{V}(s,\tilde{F}_b) &= s\dot{s} + \frac{1}{\gamma}\tilde{F}_b\dot{\tilde{F}}_b = s\cdot\left(\dot{\lambda}-\dot{\lambda}_d\right) + \frac{1}{\gamma}\tilde{F}_b(\dot{F}_b - \dot{\hat{F}}_b) \\ &= s\cdot\left(-\frac{\dot{v}_x}{v_x}(\lambda-1) - \frac{r_w}{J_w v_x}\left(r_w F_b - 2\cdot A_p R_b \mu_b P_b\right)\right) + \frac{1}{\gamma}\tilde{F}_b(-\dot{\hat{F}}_b)\end{aligned} \tag{7}$$

The above equation can be modified into the following form.

$$\begin{aligned}\dot{V}(s,\tilde{F}_b) = s\cdot&\left(-\frac{\dot{v}_x}{v_x}(\lambda-1) - \frac{r_w^2}{J_w v_x}\hat{F}_b + \frac{r_w}{J_w v_x}2\cdot A_p R_b \mu_b P_b\right) \\ &- \frac{r_w^2}{J_w v_x}s\tilde{F}_b + \frac{1}{\gamma}\tilde{F}_b(-\dot{\hat{F}}_b)\end{aligned} \tag{8}$$

By selecting the control law and the adaptive law as follows,

Control Law

$$P_b = \frac{1}{2A_p R_b \mu_b}\left[\frac{J_w \dot{v}_x}{r_w}(\lambda-1) + r_w\hat{F}_b - \frac{J_w v_x}{r_w}k\,\mathrm{sgn}(s)\right] \tag{9}$$

Adaptation Law

$$\dot{\hat{F}}_b = -\gamma\frac{r_w^2}{J_w v_x}s = -\gamma\frac{r_w^2}{J_w v_x}(\lambda-\lambda_d) \tag{10}$$

the time derivative of Lyapunov function, Equation (8), becomes negative semi-definite and the Lyapunov stability condition is satisfied.

$$\dot{V}(s,\tilde{F}_b) = -ks\,\mathrm{sgn}(s) = -k|s| \tag{11}$$

where k is the control gain of the sliding mode controller.

In the control input, Equation (9), a nominal value of the brake disk-pad friction coefficient, $\mu_{b,n}$, is utilized and its uncertainty is considered later.

$$P_b = \frac{1}{2A_p R_b \mu_{b,n}}\left[\frac{J_w \dot{v}_x}{r_w}(\lambda-1) + r_w\hat{F}_b - \frac{J_w v_x}{r_w}k\,\mathrm{sgn}(s)\right] \tag{12}$$

As a result, the tire braking force is estimated from the adaptive law in Equation (10) and the braking pressure control input can be determined from the control law in Equation (12) to compensate the error between the current slip and target slip.

3.3. Robustness

In Equation (12), the control gain, k, should be designed such that the control performance is robust to the estimation errors in braking force and brake disk-pad friction coefficient. The control gain is determined from the following sliding condition.

$$\frac{1}{2}\frac{d}{dt}s^2 \le -\frac{\mu_b}{\mu_{b,n}}\eta|s| \tag{13}$$

where η is a design parameter and a strictly positive constant. Substituting Equation (4) into Equation (13) gives the following condition.

$$\begin{aligned}&s\dot{\lambda} \le -\eta|s| \\ &s\left[-\frac{\dot{v}_x}{v_x}(\lambda-1) - \frac{r_w}{J_w v_x}\left(r_w F_b - 2A_p R_b \mu_b P_b\right)\right] \le -\frac{\mu_b}{\mu_{b,n}}\eta|s|\end{aligned} \tag{14}$$

Using the control input pressure in Equation (12) and defining the control gain, k, as follows:

$$k=N+\eta \tag{15}$$

the following sliding condition is obtained after simplification.

$$s\left[-\frac{\dot{v}_x}{v_x}(1-\lambda)\left(\frac{\mu_b}{\mu_{b,n}}-1\right)+\frac{r_w^2}{J_w v_x}\left(\frac{\mu_b}{\mu_{b,n}}\hat{F}_b-F_b\right)\right] \le \frac{\mu_b}{\mu_{b,n}}N|s| \tag{16}$$

where N is a design parameter. Because the following factors have positive values during braking,

$$-\frac{\dot{v}_x}{v_x}\ge 0,\ (1-\lambda)\ge 0,\text{ and } \frac{r_w^2}{J_w v_x}\ge 0 \tag{17}$$

the sliding condition is satisfied with the following relation.

$$N \ge \frac{\mu_{b,n}}{\mu_b}\left(\frac{-\dot{v}_x(1-\lambda)}{v_x}\left|\frac{\mu_b}{\mu_{b,n}}-1\right|+\frac{r_w^2}{J_w v_x}\left|\frac{\mu_b}{\mu_{b,n}}\hat{F}_b-F_b\right|\right) \tag{18}$$

In the above relation, two uncertainties can be considered with respect to the estimated tire braking force and disk-pad friction coefficient. It is assumed that these uncertainties are bounded,

$$\left|\frac{\mu_b}{\mu_{b,n}}-1\right|<B_1 \text{ and } \left|\frac{\mu_b}{\mu_{b,n}}\hat{F}_b-F_b\right|<B_2 \tag{19}$$

where B_1 and B_2 indicate the bound values of the uncertainties. The design parameter, N, and the control gain, k, can be selected for the robust control performance.

$$N=\frac{1}{1+B_1}\left(\frac{-\dot{v}_x(1-\lambda)}{v_x}B_1+\frac{r_w^2}{J_w v_x}B_2\right) \tag{20}$$

$$k=\frac{1}{1+B_1}\left(\frac{-\dot{v}_x(1-\lambda)}{v_x}B_1+\frac{r_w^2}{J_w v_x}B_2\right)+\eta \tag{21}$$

By substituting Equation (21) into Equation (12), the brake pressure can be determined as the control input.

In order to reduce the chattering due to the discontinuous switching of sgn(•), the discontinuous switching is replaced with continuous switching, sat(•). Then, the control input can be formulated as follows.

$$P_b=\frac{1}{2A_pR_b\mu_{b,n}}\cdot \left[\frac{J_w\dot{v}_x}{r_w}(\lambda-1)+r_w\hat{F}_b - \frac{J_w v_x}{r_w}\left\{\frac{1}{1+B_1}\left(\frac{-\dot{v}_x(1-\lambda)}{v_x}B_1+\frac{r_w^2}{J_w v_x}B_2\right)+\eta\right\}\mathrm{sat}\left(\frac{s}{\Phi}\right)\right] \tag{22}$$

where Φ is a design parameter representing the boundary layer around the s=0 sliding surface and has a small positive value.

4. OPTIMAL TRAGET SLIP ASSIGNMENT

In order to obtain the smallest stopping distance with maintaining the steerability during braking, optimal target slip needs to be determined for maximizing the braking. Figure 4 shows the typical relation between the braking force and wheel slip ratio on normal road. As shown in Figure 4, the braking force with respect to the slip ratio is a convex function and the maximum braking force is generated at the slip range between 10% and 20%. The general method to search the optimal point can be expressed as follows:

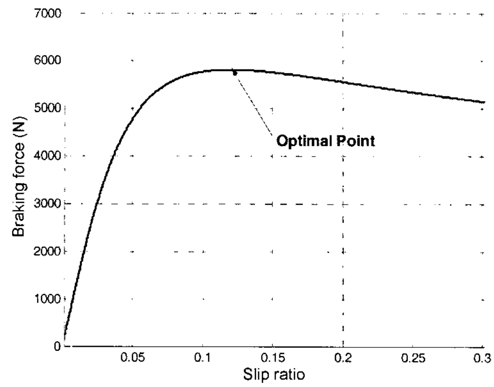

Figure 4. Braking force w.r.t slip ratio.

$$\lambda_d(k+1)=\lambda_d(k)+\alpha_k d(k) \tag{23}$$

where α_k is a step size and $d(k)$ is a search direction.

In this paper, the saturation function of the relative braking force difference is proposed as a search direction for the optimal point.

$$d(k)=\mathrm{sat}\left(\frac{\Delta F_{br}(k)}{c}\right)$$
$$\Delta F_{br}(k)=\frac{\hat{F}_b(k)-\hat{F}_b(k-1)}{\hat{F}_b(k)} \tag{24}$$

where c is a design parameter and has a positive value. $\hat{F}_b(k)$ is the estimated braking force explained in the previous section. The proposed search direction results in searching toward a point where the variation of the braking force becomes zero.

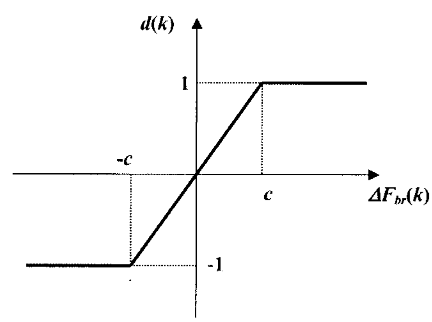

Figure 5. Search direction.

However, the real curve of the braking force with respect to the slip ratio is expected very sensitive to noise in real circumstance. Therefore, it is difficult to calculate the braking force difference accurately from the second equation in Equation (24) and to find the optimal point. In this study, the relative braking force difference is estimated using the recursive least square (RLS) method. The regression model of the relative braking force difference equation for RLS is expressed in Equation (25) and the estimation procedures are expressed in Equation (26).

$$y(k) = \varphi^T(k)\cdot\theta(k) \Rightarrow \begin{cases} y(k) = \hat{F}_b(k) - \hat{F}_b(k-1) \\ \varphi^T(k) = \hat{F}_b(k) \\ \theta(k) = \Delta F_{br}(k) \end{cases} \tag{25}$$

$$\begin{aligned} &\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[y(k) - \varphi^T(k)\hat{\theta}(k-1)] \\ &K(k) = P(k-1)\varphi(k)[FP\cdot I + \varphi^T(k)P(k-1)\varphi(k)]^{-1} \\ &P(k) = [I - K(k)\varphi^T(k)]P(k-1)/FP \end{aligned} \tag{26}$$

where FP represents a forgetting factor.

Consequently, the target slip for maximizing the braking force can be obtained from the following equation.

$$\lambda_d(k+1) = \lambda_d(k) + \alpha_k sat\left(\frac{\hat{\theta}(k)}{c}\right) \tag{27}$$

5. HILS EXPERIMENTS

5.1. Hardware-In-the-Loop Simulator

A Hardware-In-the-Loop Simulator (HILS) is built to evaluate the performance of the proposed wheel slip control system. Figure 6 shows the structure of the EHB HILS. The constructed EHB HILS is shown in Figure 7.

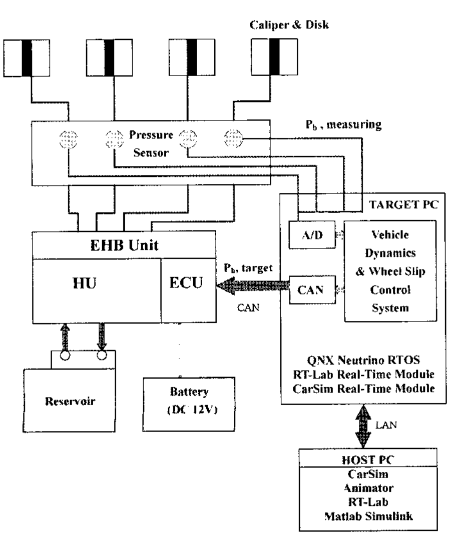

Figure 6. Structure of EHB HILS.

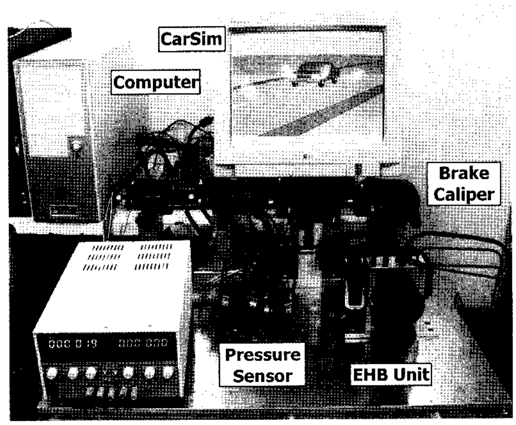

Figure 7. EHB HILS.

Table 1. Vehicle parameters.

Para.	Value	Para.	Value
m	1707 kg	J_w	0.9 kgm^2
I_z	2741.9 kgm^2	r_w	0.301 m
l_f	1.014 m	$A_{p1,2}$	3931.848 mm^2
l_r	1.6760 m	$A_{p3,4}$	2670.227 mm^2
t_f	0.770 m	$R_{b1,2}$	0.109 m
t_r	0.765 m	$R_{b3,4}$	0.107 m

As shown in Figure 6, the EHB HILS is composed of hardware (EHB unit, brake caliper, pressure sensor, etc) and software (vehicle simulation tool, real-time OS, rapid prototyping tool and proposed wheel slip control system).

The EHB unit consists of the HU (Hydraulic Unit) and ECU (Electronic Control Unit). Its pressure servo type actuator adjusts the brake pressure and has the response speed of 50 MPa/sec. The CarSim™ (2004) software, a commercial vehicle simulation tool, is used to represent a real vehicle. The adaptive wheel slip controller and the optimal target slip assignment algorithm are included using the CarSim™ and Simulink®. The real-time HILS experiments are conducted using the QNX® Neutrino® RTOS (Real-Time Operating System), CarSim™ real-time module and RT-LAB™ (2002) program which is a rapid prototyping tool. The calculated brake pressure is transmitted to the EHB ECU using the CAN (Controller Area Network) communication. The brake caliper pressure is measured using the pressure sensor and its value is sent to the controller. Table 1 shows the vehicle parameters used in experiments.

5.2. Experiment Results

The experiment results on normal road are shown in Figures 8–10. The estimation result of the tire braking force is illustrated in Figure 8 and show a little steady-

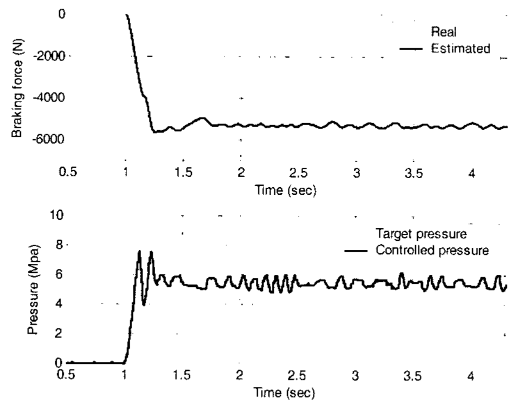

Figure 8. Braking force and pressure on normal road.

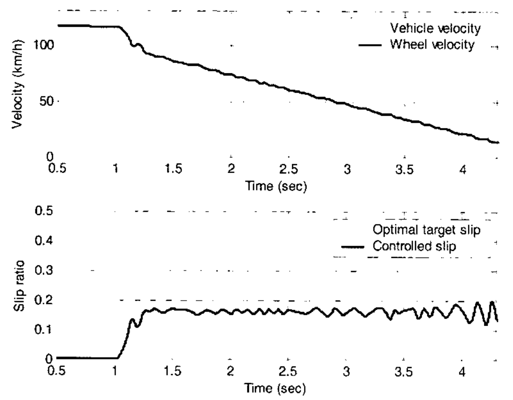

Figure 9. Velocity and wheel slip on normal road.

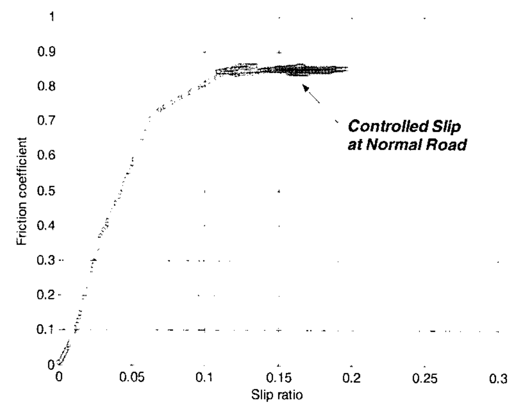

Figure 10. μ-slip curve on normal road.

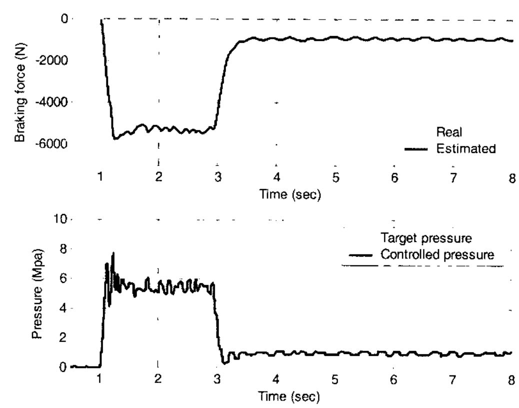

Figure 11. Braking force and pressure on μ-change road.

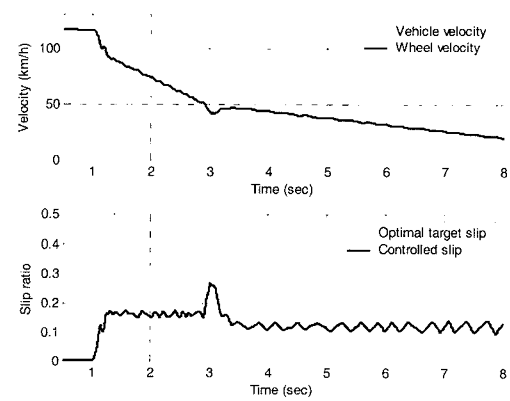

Figure 12. Velocity and wheel slip on μ-change road.

state error. The target pressure from the proposed wheel slip control system and the controlled pressure by the EHB unit are also illustrated in Figure 8. The available maximum pressure of the EHB unit is 15 Mpa and it is confirmed that the sizes of the control input is within the pressure limit of the EHB unit. The vehicle/wheel velocities and the wheel slip control result are shown in Figure 9. As shown in Figure 9, the optimal target slip on normal road is determined as about 16%. The wheel slip follows the target slip value accurately even with the braking force estimation error. The μ-slip curve during this experiment is illustrated in Figure 10. The wheel slip is maintained around the optimal point for maximizing the braking force.

Figures 11–13 show the experiment results on μ-change road, where the road friction coefficient is changed from the normal condition to the slippery condition at 2.9 sec simulation time. The estimation result of the tire braking force and the control input braking pressure are illustrated in Figure 11. Figure 12 shows the vehicle/wheel velocities and the wheel slip control result. The optimal target slip is reduced from 16% to 12% when the road condition is changed from the normal road to the slippery road. The wheel slip follows the target slip values even with the road condition variation. The μ-slip curve during this maneuvering is illustrated in Figure 13. The wheel slip is maintained around the optimal points for maximizing the braking force.

6. CONCLUSIONS

A wheel slip control system for maximizing the braking

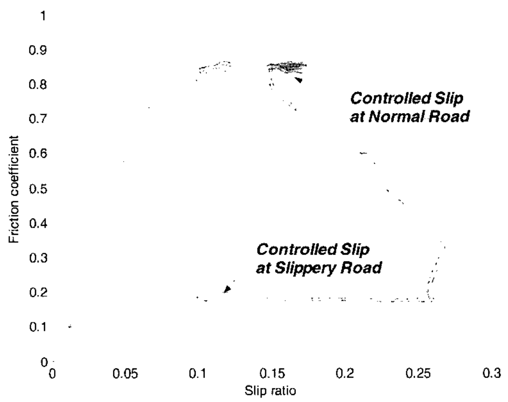

Figure 13. μ-slip curve on μ-change road.

force is developed for the EHB brake-by-wire system. The proposed wheel slip control system is composed of the adaptive sliding mode controller and the target slip assignment algorithm. Based on the Lyapunov stability theory, the adaptive law and the control law are designed for estimating the braking force and for calculating the control input braking pressure, respectively. The controller determines the size of the brake pressure as the control input utilizing the estimated tire braking forces and is designed to be robust to the estimation errors. The target slip assignment algorithm is proposed using the estimated braking force and searches for the optimal target slip corresponding to the maximum braking force. The EHB HILS is constructed in order to evaluate the proposed wheel slip control system and its performance is verified in HILS experiments. The experiment results demonstrate the effectiveness of the proposed system.

ACKNOWLEDGEMENT–The support of the Ministry of Commerce, Industry and Energy in Korea (No. 10005253) is gratefully acknowledged.

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