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INTUITIONISTIC FUZZY SEMIPRIME IDEALS OF A SEMIGROUPS

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ABSTRACT. We introduce the concept of intuitionistic fuzzy semiprimality of a semigroup which is an extension of semiprimality in it. And we obtain a characterization of a semigroup that is a semilattice of simple semigroups in terms of intuitionistic fuzzy semiprime interior ideals.

0. INTRODUCTION

The notion of fuzzy sets was introduced by Zadeh in [21]. Since its inception, the theory of fuzzy sets has developed in many directions and is finding applications in a wide variety of fields. In particular, Kuroki [16, 17] have applied the notion of fuzzy sets to semigroup theory.

In 1986, Atanassov [1] introduced the concept of intuitionistic fuzzy sets as the generalization of fuzzy sets. Recently, Çoker and his colleagues [5, 6, 7], Hur and his colleagues [12], and Lee and Lee [18] applied the notion of intuitionistic fuzzy sets to topology. In particular, Hur and his colleagues [11] applied one to topological group. Moreover, may researchers [2, 3, 8-10, 13, 14] applied the notion of intuitionistic fuzzy set to algebra.

In this paper, first we introduce the concept of intuitionistic fuzzy interior ideals of a semigroup S which is an extension of fuzzy interior ideals(and hence of interior ideals) of S, and obtain some properties of such ideals. Next we give the notion of intuitionistic fuzzy semiprimality of intuitionistic fuzzy set in a semigroup S which is also an extension of a semiprime subset of S. Finally we obtain a characterization of a semigroup that is a semilattice of simple semigroups in terms of intuitionistic fuzzy semiprime interior ideals.

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1. Preliminaries

We will list some concept and one result needed in the later sections.

For sets X, Y and Z, $f = (f_1, f_2) : X \to Y \times Z$ is called a *complex mapping* if $f_1 : X \to Y$ and $f_2 : X \to Z$ are mappings.

Throughout this paper, we will denote the unit interval $[0,1]$ as I. And for an ordinary subset A of a set X, we will denote the characteristic function of A as χ_A.

Definition 1.1 ([1, 5]). Let X be a nonempty set. A complex mapping $A = (\mu_A, \nu_A) : X \to I \times I$ is called an *intuitionistic fuzzy set* (in short, IFS) in X if $\mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$, where the mapping $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each $x \in X$ to A, respectively. In particular, $0_\sim$ and $1_\sim$ denote the *intuitionistic fuzzy empty set* and the *intuitionistic fuzzy whole set* in a set X defined by $0_\sim(x) = (0,1)$ and $1_\sim(x) = (1,0)$ for each $x \in X$, respectively.

We will denote the set of all IFSs in X as IFS(X).

Definition 1.2 ([1]). Let X be a nonempty sets and let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be IFSs in X. Then

(1) $A \subset B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.

(2) $A = B$ if and only if $A \subset B$ and $B \subset A$.

(3) $A^c = (\nu_A, \mu_A)$.

(4) $A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$.

(5) $A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$.

(6) $[\ ]A = (\mu_A, 1 - \mu_A)$, $< > A = (1 - \nu_A, \nu_A)$.

Definition 1.3 ([5]). Let $\{A_i\}_{i \in J}$ be an arbitrary family of IFSs in X, where $A_i = (\mu_{A_i}, \nu_{A_i})$ for each $i \in J$. Then

(1) $\bigcap A_i = (\wedge \mu_{A_i}, \vee \nu_{A_i})$.

(2) $\bigcup A_i = (\vee \mu_{A_i}, \wedge \nu_{A_i})$.

Let S be a semigroup. A nonempty subset A of S is called a *subsemigroup* of S if $A^2 \subset A$, and is called a *left* [resp. *right*] *ideal* of S if $SA \subset A$ [resp. $AS \subset A$]. By an *ideal*, we mean a subset of S which is both left and a right ideal of S. We will denote the set of all left ideals [resp. right ideals and ideals] of S as LI(S) [reps. RI(S) and I(S)]. A semigroup S is said to be *simple* if it contains no proper ideal. A subsemigroup A of S is called a *bi-ideal* of S if $ASA \subset A$. We will denote the set of all bi-ideals of S as BI(S). It is well-known [4, p. 84] that a semigroup S is a group if and only if it contains no proper bi-ideal.

Definition 1.4 ([8]). Let S be a semigroup and let $A \in$ IFS(S). Then A is called an :

(1) *intuitionistic fuzzy subsemigroup* (in short, *IFSG*) of S if $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$ for any $x, y \in S$.

(2) *intuitionistic fuzzy left ideal* (in short, *IFLI*) of S if $\mu_A(xy) \geq \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(y)$ for any $x, y \in S$.

(3) *intuitionistic fuzzy right ideal* (in short, *IFRI*) of S if $\mu_A(xy) \geq \mu_A(x)$ and $\nu_A(xy) \leq \nu_A(x)$ for any $x, y \in S$.

(4) *intuitionistic fuzzy (two-sided) ideal* (in short, *IFI*) of S if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right ideal of S.

We will denote the set of all IFSGs[resp. IFLIs, IFRIs and IFIs] of S as IFS(S) [resp. IFLI(S), IFRI(S) and IFI(S)].

Result 1.A ([8, Proposition 3.8]). *Let A be a non-empty subset of a semigroup S.*

(1) *A is a subsemigroup of S if and only if $(\chi_A, \chi_{A^c}) \in$ IFSG(S).*

(2) *A is a left[resp. right] ideal of S if and only if $(\chi_A, \chi_{A^c}) \in$ IFLI(S) [resp. IFRI(S)].*

(3) *A is an ideal of S if and only if $(\chi_A, \chi_{A^c}) \in$ IFI(S).*

Definition 1.5 ([13]). Let S be a semigroup and let $A \in$ IFSG(S). Then A is called an *intuitionistic fuzzy bi-ideal* (in short, *IFBI*) of S if

$$\mu_A(xyz) \geq \mu_A(x) \wedge \mu_A(z) \text{ and } \nu_A(xyz) \leq \nu_A(x) \vee \nu_A(z) \text{ for any } x, y, z \in S.$$

We will denote the set of all IFBIs of S as IFBI(S).

Result 1.B ([13, Proposition 2.5]). *Let A be a non-empty subset of a semigroup S. Then $A \in$ BI(S) if and only if $(\chi_A, \chi_{A^c}) \in$ IFBI(S).*

2. Intuitionistic Fuzzy Interior Ideals

A subsemigroup A of a semigroup S is called an *interior ideal* of S if $SAS \subset A$. We denote the *principal interior ideal* of S generated by $x \in S$ as $\mathrm{I}[x]$, i.e., $\mathrm{I}[x] = \{x, x^2\} \cup \{x\}$ (see [20]). We will denote the set of all interior ideals of S as $\mathrm{II}(S)$.

Definition 2.1. Let S be a semigroup and let $A \in \mathrm{IFSG}(S)$. Then A is called an *intuitionistic fuzzy interior ideal* (in short, *IFII*) of S if for any $x, a, y \in S$, $\mu_A(xay) \geq \mu_A(a)$ and $\nu_A(xay) \leq \nu_A(a)$.

We will denote the set of all IFIIs of S as $\mathrm{IFII}(S)$. It is clear that $\mathrm{IFI}(S) \subset \mathrm{IFII}(S)$. But the converse inclusion does not hold.

Example 2.2. Let $S = \{a, b, c, d\}$ be the semigroup with the following multiplication table :

	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	b

We define a complex mapping $A : S \to I \times I$ as follows :

$$A(a) = (0.7, 0.2),\ A(b) = (0, 1),\ A(c) = (0.3, 0.6),\ \text{and } A(d) = (0, 1)$$

Then we can see that $A \in \mathrm{IFII}(S)$ but $A \notin \mathrm{IFI}(S)$.

Theorem 2.3. *Let A be any nonempty subset of a semigroup S. Then $A \in II(S)$ if and only if $(\chi_A, \chi_{A^c}) \in IFII(S)$.*

Proof. ($\Rightarrow$): Suppose $A \in \mathrm{II}(S)$ and let $x, a, y \in S$.
Case (i) : Suppose $a \in A$. Since $A \in \mathrm{II}(S)$, $xay \in SAS \subset A$. Then $\chi_A(xay) = 1 = \chi_A(a)$ and $\chi_{A^c}(xay) = 0 = \chi_{A^c}(a)$.
Case (ii) : Suppose $a \notin A$. Then $\chi_A(xay) \geq 0 = \chi_A(a)$ and $\chi_{A^c}(xay) \leq 1 = \chi_{A^c}(a)$. So, in all, $\chi_A(xay) \geq \chi_A(a)$ and $\chi_{A^c}(xay) \leq \chi_{A^c}(a)$. It is clear that $(\chi_A, \chi_{A^c}) \in \mathrm{IFSG}(S)$ from Result 1.A(1). Hence $(\chi_A, \chi_{A^c}) \in \mathrm{IFII}(S)$.

($\Leftarrow$): Suppose $(\chi_A, \chi_{A^c}) \in \mathrm{IFII}(S)$. Let $z \in SAS$. Then there exist $x, y \in S$ and $a \in A$ such that $z = xay$. Since $a \in A$,

$$\chi_A(z) = \chi_A(xay) \geq \chi_A(a) = 1 \text{ and } \chi_{A^c}(z) = \chi_{A^c}(xay) \leq \chi_{A^c}(a) = 0.$$

Thus $\chi_A(z) = 1$ and $\chi_{A^c}(z) = 0$. So $z \in A$, i.e., $SAS \subset A$. Hence $A \in \mathrm{II}(S)$. This complete the proof. □

Remark 2.4. Let S be a semigroup S.

(1) If $A \in \text{IFII}(S)$, then μ_A and ν_A^c are fuzzy interior ideals of S.

(2) If $A \in \text{IFII}(S)$, then $[\ ]A, <> A \in \text{IFII}(S)$ by [6, Definition 1.2].

Theorem 2.5. *Let S be a semigroup and let $A \in IFS(S)$. Then $A \in IFII(S)$ if and only if* $1_\sim \circ A \circ 1_\sim \subset A$.

Proof. $(\Rightarrow)$: Suppose $A \in \text{IFII}(S)$ and let $a \in S$.

Case (i) : Suppose $(1_\sim \circ A \circ 1_\sim)(a) = (0, 1)$. Then clearly

$$1_\sim \circ A \circ 1_\sim \subset A.$$

Case (ii) : Suppose $(1_\sim \circ A 1_\sim)(a) \neq (0, 1)$. Then

$$\begin{aligned}
\mu_{1_\sim \circ A \circ 1_\sim}(a) &= \bigvee_{a=xy} [\mu_{1_\sim}(x) \wedge \mu_{A \circ 1_\sim}(y)] = \bigvee_{a=xy} \mu_{A \circ 1_\sim}(y) \\
&= \bigvee_{a=xy} \bigvee_{y=st} [\mu_A(s) \wedge \mu_{1_\sim}(t)] = \bigvee_{a=xst} \mu_A(s) \\
&\leq \bigvee_{a=xst} \mu_A(xst) \text{ (since } A \in \text{ IFII}(S)) \\
&= \mu_A(a)
\end{aligned}$$

and

$$\begin{aligned}
\nu_{1_\sim \circ A \circ 1_\sim}(a) &= \bigwedge_{a=xy} [\nu_{1_\sim}(x) \vee \nu_{A \circ 1_\sim}(y)] = \bigwedge_{a=xy} \nu_{A \circ 1_\sim}(y) \\
&= \bigwedge_{a=xy} \bigwedge_{y=st} [\nu_A(s) \vee \nu_{1_\sim}(t)] \\
&= \bigwedge_{a=xst} \nu_A(s) \geq \bigwedge_{a=xst} \nu_A(xst) = \nu_A(a).
\end{aligned}$$

Hence, in all, $1_\sim \circ A \circ 1_\sim \subset A$.

$(\Leftarrow)$: Suppose the necessary condition holds. Let $a, x, y \in S$ and let $z = xay$. Then, by the hypothesis,

$$\mu_{1_\sim \circ A \circ 1_\sim}(z) \leq \mu_A(z) \text{ and } \nu_{1_\sim \circ A \circ 1_\sim}(z) \geq \nu_A(z).$$

On the other hand,

$$\begin{aligned}
\mu_{1_\sim \circ A \circ 1_\sim}(z) &= \bigvee_{z=st} [\mu_{1_\sim}(s) \wedge \mu_{A \circ 1_\sim}(t)] \\
&\geq \mu_{1_\sim}(x) \wedge \mu_{A \circ 1_\sim}(ay) \text{ (since } z = xay)
\end{aligned}$$

$$= \mu_{A\circ 1_\sim}(ay) \text{ (since } \mu_{1_\sim}(x) = 1)$$
$$= \bigvee_{ay=pq} [\mu_A(p) \wedge \mu_{1_\sim}(q)] \geq \mu_A(a) \wedge \mu_{1_\sim}(y) = \mu_A(a)$$

and

$$\begin{aligned}\nu_{1_\sim \circ A \circ 1_\sim}(z) &= \bigwedge_{z=st} [\nu_{1_\sim}(s) \vee \nu_{A\circ 1_\sim}(t)] \leq \nu_{1_\sim}(x) \vee \nu_{A\circ 1_\sim}(ay) \\ &= \nu_{A\circ 1_\sim}(ay) = \bigwedge_{ay=pq} [\nu_A(p) \vee \nu_{1_\sim}(q)] \\ &\leq \nu_A(a) \vee \nu_{1_\sim}(y) = \nu_A(a).\end{aligned}$$

Thus $\mu_A(xay) = \mu_A(z) \geq \mu_A(a)$ and $\nu_A(xay) = \nu_A(z) \leq \nu_A(a)$. Hence $A \in$ IFII(S). This completes the proof. □

It is clear that any ideal of a semigroup S is an interior ideal of S. It is also clear that every IFI of S is an IFII of S. The following result shows that the converse holds for a regular semigroup. A semigroup S is said to be *regular* if for each $a \in S$, there exists an $x \in S$ such that $a = axa$.

Proposition 2.6. *Let S be a regular semigroup. Then IFII(S) $\subset$ IFSG(S).*

Proof. Let $A \in$ IFII(S) and let $a, b \in S$. Since S is regular, the exist $x, y \in S$ such that $a = axa$ and $b = byb$. Then $ab = (axa)(byb) = a(xaby)b$. Thus

$$\begin{aligned}\mu_A(ab) &= \mu_A(a(xaby)b) \geq \mu_A(xaby) \text{ (Since } A \in \text{ IFII}(S)) \\ &= \mu_A(xa(by)) \geq \mu_A(a) \text{ (Since } A \in \text{ IFII}(S))\end{aligned}$$

and

$$\begin{aligned}\nu_A(ab) &= \nu_A(a(xaby)b) \leq \nu_A(xaby) \\ &= \nu_A(xa(by)) \leq \nu_A(a).\end{aligned}$$

Similarly, we have $\mu_A(ab) \geq \mu_A(b)$ and $\nu_A(ab) \leq \nu_A(b)$. So $\mu_A(ab) \geq \mu_A(a) \wedge \mu_A(b)$ and $\nu_A(ab) \leq \nu_A(a) \vee \nu_A(b)$. Hence $A \in$ IFSG(S). □

Theorem 2.7. Let S be a regular semigroup and let $A \in$ IFS(S). Then $A \in$ IFI(S) if and only if $A \in$ IFII(S).

Proof. ($\Rightarrow$): It is clear.

($\Leftarrow$): Suppose $A \in$ IFII(S) and let $a, b \in S$. Since S is regular, there exist $x, y \in S$ such that $a = axa$ and $b = byb$. Then

$$\mu_A(ab) = \mu_A((axa)b) = \mu_A((ax)(ab)) \geq \mu_A(a) \text{ (since } A \in \text{IFII}(S)),$$
$$\nu_A(ab) = \nu_A((axa)b) = \nu_A((ax)(ab)) \leq \nu_A(a),$$

and

$$\mu_A(ab) = \mu_A(a(byb)) = \mu_A(ab(yb)) \geq \mu_A(b) \text{ (since } A \in \text{IFII}(S)),$$
$$\nu_A(ab) = \nu_A(a(byb)) = \nu_A(ab(yb)) \leq \nu_A(b).$$

Hence $A \in \text{IFI}(S)$. This completes the proof. □

Corollary 2.7. Let A be a subset of a regular semigroup. Then $A \in \text{I}(S)$ if and only if $A \in \text{I}(S)$.

Note that the semigroup S given in Example 2.2 is not regular and the IFII A of S is not an IFSG of S.

3. Simple Semigroups

A semigroup S is said to be *simple* if it contains no proper ideal.

Definition 3.1 ([13]). Let S be a semigroup. Then S is said to be *intuitionistic fuzzy simple* if every IFI of S is a constant complex mapping.

Result 3.A ([13, Proposition 7.3]). *Let S be a semigroup. Then S is simple if and only if S is intuitinistic fuzzy simple.*

Theorem 3.2. *Let S be a semigroup. Then S is simple if and only if Every IFII of S is a constant complex mapping.*

Proof. ($\Rightarrow$): Suppose S is simple. Let $A \in \text{IFII}(S)$ and let $a, b \in S$. Since S is simple, by Lemma I.3.9 in [19], there exist $x, y \in S$ such that $a = xby$. Since $A \in \text{IFII}(S)$,

$$\mu_A(a) = \mu_A(xby) \geq \mu_A(b) \text{ and } \nu_A(a) = \nu_A(xby) \leq \nu_A(b).$$

By the similar arguments, we have $\mu_A(b) \geq \mu_A(a)$ and $\nu_A(b) \leq \nu_A(a)$. Thus $A(a) = A(b)$. Hence A is a constant complex mapping.

($\Leftarrow$): Suppose the necessary condition holds. Let $A \in \text{IFI}(S)$. Then clearly $A \in \text{IFII}(S)$. By the hypothesis, A is a constant complex mapping. Thus S is intuitionistic fuzzy simple. Hence, by Result 3.A, S is simple. This complete the proof. □

4. INTUITIONISTIC FUZZY SEMIPRIMALITY

A subset A of a semigroup S is said to be *semiprime*[4, p.121] if $a^2 \in A$, $a \in S$ imply $a \in A$. We will denote the set of all semiprimes of S as SP(S).

Definition 4.1. Let S be a semigroup and let $A \in$ IFS(S). Then A is said to be *intuitionistic fuzzy semiprime*(in short, *IFSP*) if for each $a \in S$, $\mu_A(a) \geq \mu_A(a^2)$ and $\nu_A(a) \leq \nu_A(a^2)$.

We will denote the set of all IFSPs of S as IFSP(S).

The following shows that the concept of intuitionistic fuzzy semiprimality in a semigroup is an extension of semiprimality.

Theorem 4.2. *Let A be a nonempty subset of a semigroup S. Then $A \in SP(S)$ if and only if $(\chi_A, \chi_{A^c}) \in IFSP(S)$.*

Proof. ($\Rightarrow$): Suppose $A \in$ SP(S) and let $a \in S$.
Case (i) : Suppose $a^2 \in A$. Since $A \in$ SP(S), $a \in A$. Then

$$\chi_A(a) = 1 = \chi_A(a^2) \text{ and } \chi_{A^c}(a) = 0 = \chi_{A^c}(a^2).$$

Case (ii) : Suppose $a^2 \notin A$. Then

$$\chi_A(a) \geq 0 = \chi_A(a^2) \text{ and } \chi_{A^c}(a) \leq 1 = \chi_{A^c}(a^2).$$

Hence, in all, $(\chi_A, \chi_{A^c}) \in$ IFSP(S).
($\Leftarrow$): Suppose $(\chi_A, \chi_{A^c}) \in$ IFSP(S). Let $a^2 \in A$ and let $a \in S$. Since

$$(\chi_A, \chi_{A^c}) \in IFSP(S),$$

$$\chi_A(a) \geq \chi_A(a^2) = 1 \text{ and } \chi_{A^c}(a) \leq \chi_{A^c}(a^2) = 0.$$

Then $\chi_A(a) = 1$ and $\chi_{A^c}(a) = 0$. Thus $a \in A$. Hence $A \in$ SP(S). This completes the proof. □

Theorem 4.3. *Let S be a semigroup and let $A \in IFSG(S)$. Then $A \in IFSP(S)$ if and only if $A(a) = A(a^2)$ for each $a \in S$.*

Proof. ($\Leftarrow$): It is clear.
($\Rightarrow$): Suppose $A \in$ IFSP(S) and let $a \in S$. Then, by the hypothesis,

$$\mu_A(a) \geq \mu_A(a^2) \geq \mu_A(a) \wedge \mu_A(a) = \mu_A(a)$$

and

$$\nu_A(a) \leq \nu_A(a^2) \leq \nu_A(a) \vee \nu_A(a) = \nu_A(a).$$

So $A(a) = A(a^2)$. This completes the proof. □

Result 4.A ([15, Theorem 6.2]). *Let S be a semigroup. Then S is a union of groups if and only if every bi-ideal of S is semiprime.*

A semigroup S is said to be *completely regular* if for each $a \in S$ there exists an $x \in S$ such that $a = axa$ and $ax = xa$. It is clear that every completely regular semigroup is regular.

Result 4.B ([13, Proposition 5.2]). *Let S be a semigroup. Then the following are equivalent :*

(1) *S is completely regular.*

(2) *For each $A \in IFBI(S)$, $A(a) = A(a^2)$ for each $a \in S$.*

(3) *For each $B \in IFLI(S)$ and each $C \in IFRI(S)$, $B(A) = B(a^2)$ and $C(a) = C(a^2)$ for each $a \in S$.*

The following is the immediate result of Theorem 4.3, Results 4.A and 4.B.

Theorem 4.4. *Let S be a semigroup. Then the following are equivalent :*

(1) *S is a union of groups.*

(2) *Every bi-ideal of S is semiprime.*

(3) *Every IFBI of S is IFSP.*

A semigroup S is said to be *left*[resp. *right*] *regular* if for each $a \in S$ there exists an $x \in S$ such that $a = xa^2$ [resp. $a = a^2x$].

Result 4.C ([13, Propositions 5.1 and 5.1$'$]). *Let S be a semigroup. Then S is left[resp. right] regular if and only if for each $A \in IFLI(S)$ [resp. $IFRI(S)$], $A(a) = A(a^2)$ for each $a \in S$.*

The following is the immediate result of Theorem 4.3 and Result 4.C.

Theorem 4.5. *Let S be a semigroup. Then S is left [resp. right] regular if and only if for every IFLI [resp. IFRI] of S is IFSP.*

5. Intra-regular Semigroups

A semigroup S is said to be *intra-regular* if for each $a \in S$, there exist $x, y \in S$ such that $a = xa^2y$.

Such a semigroup is characterized as follows : The equivalence of (1), (2) and (3) is due to Theorem 4.4 of [4, p.123], and of (1) and (4) is due to [13, Proposition 4.1].

Lemma 5.1. *Let S be a semigroup. Then the following are equivalent.*

(1) *S is intra-regular.*
(2) *S is a union of simple semigroups.*
(3) *Every ideal of S is semiprime.*
(4) *For each $A \in IFI(S)$ and each $a \in S$, $A(a) = A(a^2)$.*

Lemma 5.2. *Let S be an intra-regular semigroup and let $A \in IFS(S)$. Then $A \in IFI(S)$ if and only if $A \in IFII(S)$.*

Proof. ($\Leftarrow$): It is clear.

($\Rightarrow$): Suppose $A \in$ IFII(S) and let $a, b \in S$. Since S is intra-regular, there exist $x, y, u, v \in S$ such that $a = xa^2y$ and $b = ub^2v$. Since $A \in$ IFII(S),

$$\mu_A(ab) = \mu_A((xa^2y)b) = \mu_A((xa)a(yb)) \geq \mu_A(a),$$
$$\nu_A(ab) = \nu_A((xa^2y)b) = \nu_A((xa)a(yb)) \leq \nu_A(a),$$

and

$$\mu_A(ab) = \mu_A(a(ub^2v)) = \mu_A((au)b(bv)) \geq \mu_A(b),$$
$$\nu_A(ab) = \nu_A(a(ub^2v)) = \nu_A((au)b(bv)) \leq \nu_A(b)$$

Thus $A \in$ IFI(S). This completes the proof. □

Corollary 5.2. *Let A be a subset of an intra-regular semigroup S. Then $A \in I(S)$ if and only if $A \in II(S)$.*

Theorem 5.3. *Let S be a semigroup and let $A \in IFS(S)$. Then the following are equivalent :*

(1) *S is intra-regular.*
(2) *Every IFI of S is IFSP.*
(3) *Every interior ideal of S is semiprime.*
(4) *Every IFII of S is IFSP.*
(5) *For each $A \in IFII(S)$ and each $a \in S$, $A(a) = A(a^2)$.*

(6) *For each* $a \in S$, $I[a] = I[a^2]$.

Proof. From Theorem 4.3 and Lemma 5.1, it is clear that (1) $\Leftrightarrow$ (2). It follows that (1) and (3) are equivalent from Lemma 5.1 and Corollary 5.2. It is clear that (2) implies (4) from Lemma 5.2. It follows from Theorem 2.3 and Theorem 4.2 that (4) implies (3). It is clear that (5) implies (4). From Theorem 4.3 it is clear that (4) implies (5). It follows from Corollary 5.2 and Theorem 4.4 in [4] that (1) implies (6). It is sufficient to show that (6) implies (1). Suppose that condition (6) holds and let $a \in S$. Then

$$a \in I[a] = I[a^2] = \{a^2, a^4\} \cup Sa^2S.$$

So S is intra-regular. This completes the proof. □

Proposition 5.4. *Let S be an intra-regular semigroup and let $A \in IFII(S)$. Then $A(ab) = A(ba)$ for any $a, b \in S$.*

Proof. Let $A \in \text{IFII}(S)$ and let $a, b \in S$. Then, by Theorem 5.3,

$$\mu_A(ab) = \mu_A((ab)^2) = \mu_A(a(ba)b) \geq \mu_A(ba) = \mu_A((ba)^2) = \mu_A(b(ab)a) \geq \mu_A(ab)$$

and

$$\nu_A(ab) = \nu_A((ab)^2) = \nu_A(a(ba)b) \leq \nu_A(ba) = \nu_A((ba)^2) = \nu_A(b(ab)a) \leq \nu_A(ab).$$

Hence $A(ab) = A(ba)$. □

A semigroup S is said to be *archimedean*[19, p.49] if for any $a, b \in S$, there exists a positive integer n such that $a^n \in SbS$.

Proposition 5.5. *Let S be an archimedean semigroup. Then every intuitionistic fuzzy semiprime ideal of S is a constant complex mapping.*

Proof. Let A be any intuitionistic fuzzy semiprime ideal of S and let $a, b \in S$. Since S is archimedean, there exist $x, y \in S$ such that $a^n = xby$ for some positive integer n. Then

$$\mu_A(a) = \mu_A(a^n) = \mu_A(xby) \geq \mu_A(b)$$

and

$$\nu_A(a) = \nu_A(a^n) = \nu_A(xby) \leq \nu_A(b).$$

By the similar arguments, we have $\mu_A(b) \geq \mu_A(a)$ and $\nu_A(b) \leq \nu_A(a)$. Thus $A(a) = A(b)$. Hence A is a constant complex mapping. □

Corollary 5.6. *Let S be an archimedean semigroup. Then it contains no proper semiprime ideal.*

Theorem 5.6. *Let S be a semigroup. Then S is simple if and only if S is intra-regular and archimedean.*

Proof. ($\Rightarrow$): Suppose S is simple. Then it follows from Result 3.A that every IFI of S is a constant complex mapping. Thus $A(a) = A(a^2)$ for each $a \in S$. So, by Lemma 5.1, S is intra-regular. Now let $a, b \in S$. Then, by Lemma I.3.9 in [19], there exist $x, y \in S$ such that $a^1 = a = xby$. So S is archimedean. Hence the necessary conditions hold.

($\Leftarrow$): Suppose the necessary conditions hold and let $A \in \mathrm{IFI}(S)$. Then, by Lemma 5.1, $A \in \mathrm{IFSP}(S)$. Since S is archimedean, by Proposition 5.5, A is a constant complex mapping. Thus S is intuitionistic fuzzy simple. Hence, by Result 3.A, S is simple. This completes the proof. □

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