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MODELING TRANSMISSION ERRORS OF GEAR PAIRS WITH MODIFIED TEETH FOR AUTOMOTIVE TRANSMISSIONS

H. W. LEE[1)], M. W. PARK[1)*], S. H. JOO[2)], N. G. PARK[1)] and M. H. BAE[3)]

[1)]School of Mechanical Engineering, Pusan National University, Busan 609-735, Korea
[2)]GM Daewoo Auto & Technology Company, Changwon, Gyeongnam 641-745, Korea
[3)]Department of Automotive & Mechanical Engineering, Changwon College, Gyeongnam 641-773, Korea

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ABSTRACT−A tooth profile modification for loaded gears is used to avoid a tooth impact. Since a tooth profile error causes amplification of the cumbersome whine noise in automotive gear transmissions, an optimal quantity of tooth profile modifications must be obtained for good performance in the vibration sense. In this paper, a tooth profile modification curve considering profile manufacturing errors and elastic deformation of the gear tooth is formulated; in addition, transmission errors of the gear system with modified teeth are verified. The equivalent excitation due to transmission errors is formulated. For experimental evaluation of the transmission error, the transmission error for a simple gear system was measured by two rotational laser vibrometers. Finally, we perform a comparative analysis between the calculated and measured responses to the excitations due to the transmission error to verify the practicability of the application to automotive transmissions.

KEY WORDS : Transmission error, TE (Transmission error), Tooth profile modification, Transmission, Whine noise, Laser vibrometer

1. INTRODUCTION

The severe noise sensed by passengers inside a vehicle is the gear whine noise, which is generated by an excitation source of the gear system. This excitation source is a quantity of the differentiation from the ideal tooth profile in the mating gears and has a frequency band between 300 Hz and 3,000 Hz (Mitchell *et al.*, 1983). The major excitation sources of gear whine noise are known to be the transmission error (TE) due to tooth profile modifications, profile manufacturing error, and elastic deformation (torsion/bending of a shaft, housing deformation and gear tooth deformation) of the components caused by mean transmission torque. The excitation frequency is a high gear mesh frequency. Kubo (1990) investigated the relationship between gear noise and profile error curves by measuring TE for the curves modularized for 4 types of transmission gears. KIM *et al.* (2007) showed that the modification of the tooth profile yields a reduction of TE, which is Mark (1987) analyzed TE considering pitch error by utilizing a gear mesh analysis and a kinematical analysis for TE from a profile manufacturing error. Graber (1994) carried out a comparative analysis both for experimental results and analytical results for TE of spur gears and helical gears with several tooth profile modifications and gearing rates. Choi *et al.* (1990) studied the relationship between TE due to tooth stiffness and tooth deformation of spur gears and helical gears under loads. Smith (1990) suggested the method of using rotary encoders and tangential accelerometers to measure TE, while Kato (1993) suggested the method of using laser velocimeters. Munro (1990) determined that TE due to tooth deformation of a loaded spur gear is the major noise and vibration source, and he expressed TE measured as a time and frequency domain response. Munro *et al.* (1990) studied the optimum quantities and ranges of tooth profile modifications in the profile direction to avoid tooth impacts due to tooth deformation of loaded spur gears. Tavakoli *et al.* (1983) studied optimum values of tooth profile modifications to minimize TE of spur gears.

In this paper, formulation of the tooth modification curve considering profile manufacturing errors and loading deformations of the gear tooth is suggested and the TE of the gear system with modified teeth is evaluated. The model of the tooth modification curve is formulated as a B-spline curve so that it is practical to use in industrial sites and its application is easier than other models. A set of gear pairs is mathematically modeled and the equivalent excitation in the gear vibration model is formulated. For experimental evaluation of the derived TE func-

**Corresponding author.* e-mail: munu@pusan.ac.kr

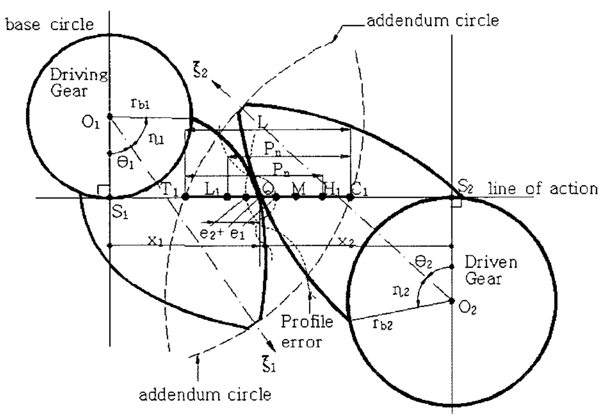
tion, a simple geared system has been set up in which the gears are designed to have the pre-designed tooth profile modifications. The geared system was manufactured by CNC Wire Cutting Machine. Under slow speed operation, TE of the gear pair has been measured by two rotational laser vibrometers and compared with the calculated value. In addition, to verify a practicability for the application to automotive transmissions, we perform a comparative analysis between the calculated and measured responses to the excitations due to the transmission errors.

2. MODELLING OF TRANSMISSON ERROR

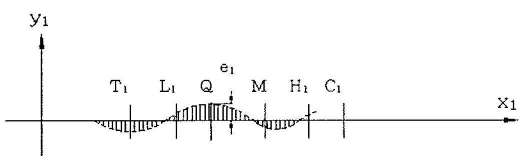
2.1. Formulation of Tooth Curve

To avoid tooth impact on a loaded gear, removing material at the tip and/or root of the involute tooth profiles is considered. This process is known as tooth profile modification, which is the modification in the profile and lead directions.

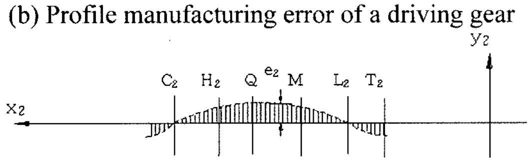
In this paper, the modification in the profile direction, which is the pressure angle modification, is considered and the tooth modification curves considering manufacturing errors (profile errors) and tooth deformation are determined. The first objective of the tooth modification curves is to set a contact zone for the teeth located in the middle of the face width. After defining the range of tooth profile modifications, the tooth modification curves are designed to locate the objective tooth curves in this range.



(a) Schematics of involute gears



(b) Profile manufacturing error of a driving gear



(c) Profile manufacturing error of a driven gear

Figure 1. Kinematical relationship of a tooth modified gear system.

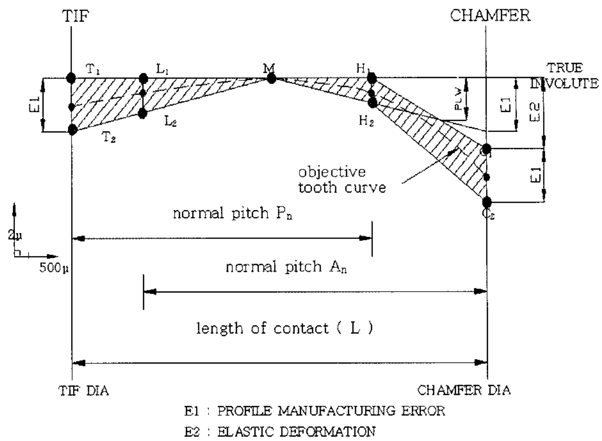


Figure 2. Range of the tooth modification in the profile direction.

Figure 1 shows the kinematical relationship of a tooth modified gear system. Two involute tooth profiles are tangent to the line of action and in contact with the point Q. The point T_1, which is the intersection point of both the addendum circle and the line of action, is a starting point for the true involute form (TIF-Dia). Here, the contact zone is from T_1 to C_1.

Figure 2 shows the range of tooth profile modifications. This range is defined as several boundary lines from the mid-point of the teeth contact (MPTC) to the TIF-Dia and the diameter of chamfer (Chamfer-Dia) gotten relief as a profile error (E_1), from the highest point of single teeth contact (HPSTC) to Chamfer-Dia gotten relief as tooth deformation (E_2) and from H_2 to Chamfer-Dia gotten relief as E_1+E_2.

Considering the upper limit of the tooth curve, which is the B-spline curve passing through the points T_1, L_1, M, H_1 and C_1, and the lower limit of the tooth curve, which is the B-spline curve passing through the point T_2, L_2, M, H_2 and C_2, the objective tooth curve is defined as the mean value of both the upper limit of the tooth curve and the lower limit of the tooth curve. Thus, the profile error function $e_1(x_1)$ of a pinion (a driving gear) in contact with the point Q is defined as:

$$e_1(x_1)=A_1+B_1x_1+C_1x_1^2+D_1x_1^3+E_1x_1^4\,. \tag{1}$$

Similarly, the profile error function $e_2(x_2)$ of a gear (a driven gear) is defined as:

$$e_2(x_2)=A_2+B_2x_2+C_2x_2^2+D_2x_2^3+E_2x_2^4\,, \tag{2}$$

$$\text{where, } x_2=\overline{S_1S_2}-x_1\,. \tag{3}$$

In Figure 1, the rotation angle of the driving gear is at a counterclockwise angle from the horizontal line (O_1S_1), to the centerline of the tooth (ξ_1 axis), where the relationship between the distance from the point on the line of action to the point of teeth contact Q is:

$$x_1=r_{b1}(\theta_1+\eta_1),\ \theta_{10}\le\theta_1\le\theta_{11}, \tag{4}$$

$$\theta_{10}=\frac{S_1T_1}{r_{b1}}-\eta_1 \tag{5}$$

$$\theta_{11}=\frac{S_1C_1}{r_{b1}}-\eta_1. \tag{6}$$

Thus, the profile error functions of a gear pair are defined as a parameter of θ_1 from Equation (1)~(6).

2.2. Formulation of Transmission Error

The analytical rotation angle of the driven gear must be $(N_1/N_2)\theta_1$ when the driving gear rotates an angle of θ_1, but a difference in both the actual rotation angle θ_2 and the analytical rotation angle occurred. The difference is defined as TE $\Delta\theta_2$, which is expressed in the following equation:

$$\Delta\theta_2=\theta_2-\frac{N_1}{N_2}\theta_1. \tag{7}$$

In Figure 1, the tooth profiles of the mating gears with a purely involute curve are tangent to the point Q. Using the assumption that the driving gear is fixed, the modified teeth profiles have errors as a quantity of the modifications e_1, e_2 and the tooth profile is the modified one and not a purely involute curve. In addition, an error in the rotation angle of the driven gear occurrs as the sum of the errors. Thus, TE, $\Delta\theta_2$ can be expressed as follows:

$$\Delta\theta_2=\frac{e_t^{(i)}}{r_{b2}}, \tag{8}$$

where $e_t^{(i)}$ is e_1+e_2 as the i-th quantity of the modification for a gear pair and r_{b2} is a base circle of the driven gear. Substitution of Equations (2) and (3) into Equation (8) yields TE as a function of parameter θ_1.

Involute gears usually do not have a single tooth contact in the length of the contact. In general, a gearing ratio is defined as:

$$\varepsilon=N+\alpha. \tag{9}$$

In Figure 3, the gears have N+1 teeth in contact in the region of A and N teeth in contact in the region of B, where, N is an integer and α is a positive number less than 1.

While a pair of gears is rotating in mesh, mating teeth pass through the line of action. Since neighboring teeth pass at regular intervals in the length of contact, the sum of the modification quantity for a gear pair is the maximum value of each overlapped quantity of the modifications.

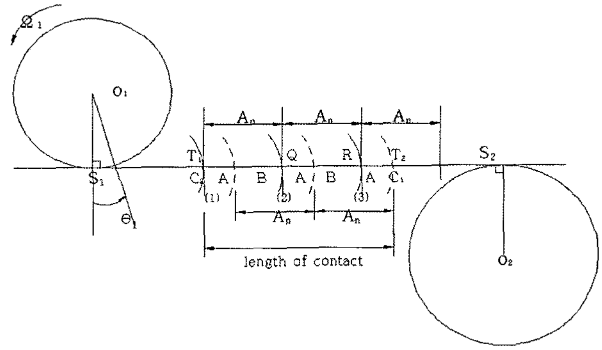


Figure 3. Range for multiple teeth in contact.

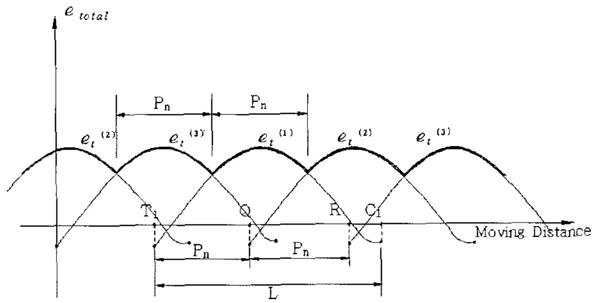


Figure 4. Range for multiple teeth in contact.

Figure 4 shows the sum of the modification quantity e_{total}. Thus,

$$e_{total}=\max\{e_t^{(1)}, e_t^{(2)}, e_t^{(3)},\dots, e_t^{(N)}\}. \tag{10}$$

The gears are driven at constant speed. We substitute $\theta_1=\Omega_1 t$, Equation (3) and Equation (4) into Equations (4), (1) and (2), respectively. Combining Equations (8) and (10), we have $e_{total}(t)$, which is a periodic function with a period of $2\pi/Z_1\Omega_1$. Here, Ω_1 and Z_1 represent rotation speed and number of teeth on the driving gear, respectively.

2.3. Relationship between Transmission Error and Equivalent Excitation

While a pair of gears is rotating in mesh, the profile error takes a role of displacement excitation as the quantity of the difference from a pure involute curve in the direction of the line of action. Thus, we verify the relationship between TE and equivalent excitation, considering a mathematical model of a gear pair which is shown in Figure 5. We suppose that the effect of tooth deformation is equivalent to springs in the direction of the line of action.

Figure 6 shows a dynamic model of a gear pair. The point C is an estimated contact point of the pure involute profiles without the modifications. Since the tooth profile modifications make displacement on the contact point act forcibly, the excitation is a displacement one. Thus, an

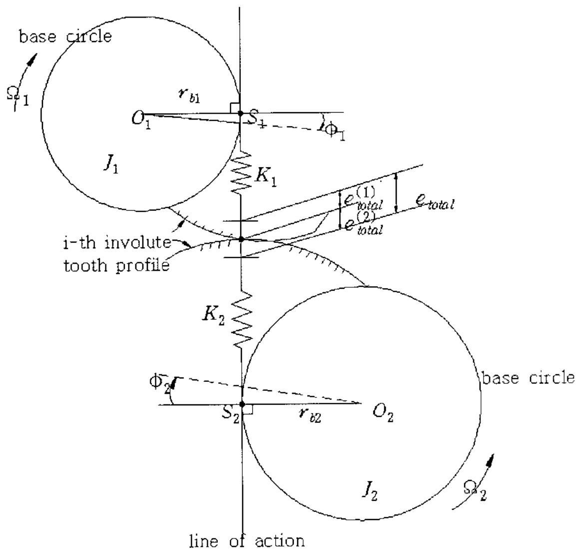


Figure 5. A mathematical model of the gear pair.

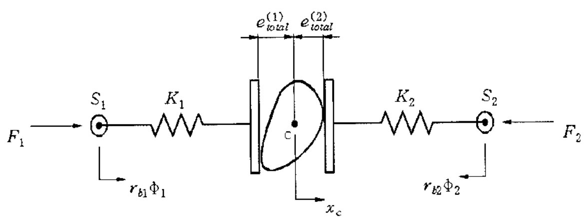


Figure 6. A dynamic model of the gear pair.

ideal cam mechanism at the point C is considered and we suppose that the compressible forces acting on the points S_1 and S_2 are F_1 and F_2, respectively. Then, we have as follows:

$$F_1=K_1(r_{b1}\phi_1 + e_{total}^{(1)} - x_c) \tag{11}$$

$$F_2=K_2(r_{b2}\phi_2 + e_{total}^{(2)} - x_c) \tag{12}$$

$$F_1=F_2\,. \tag{13}$$

Combining Equations (11), (12) and (13) and eliminating x_c, we have as follows:

$$F_1=F_{eq}(r_{b1}\phi_1+r_{b2}\phi_2+e_{total})\,. \tag{14}$$

Moment equilibrium equations between the driving gear and driven gear shown in Figure 5 are defined as:

$$J_1\ddot{\phi}_1+r_{b1}^2K_{eq}\phi_1+r_{b1}r_{b2}K_{eq}\phi_2=T_1(t) \tag{15}$$

$$J_2\ddot{\phi}_3+r_{b1}r_{b2}K_{eq}\phi_1+r_{b1}^2K_{eq}\phi_2=T_1(t) \tag{16}$$

Excitation torques ($T_1(t)$ and $T_2(t)$) are expressed as equivalent torques due to TE in the driving gear and driven gear, respectively. The torques are given as:

$$T_1(t)=r_{b1}K_{eq}e_{total}(t)=r_{b1}r_{b2}K_{eq}\Delta\theta_2 \tag{17}$$

$$T_2(t)=r_{b2}K_{eq}e_{total}(t)=r_{b2}^2K_{eq}\Delta\theta_2 \tag{18}$$

Considering the simple mathematical model shown in Figure 5 and applying Equations (17) and (18) to this model, the excitations due to TE are in proportion to TE of the gears. This result can be applied to bending and axial vibration of complicated gear systems since transmission forces of teeth contact defined as reaction forces due to displacement excitation and elastic force due to the tooth modifications are acting as not only rotation torque but radial or axial force of the axis.

3. CALCULATION AND EVALUATION OF TRANSMISSION ERROR

3.1. Development of Program

TE is quantitatively calculated by input parameters that are specifications of a gear pair, rotation speed, profile error and tooth deformation. Because this is a complicated kinematical calculation, further development of a practical program is needed.

In this paper, we develop the program to calculate TE as a time and frequency domain response. A flow chart of the program is shown in Figure 7.

3.2. Experimental Evaluation

Evaluation of TE needs synchronous sensors to measure the rotating displacement of the driving gear and driven gear. A laser vibrometer is an effective piece of equipment for conducting non-contact measurements, and it can scan pure rotation displacement of an object with a deviation from roundness. Figure 8 shows the principles

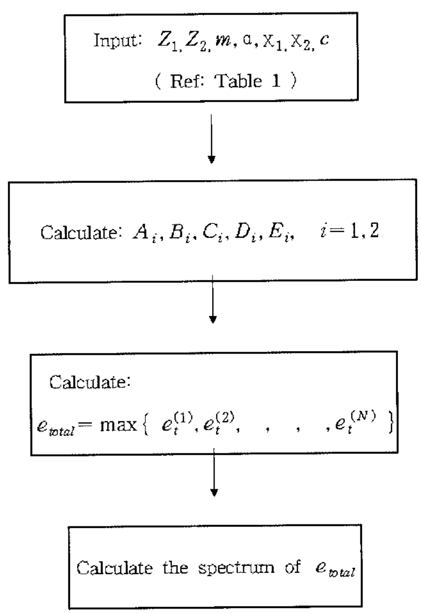


Figure 7. A flow chart of the program.

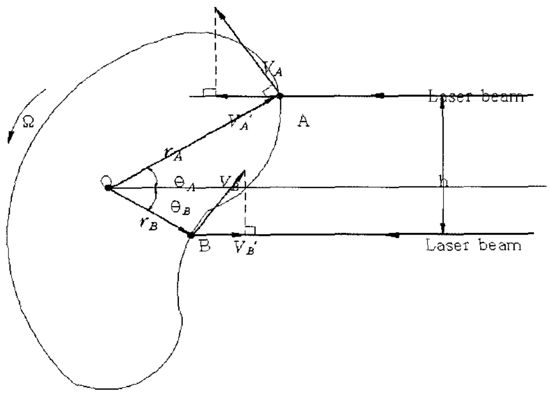


Figure 8. The principle of a laser vibrometer.

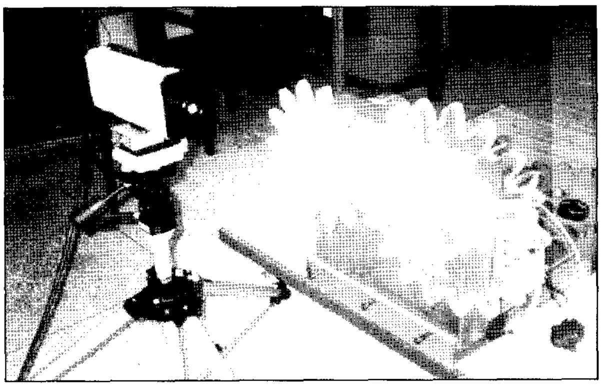


Figure 9. A test-rig to measure the transmission error.

of a laser vibrometer. As signals are detected from reflected light of two beams with tangential velocities V_A' and V_B' at the scanning points A and B, the following is satisfied:

$$V_A' - V_B' = \Omega h. \tag{19}$$

If distance and horizontality between the two beams are kept, we can check the pure rotation speed of an object with deviation from roundness.

To manufacture the gear system for experimental evaluation, a numerical solution for the modification curves with a predefined modification quantity was established and a pair of gears was machined by CNC Wire Cutting Machine.

Figures 9 and 10 show a test-rig which consists of a pair of gears, a DC servo motor, a flat belt-pulleys mechanism, a friction brake, two sets of laser vibrometers and a FFT. The pair of gears was mounted on a very stiff structure. The driving shaft was connected to the DC servo motor by the flat belt-pulleys mechanism and the driven shaft was connected to the friction brakes for a braking force.

Two rotational laser vibrometers were used to measure vibration angles of the driving and driven shafts. Reflection tape was put on a rotating reflection zone to make light reflected from the shafts clear. We measured the TE by recoding the vibration angle using a 2-channel FFT.

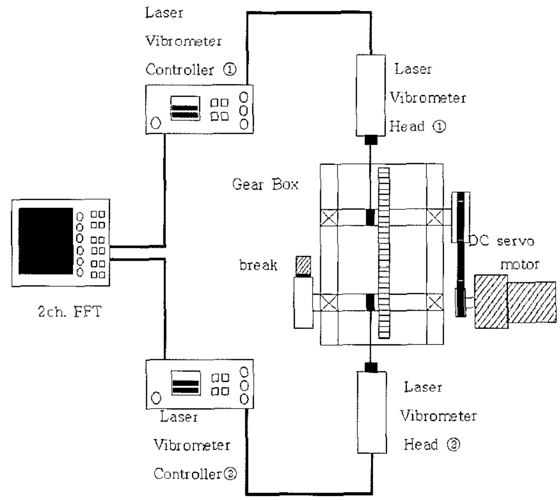


Figure 10. A schematic of the test-rig.

Table 1. Specifications of the gear pair for experiment.

Input parameters	Driving gear	Driven gear
Number of teeth (Z)	24	12
Module (m)	10 (mm)	10 (mm)
Tool pressure angle (α)	20°	
Addendum of the rack (A_r)	1.25m	1.25m
Radius of the rack tip (r_{rt})	0.3m	0.3m
Addendum of the gear (a)	m	m
Working center distance (c)	184 (mm)	
Backlash (B)	0.3 (mm)	
Pitch radius (R_p)	120.00 (mm)	60.00 (mm)
Standard center distance (c_s)	180.00 (mm)	
Working pitch radius (R_{pb})	122.67 (mm)	61.33 (mm)
Working pressure angle (α)	23.2°	
Profile shift value (e)	0.00 (mm)	3.87 (mm)

Specifications of the gear pair used for the experiments are listed in Table 1. The modification quantity that applies to the gear pair is determined as in Figure 11. The tooth profiles of the driving and driven gear modified by the determined modification quantity are shown in Figure 12. Mesh of the modified tooth profiles was checked before machining the gears.

The driving gear was rotated at a slow speed of 30 rpm to minimize the effects of inertia and a light brake was set in the driven gear to remove signals from the teeth impacts. The time domain signals for the vibration angles of the driving and driven gears are shown in Figure 13.

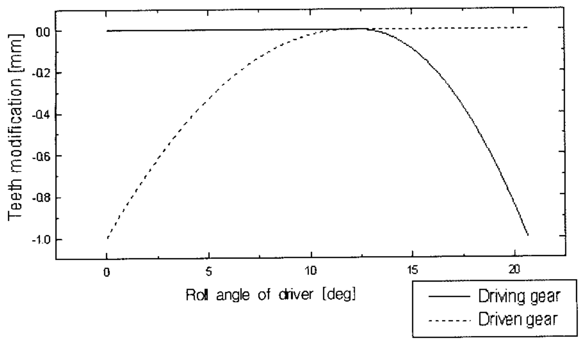
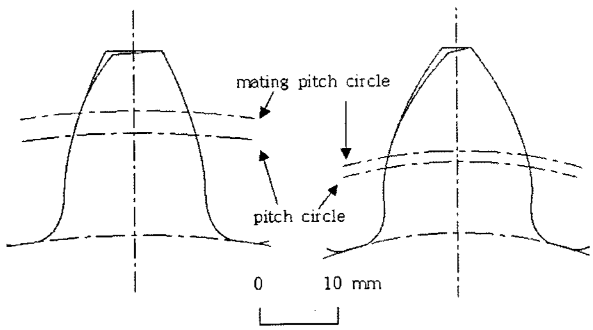


Figure 11. The tooth modification values of the two gears used in the experiment.



(a) Tooth shape of driving gear (b) Tooth shape of driven gear

Figure 12. The teeth shapes.

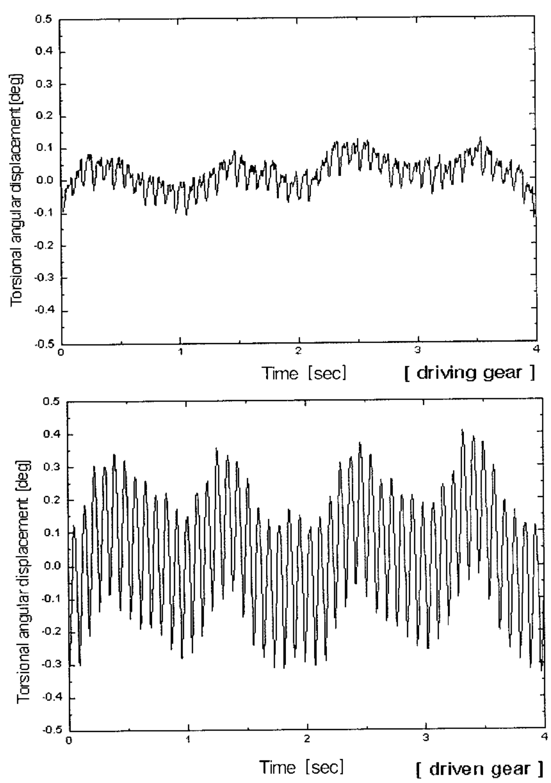


Figure 13. Angular displacement of the gears.

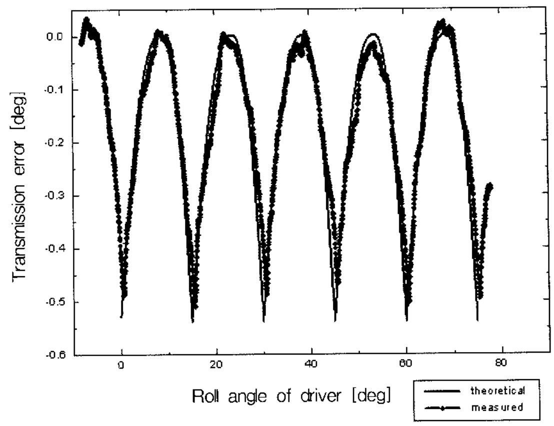


Figure 14. Transmission errors of the gear pair.

The mean rotation speed was irregular; that is, it was caused by a disturbance torque of the DC servo motor. The disturbance torque did not act as a disturbance signal because the self-excited signal was removed because the TE was defined as a relative displacement between the two gears.

A comparison of the TE calculated by the developed program and measured value is shown in Figure 14.

4. APPLICATION TO A COMMERCIAL AUTOMOTIVE TRANSMISSION

To verify practicability of the developed program for application to automotive transmissions, we analytically calculated a vibration response to the excitations due to the TE and performed a comparative analysis between the calculated and measured responses. This comparative analysis signifies that the gear whine noise can be reduced by prediction and analysis of the TE, which is the major excitation source of the noise.

4.1. Experiment on Excitations due to TE of Automotive Manual Transmission

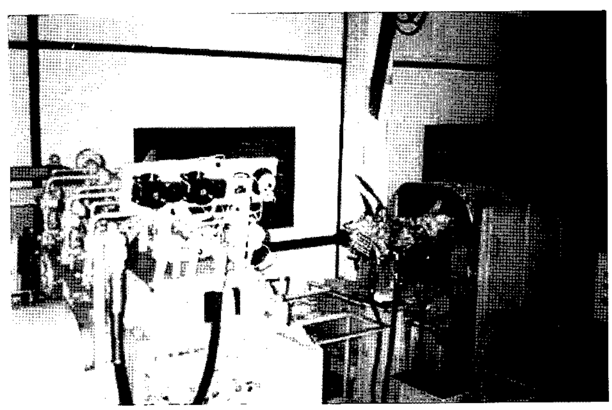


Figure 15. A test-rig of the automotive transmission.

Table 2. Specifications of the automotive manual transmission (3rd speed).

Gear no.	Node no.	No. of teeth	Module	Face width (mm)	Pressure angle (°)	Helix angle
G1	11	23	2.0	25.5	17.5	34°4'7" (LH)
G2	55	38	2.0	23.5	↑	34°4'7" (RH)
G3	21	26	2.25	23	↑	31°19'38" (LH)
G4	58	30	2.25	22	↑	31°19'38" (RH)

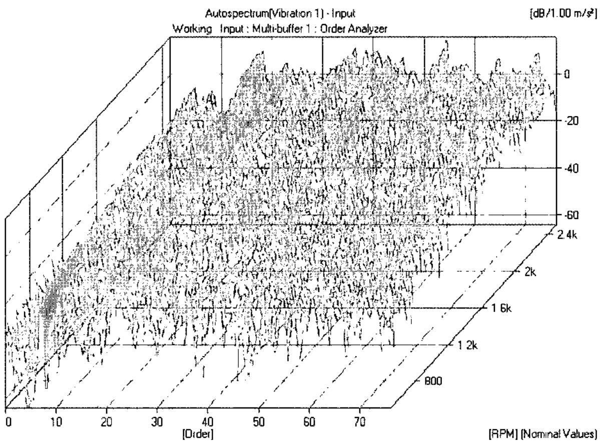


Figure 16. Waterfall diagram of the manual automotive transmission (3rd speed).

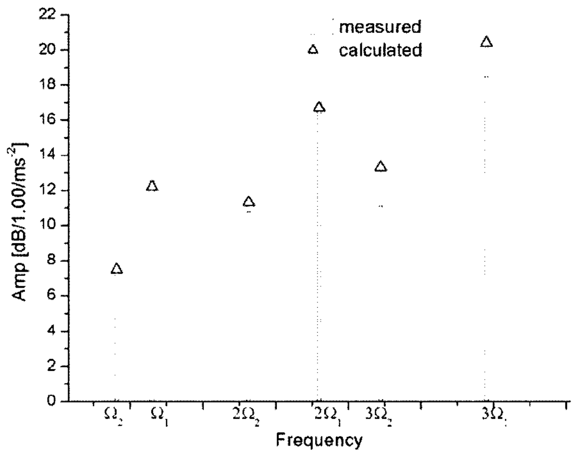


Figure 17. Spectrums of vibration responses due to TE (in the case of 2060 rpm).

A test-rig of the manual transmission for a rear wheel drive car is shown in Figure 15. The test-rig consists of a 120 kw motor, a dynamometer, a controller, two gearboxes and two torque meters. The controller of the test-rig adds the maximum torque of each speed of the transmission and increases rotation speed. A 3-axis accelerometer is attached to the upside of the transmission housing, and a tachometer is installed in the motor shaft. Vibration signals and rotation speeds are measured by the accelerometer and tachometer, respectively. Specifications of the transmission gears for 3rd speed are listed in Table 2.

Figure 16 shows the waterfall diagram of the transmission in 3rd speed, which illustrates that the measured frequency orders of the major vibration signals are 18.15X, 23X, 36.3X, 46X, 54.45X and 69X. The orders of 23X and 18.15X are the input and output gear mesh frequencies of the gear pairs P1 (Gears No. 1 and 2) and P2 (Gears No. 3 and 4), respectively. The orders of 36.3X, 46X, 54.45X and 69X are the harmonics, respectively.

Table 3. The values of the teeth modification.

Gear pair	Gear No.	The teeth modification values (μm)	
		E1	E2
P1	G1	11	15
	G2	11	15
P2	G3	19	25
	G24	19	25

4.2. Results of Comparative Analysis

Figures 17 and 18 illustrate the results of the comparative

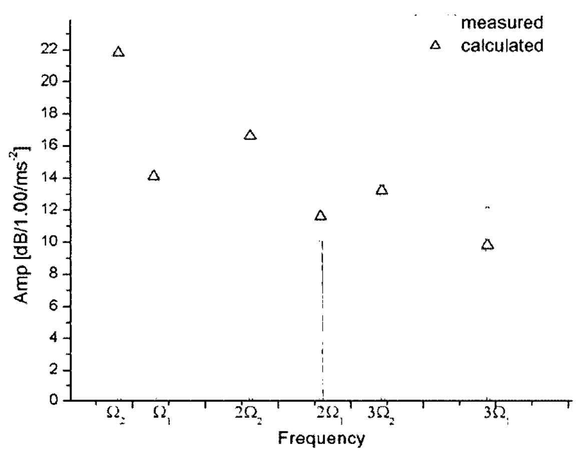


Figure 18. Spectrums of vibration responses due to TE (in the case of 2420 rpm).

analysis between the calculated and measured responses of the automotive transmission due to the TE. The analysis is carried out for the cases of critical speeds of 2060 and 2420 rpm. According to the results, the maximum difference between the calculated and measured responses is 20%, but trends of the changes of the input and output gear mesh frequencies and their harmonics show a relatively good agreement. We consider the causes of the difference in the responses are the assumption of the teeth modification values (listed in Table 3) which are used in industry, the transmission housing which is a rigid body. the viscosity effect of the gear lubrication oil that we disregarded.

Therefore, the developed program is practical to use to develop the optimal gear teeth profile for gear noise reduction by prediction and analysis of the TE, which is a major excitation source of the noise. In addition, this program can be good helpful in reduing costs and time periods for manufacturing automotive transmissions.

5. CONCLUSIONS

The formulation of tooth profile modifications and TE of gear systems was suggested for an effective reduction of the noise in automotive transmissions. The summary is expressed as follows:

(1) A program calculating the TE of the gear system by formulation of the tooth profile modification curve was developed.
(2) The equivalent excitation due to TE was formulated.
(3) TE of the gear pair was measured by two rotational laser vibrometers and compared with the calculated value, and the results show good agreement.
(4) The developed program is practical for application to automotive transmissions for gear noise reduction by prediction and analysis of the TE.

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