Load Flow Analysis for Distribution Automation System based on Distributed Load Modeling

Xia Yang†, Myeon-Song Choi*, Il-Hyung Lim* and Seung-Jae Lee*

Abstract – In this paper, a new load flow algorithm is proposed on the basis of distributed load modeling in radial distribution networks. Since the correct state data in the distribution power networks is basic for all distribution automation algorithms in the Distribution Automation System (DAS), the distribution networks load flow is essential to obtain the state data. DAS Feeder Remote Terminal Units (FRTUs) are used to measure and acquire the necessary data for load flow calculations. In case studies, the proposed algorithm has been proven to be more accurate than a conventional algorithm; and it has also been tested in a simple radial distribution system.

Keywords: DAS, Distribution System, Load flow

1. Introduction

Power flow is a crucial part of power system design procedures, and it is categorized into transmission power flow and distribution power flow. The power flow problem consists of a given transmission network where all lines are represented by a Pi-equivalent circuit and transformers by an ideal voltage transformer in series with an impedance. Generators and loads represent the boundary conditions of the solution. In addition, generator or load real and reactive power involves products of voltage and current [1]. However, distribution networks commonly have some special features such as: insufficient information of loads; being radial with sometimes weakly-meshed topology; or high resistance to reactance r/x ratios. Those features need to be taken into account while carrying out the load flow analysis.

The conventional load flow methods developed essentially for solving transmission networks may encounter convergence problems when applied to distribution networks, due to their high r/x ratio, which deteriorates the diagonal dominance of the Jacobian matrix in the Newton method. Moreover, the loads in distribution networks are distributed and unknowable, since they are always varying with a series of change of daily condition. Therefore, some new methods for distribution systems should be discovered to help solve those issues.

So far, a significant amount of work has been carried out on load flow analysis in distribution networks, but very little work has been focused on the problem of the loads. Various three-phase load flow methods for distribution systems have been reviewed in [2], such as implicit Z-bus method, modified Gauss-Seidel method, network topology based method, forward-backward substitution method, and ladder network theory, etc. Forward sweeping method has been proposed for solving radial distribution networks by evaluating the total real and reactive power fed through any node [3]. Another method based on both the forward and backward sweep has been presented [4]. A simple method is based on the evaluation of an algebraic expression of receiving-end voltage [5]. A phase-decoupled load flow method ensures a fast convergence [6]. A modified Newton-Raphson method for power flows solution is presented in [7]. In [8], a unique three-phase power flow solution method is presented for large distribution systems.

On all accounts, those researches are based on concentrated load modeling, and all focus on the same objective, which is to determine the real and reactive powers flowing in each line and then to achieve the voltage magnitude and phase angle at each node.

In the proposed algorithm, due to the application of the Distribution Automation System (DAS), Feeder Remote Terminal Units (FRTUs) can automatically collect data provided for load flow calculations. Moreover, the method of four terminal constants is used to simplify those calculations. This analysis is based on a distributed load modeling in a distribution system, which is different from the concentrated load modeling. Thus, the results of case studies have demonstrated that the proposed algorithm is more accurate than a conventional one based on the concentrated load modeling; furthermore, the proposed algorithm has also been tested in a simple radial distribution system.

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2. Proposed Algorithm

2.1 Problem in Distribution Load Flow

The main objective of the distributed load flow is to discover the magnitude and phase angle of the voltage and current at each node in a power system. In DAS, Feeder Remote Terminal Units (FRTUs) play a critical role to measure and acquire the magnitude and phase angle of the voltage and current at each node. However, the problem is that the voltage data is incorrect with quite a large error whereas the current data is near true value. Therefore, the proposed algorithm contributes to solve this issue based on a new load modeling and the method of four terminal constants.

2.2 Overview

Fig. 1 shows a simple distribution system diagram which consists of 5 nodes with FRTU respectively. Given the magnitude and phase angle of voltage and current at node 0, those data at node 1 can be calculated through the proposed algorithm. Then the same process would be handled in the next section between nodes 1 and 2. As moving sections one by one, the magnitude and phase angle of voltage and current at each node would be obtained.

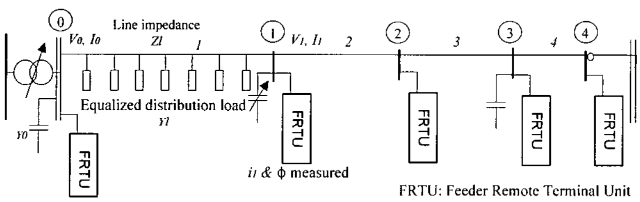

Fig. 1. A simple distribution system

The process in the first section between nodes 0 and 1 is taken out to be analyzed in detail.

At first, listing the given conditions below:

1. The phasor of voltage and current at node 0;
2. Line impedance in the first section;
3. The current magnitude and the angle of power factor at node 1, both data are measured by FRTU.

Then, the following data are supposed to be solved:

1. Distributed load admittance in the first section;
2. The magnitude and phase angle of voltage and current at node "1".

The purpose of the proposed algorithm is to compute the magnitude and phase angle of voltage and current at each node in the system.

2.3 Distributed Load Modeling

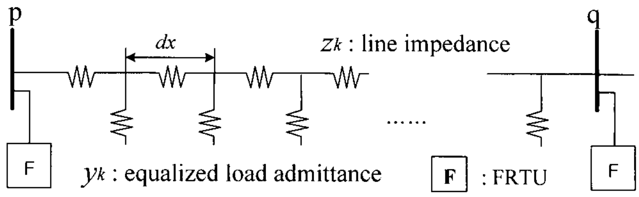

Fig. 2. Distributed load modeling in one-section feeder

Fig. 2 shows a new modeling of a distribution system that considers the load being equalized for distribution along the line.

The load information is very important for load flow analysis, especially when it comes to a complicated radial distribution system, where the different loads are distributed in the different lines. The consumption of electric power for loads is varying with the voltage level, so the load can affect the voltage magnitude. In the proposed algorithm, the modeling of the distributed load and the distributed current is utilized.

2.4 Process of Load Flow Calculation

The one-section modeling between nodes p and q is shown in Fig. 2. Line impedance zk and equalized load admittance yk are distributed at every dx in the line. The basic equations of voltage drop and current drop are illustrated as follows.

$$dV(x) = -I(x)zdx \tag{1}$$

$$dI(x) = -V(x)ydx \tag{2}$$

where, zdx is line impedance in the unit length, ydx is load admittance in the unit length.

Through a series of differential calculations, the solutions of (1) and (2) are obtained below.

$$V(x) = C_1 \cosh\gamma_k x + C_2 \sinh\gamma_k x \tag{3}$$

$$I(x) = C_3 \sinh\gamma_k x + C_4 \cosh\gamma_k x \tag{4}$$

where, $\gamma_k = \sqrt{z_k y_k}$ is the characteristic constant of the line.

Taking into account the boundary conditions at the source side, (3) and (4) are able to be further expressed below.

$$V(x) = V_p \cosh\gamma_k x - \frac{\gamma_k}{y_k} I_p \sinh\gamma_k x \tag{5}$$

$$I(x) = -\frac{y_k}{\gamma_k} V_p \sinh\gamma_k x + I_p \cosh\gamma_k x \tag{6}$$

Assuming Lk is the length of one section, in terms of (5) and (6), the phasor of voltage and current at the load side would be achieved in the case of $x=Lk$, which are shown in (7) and (8).

$$V_q = V_p \cosh\beta_k - Z_k I_p \sinh\beta_k \tag{7}$$

$$I_q = -\frac{V_p}{Z_k}\sinh\beta_k + I_p \cosh\beta_k \tag{8}$$

where, $Z_k = \sqrt{z_k / y_k}$, $\beta_k = \gamma_k L_k$

In (7) and (8), except the equalized load admittance yk, the other unknown variables can be computed.

If yk is obtained, Vq would be achieved easily. Thus, to get yk, some information at the load-side node is needed.

i) The current magnitude is measured by FRTU
ii) The phase angle difference between the voltage and the current is acquired by FRTU, and it is the same as the angle of power factor.

Moreover, there are two cases in the distribution line. The first case is the line connected with a feeder end; the second case is the line not connected with a feeder end. Two kinds of conditions should be respectively found out in accordance with both cases.

In the first case, the current at the end of the feeder is zero as revealed in (9).

$$I_q = -\frac{V_p}{Z_k}\sinh\beta_k + I_p \cosh\beta_k = 0 \tag{9}$$

Substituting (7) and (8) into (9), distributed load admittance yk can be computed through the Newton-Raphson method.

In the second case, the two equations shown in (10) and (11) are found out:

$$i_q^{\,2} = I_q \cdot I_q^{\,*} \tag{10}$$

$$(V_q I_q^{\,*})^2 = v_q^{\,2} i_q^{\,2} \cos^2\varphi_q \tag{11}$$

Substituting (7) and (8) into (10) and (11) further, the following equations can be obtained.

$$i_q^2 = (I_p \cosh\beta_k - \frac{V_p}{Z_k}\sinh\beta_k)(I_p \cosh\beta_k - \frac{V_p}{Z_k}\sinh\beta_k)^* \tag{12}$$

$$\left[\left(V_p \cosh\beta_k - Z_k I_p \sinh\beta_k\right)\cdot\left(I_p \cosh\beta_k - \frac{V_p}{Z_k}\sinh\beta_k\right)\right]^2 =$$
$$i_q^2\left(\cos\varphi_q\right)^2 (V_p \cosh\beta_k - Z_k I_p \sinh\beta_k)(V_p \cosh\beta_k - Z_k I_p \sinh\beta_k)^* \tag{13}$$

where,

i_q is the current magnitude at load-side node q;

φ_q is the angle of power factor at load-side node q.

In the same way, yk can be solved through the Newton-Raphson method in terms of (12) and (13).

For both cases, finally yk is substituted into (7) and (8) to calculate Vq and Iq which are the magnitude and phase angle of voltage and current at the load-side node.

2.5 Four Terminal Constants

But in a complicated radial system, it is very hard to obtain the solution of load flow for each node. Thus the method of four terminal constants has been used to simplify the process of load flow calculation.

Actually, (7) and (8) can be further expressed in matrix form.

$$\begin{pmatrix} V_q \\ I_q \end{pmatrix} = \begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} \begin{pmatrix} V_p \\ I_p \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} = \begin{pmatrix} \cosh\beta_k & -Z_k \sinh\beta_k \\ -\frac{1}{Z_k}\sinh\beta_k & \cosh\beta_k \end{pmatrix} \tag{15}$$

The coefficient matrix (15) is called four terminal constants, which is a basal form. In radial distribution networks, there are many different configurations at different nodes, so the extensional forms of four terminal constants should be analyzed.

2.5.1 Node with outgoing current

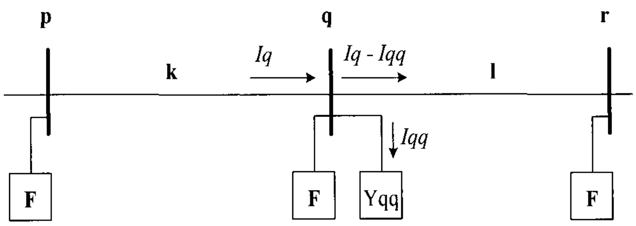

Fig. 3. Node with outgoing current

Fig. 3 shows the case of a node with outgoing current.

$$I_{qq} = Y_{qq} V_q \tag{16}$$

where, $\boldsymbol{Iqq}$ is an outgoing current at the node q.

Note that the incoming current $\boldsymbol{Iq}$ should subtract the outgoing current $\boldsymbol{Iqq}$, so the voltage and current at the node r can be found out as in (17) according to (14).

$$\begin{pmatrix} V_r \\ I_r \end{pmatrix} = \begin{pmatrix} \cosh\beta_l & -Z_l \sinh\beta_l \\ -\frac{1}{Z_l}\sinh\beta_l & \cosh\beta_l \end{pmatrix} \begin{pmatrix} V_q \\ I_q - I_{qq} \end{pmatrix} \tag{17}$$

Thus the extensional form of four terminal constants is expressed in (18).

$$\begin{pmatrix} A_l & B_l \\ C_l & D_l \end{pmatrix} = \begin{pmatrix} \cosh\beta_l + Y_{qq} Z_l \sinh\beta_l & -Z_k \sinh\beta_l \\ -\frac{1}{Z_l}\sinh\beta_l - Y_{qq}\cosh\beta_l & \cosh\beta_l \end{pmatrix} \tag{18}$$

2.5.2 Node with lateral

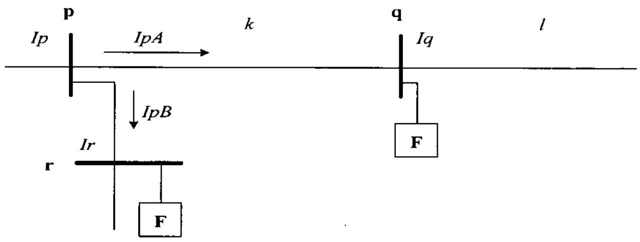

Fig. 4. Node with lateral

In radial distribution networks, a node with lateral is a common case as shown in Fig. 4. In this case, the current distribution factors of the branch lines should be computed by the weight ratio of the loads, which can be estimated by the proportion of load capacities. If there are more load capacities, the load weight ratio is supposed to be larger. In Fig. 4, the following equations are found out.

$$\boldsymbol{I}_{pA} \triangleq \boldsymbol{K}_{pq}\boldsymbol{I}_p \tag{19}$$

$$\boldsymbol{I}_{pB} \triangleq \boldsymbol{K}_{pr}\boldsymbol{I}_p \tag{20}$$

$$\boldsymbol{K}_{pq} + \boldsymbol{K}_{pr} = 1 \tag{21}$$

where, at the node p,

Ip is the incoming load current;
IpA is the load current flowing in section k;
IpB is the load current flowing in branch section l.
Kpq is the current distribution factor in section k;
Kpr is the current distribution factor in section l.

So the voltage and current at node q can be found out as in (22) according to (14), and the extensional form of four terminal constants is presented in (23).

$$\begin{pmatrix} V_q \\ I_q \end{pmatrix} = \begin{pmatrix} \cosh\beta_k & -Z_k \sinh\beta_k \\ -\frac{1}{Z_k}\sinh\beta_k & \cosh\beta_k \end{pmatrix} \begin{pmatrix} V_p \\ K_{pq} I_p \end{pmatrix} \tag{22}$$

$$\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} = \begin{pmatrix} \cosh\beta_k & -K_{pq} Z_k \sinh\beta_k \\ -\frac{1}{Z_k}\sinh\beta_k & K_{pq}\cosh\beta_k \end{pmatrix} \tag{23}$$

2.5.3 Node with transformer

If the voltage drop at the high-voltage distribution line is greater than 5%, a Step Voltage Regulator (SVR) placed somewhere along the feeder could be utilized to maintain the voltage on a certain nominal level, because the SVR is a special transformer called an autotransformer having the ability to automatically change its turn ratio.

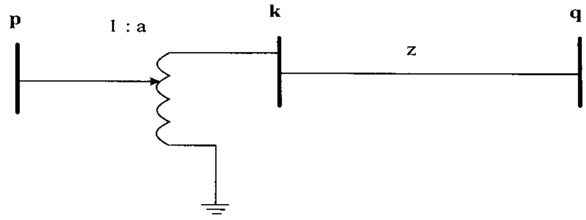

Fig. 5. Node with transformer

The equivalent circuit modeling of the SVR is shown in Fig. 5, where z is the impedance of the transformer and in which the ratio of the transformer is 1:a. So the extensional form of four terminal constants is achieved below.

$$\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} = \begin{pmatrix} a & -\frac{z}{a} \\ 0 & \frac{1}{a} \end{pmatrix} \tag{24}$$

In complicated radial distribution networks, the extensional forms of four terminal constants can be integrated due to the aforementioned three cases, thus load flow calculations could be greatly simplified.

3. Case Study

3.1 Comparison

In this case study, the comparison among the simulation result, proposed algorithm, and a conventional algorithm [3] are demonstrated. The proposed algorithm considers that the load is equalized to be distributed in the line, but the conventional algorithm regards the load concentrated at the end of each line section. Due to the different forms of load modeling, both algorithms would achieve diverse results on the analysis of load flow. Fig. 6 shows a single line diagram of a main feeder in the distribution system. There are three line sections, all 5 [km] in length.

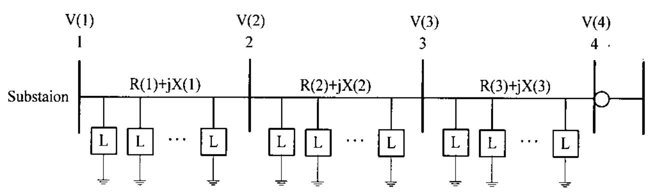

Fig. 6. Single line diagram of a main feeder

Table 1 tabulates the comparison results among the simulation result, proposed algorithm, and the conventional algorithm.

Table 1. Comparison results

V [p.u] \ Case	Simulation	Proposed	Conventional
V(1)	1.0000	1.0000	1.0000
V(2)	0.9102	0.9103	0.8815
V(3)	0.8647	0.8647	0.8115
V(4)	0.8510	0.8510	0.7795

As shown in Table 1, the simulation result is almost identical to the proposed algorithm, but has a bigger error with the conventional algorithm. The reason is the difference of the load modeling. The conventional algorithm considers the load lumped at each node but the proposed algorithm considers the load distributed along the line.

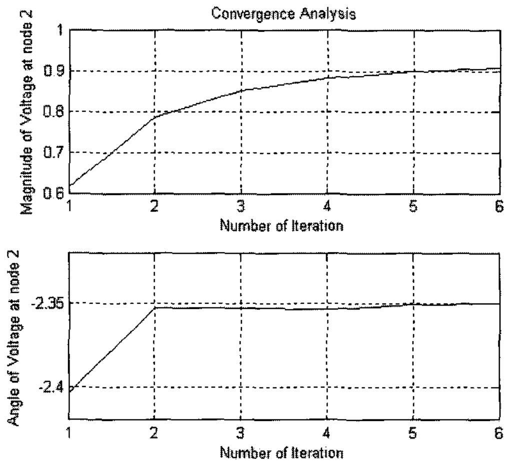

Fig. 7. Convergence analysis

The Newton-Raphson method ensures a rapid convergence. Fig. 7 presents the convergence curve on estimating the voltage magnitude and phase angle at node 2 in Fig. 6.

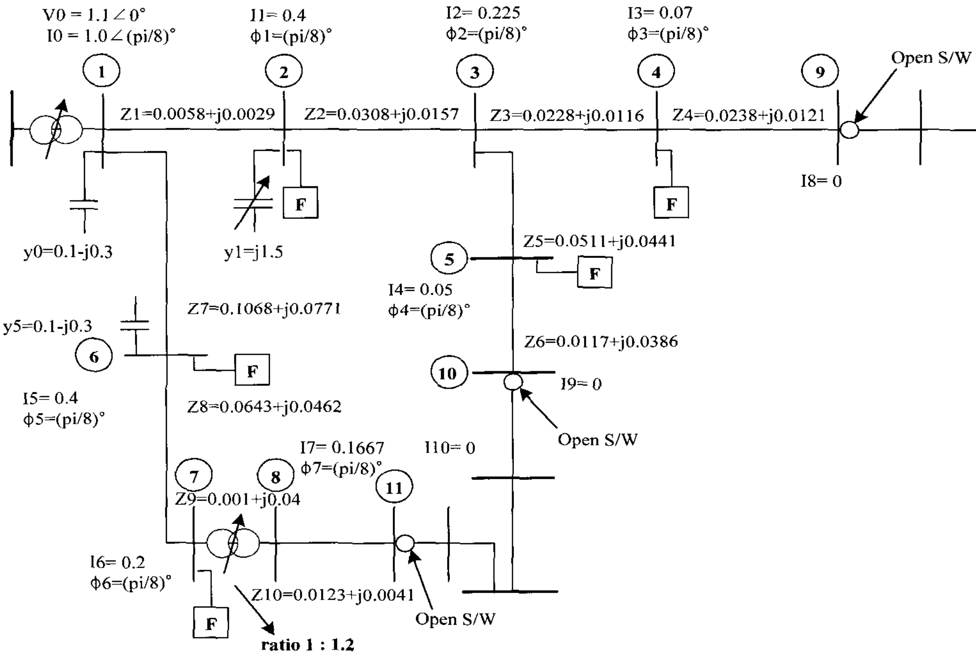

Fig. 8. A simple radial distribution system

3.2 Test in a radial distribution system

In this case, the proposed algorithm has been testified in a simple radial distribution system with 11 nodes as shown in Fig. 8, where all line impedances and the transformer impedance have been given. In addition, the ratio of the transformer is 1:1.2. The angle of power factor is pi/8; and the compensation factor of the capacitances is 0.03.

Table 2 lists the estimation results, where the magnitude and phase angle of voltage and current at each node have been calculated through the proposed algorithm.

Table 2. Estimation results

Node	V	V_θ	I	I_θ
1	1.0000	0.0000	1.0000	-0.5236
2	0.9971	-0.0001	0.4000	-0.3050
3	0.9749	0.0215	0.2250	-0.2834
4	0.9726	0.0210	0.0750	-0.2838
5	0.9698	0.0194	0.0500	-0.3733
6	0.9417	-0.0097	0.4000	-0.4024
7	0.9186	-0.0156	0.2000	-0.4083
8	1.0996	-0.0211	0.1667	-0.4083
9	0.9716	0.0209	0.0000	-0.1604
10	0.9692	0.0186	0.0000	0.5045
11	1.0985	-0.0211	0.0000	-0.4636

4. Conclusion

Due to the distributed load modeling and the application of FRTUs in DAS, the proposed algorithm contributes more accurate calculation for load flows than the conventional one with the concentrated load modeling. Meanwhile, the method of four terminal constants greatly simplifies the processes. The validity of the proposed algorithm has been testified by the simulations.

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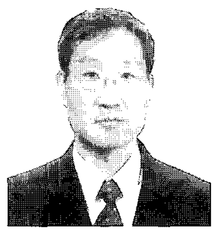

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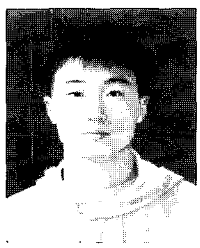

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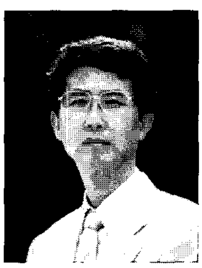

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