퍼지 및 지능시스템학회 논문지 2007, Vol. 17, No. 4 , pp. 557-562

Interval-valued Fuzzy Soft Sets

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Abstract

This paper extends the work of Maji et al. (2001) to present the concept of interval-valued fuzzy soft sets and to present an algorithm for finding where the degree of membership are represented by interval values in $[0, 1]$. The proposed method is more flexible than the one presented in Maji et al. (2001) due to the fact that it allows the degrees of membership of object for parameters to be represented by interval-values rather than crisp real values between zero and one.

Key words : Interval-valued fuzzy sets, Interval-valued fuzzy soft sets.

1. Introduction

Most of problems in real life situation such as economics, engineering, environment, social sciences and medical sciences not always involve crisp data. So we cannot successfully use the traditional methods because of various types of uncertainties present in these problems. Since Zadeh [17] introduced fuzzy sets in 1965, a lot of new theories treating imprecision and uncertainty have been introduced. Some of these theories are extensions of fuzzy set theory and the others try to handle imprecision and uncertainty in different ways. Kerre [8] has given a summary of the links that exist between fuzzy sets and other mathematical models such as flou sets [16], two-fold fuzzy sets [4] and L-fuzzy sets [6].

The theories such as probability theory, fuzzy set theory [13, 17, 18], intuitionistic fuzzy set theory [1-3], vague set theory [5], interval mathematics theory [3,7] and rough set theory [14, 15], which can be considered as mathematical tools for dealing with uncertainties, have their inherent difficulties (see [12]). The reason for these difficulties is possibly the inadequacy of parameterization tool of the theories. Molodtsov [12] introduced soft sets as a mathematical tool for dealing with uncertainties which is free from the above-mentioned difficulties. Since the soft set theory offers mathematical tool for dealing with uncertain, fuzzy and not clearly defined objects, it has a rich potential for applications to problems in real life situation. Maji et al. [10, 11] made a theoretical study of the soft set theory. In [9], Maji et al. also introduced fuzzy soft sets, where the relevant values (degrees of membership of object for parameters) are represented by real values between zero and one, and presented an application of fuzzy soft set in decision making problem. However, the fuzzy soft sets presented in [9] all assume that the relevant values (degrees of membership) in a fuzzy soft set are represented by crisp real values between zero and one. If we can allow the relevant values (degrees of membership) in a fuzzy soft set to be represented by intervals in $[0, 1]$ rather than crisp real values between zero and one, then there is room for more flexibility.

In this paper, we extend the work of [9] to present the concept of interval-valued fuzzy soft sets based on [4, 7] and present an application of interval-valued fuzzy soft sets in a decision making problem. The proposed mathematical tool is more flexible than the one presented in [9] due to the fact that it allows the degrees of membership to be represented by interval-values rather than crisp real values between zero and one.

2. Interval-valued fuzzy sets

Definition 2.1. [4, 7] An interval-valued fuzzy set A on a unverse X is a map $X \to \mathrm{Int}([0, 1])$, where $\mathrm{Int}([0, 1])$ stands for the sets of all closed subintervals of $[0, 1]$.

The basic operations such as union, intersection and complement are defined as follows: let A, B be two

접수일자 : 2007년 6월 22일
완료일자 : 2007년 7월 20일

interval-valued fuzzy sets in X, then

- $A \cup B(x) = [\max(\inf A(x), \inf B(x)), \max(\sup A(x), \sup B(x))]$, for all $x \in X$,
- $A \cap B(x) = [\min(\inf A(x), \inf B(x)), \min(\sup A(x), \sup B(x))]$, for all $x \in X$,
- $A \subset B \Leftrightarrow \inf A(x) \leq \inf B(x)$, $\sup A(x) \leq \sup B(x)$, for all $x \in X$,
- $A = B \Leftrightarrow A \subset B,\ B \subset A$,
- $A^c(x) = [1 - \sup A(x), 1 - \inf A(x)]$, for all $x \in X$.

3. Interval-valued fuzzy soft sets

Let U be an initial unverse set and E be a set of parameters.

Definition 3.1. A pair (F, A) is called interval-valued fuzzy soft set (over U) if $A \subset E$ and F is a mapping of A into the set of all interval-valued fuzzy sets of U.

In other words, the interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy sets of the set U. Viewing U as the collection of objects of interest and E as the collection of all possible characteristics, or properties of objects in U, an interval-valued fuzzy soft set is a specification of objects in U with respect to a sub-collection A of properties in E.

Now, we give an example of an interval-valued fuzzy soft set.

Example 3.2. Suppose that U is the set of houses under consideration and E is the set of parameters such that each parameter is an interval-valued fuzzy word or a sentence involving interval-valued fuzzy words. Let $E = \{$expensive, beautiful, wooden, cheap, in the green surroundings, modern, in good repair, in bad repair$\}$. In this case, to define an interval-valued fuzzy soft set means to point out *expensive* houses, *beautiful* houses, and so on. The interval-valued fuzzy soft set (F, E) describes "the attractiveness of the houses" which Mr. X (say) is going to buy.

For our next discussion, we consider the same type of example which is considered above. Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be the set of six houses under consideration and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the set of five parameters, where e_1 stands for 'expensive', e_2 stands for 'beautiful', e_3 stands for 'wooden', e_4 stands for 'cheap' and e_5 stands for 'in the green surrounding'. Suppose that

$$
\begin{aligned}
F(e_1) &= \{h_1/[0.5, 0.8],\ h_2/[1, 1],\ h_3/[0.4, 0.7],\ h_4/[1, 1],\ h_5/[0.3, 0.6],\ h_6/[0, 0.7]\},\\
F(e_2) &= \{h_1/[1, 1], h_2/[0.4, 0.7], h_3/[1, 1], h_4/[0.4, 0.8],\ h_5/[0.6, 0.8],\ h_6/[0.8, 0.9]\},\\
F(e_3) &= \{h_1/[0.2, 0.5],\ h_2/[0.3, 0.6],\ h_3/[1, 1],\ h_4/[1, 1],\ h_5/[1, 1],\ h_6/[0, 0.6]\},\\
F(e_4) &= \{h_1/[1, 1],\ h_2/[0, 0.6],\ h_3/[1, 1],\ h_4/[0.4, 0.7],\ h_5/[0.6, 0.8],\ h_6/[0.8, 0.9]\},\\
F(e_5) &= \{h_1/[1, 1],\ h_2/[0.1, 0.6],\ h_3/[0.5, 0.8],\ h_4/[0.3, 0.8],\ h_5/[0.2, 0.4],\ h_6/[0.3, 0.6]\}.
\end{aligned}
$$

The interval-valued fuzzy soft set (F, E) is a parameterized family $\{F(e_i) : i = 1, 2, 3, 4, 5\}$ of all interval-valued fuzzy sets of the set U and gives us a collection of approximate description of an object. The mapping F, here, is "houses $(\cdot)$" where dot$(\cdot)$ is to be filed up by a parameter $e \in E$. For example, $F(e_1)$ means "houses (expensive)" whose functional value is the interval-valued fuzzy set

$$\{h_1/[0.5, 0.8], h_2/[1, 1],\ h_3/[0.4, 0.7],\ h_4/[1, 1],\ h_5/[0.3, 0.6],\ h_6/[0, 0.7]\}.$$

Thus, we can view the interval-valued fuzzy soft set (F, E) as a collection of interval-valued fuzzy approximations which are interval-valued fuzzy sets as follows: $(F, E) = \{$ expensive houses $= \{h_1/[0.5, 0.8],\ h_2/[1, 1],\ h_3/[0.4, 0.7],\ h_4/[1, 1],\ h_5/[0.3, 0.6],\ h_6/[0, 0.7]\}$, beautiful houses $= \{h_1/[1, 1],\ h_2/[0.4, 0.7],\ h_3/[1, 1],\ h_4/[0.4, 0.8],\ h_5/[0.6, 0.8],\ h_6/[0.8, 0.9]\}$, wooden houses $= \{h_1/[0.2, 0.5], h_2/[0.3, 0.6], h_3/[1, 1], h_4/[1, 1], h_5/[1, 1], h_6/[0, 0.6]\}$, cheap houses $= \{h_1/[1, 1],\ h_2/[0, 0.6],\ h_3/[1, 1],\ h_4/[0.4, 0.7],\ h_5/[0.6, 0.8],\ h_6/[0.8, 0.9]\}$, houses in the green surroundings $= \{h_1/[1, 1],\ h_2/[0.1, 0.6],\ h_3/[0.5, 0.8], h_4/[0.3, 0.8], h_5/[0.2, 0.4], h_6/[0.3, 0.6]\}$ $\}$, where each ith approximation has two parts:

(i) a predicate p_i,

(ii) an approximate interval-valued fuzzy set v_i.

For example, for the approximation "expensive houses $= \{h_1/[0.5, 0.8], h_2/[1, 1], h_3/[0.4, 0.7], h_4/[1, 1], h_5/[0.3, 0.6], h_6/[0, 0.7]\}$", we have

(i) the predicate name is 'expensive houses',

(ii) the approximate interval-valued fuzzy set is $\{h_1/[0.5, 0.8], h_2/[1, 1],\ h_3/[0.4, 0.7],\ h_4/[1, 1],\ h_5/[0.3, 0.6], h_6/[0, 0.7]\}$.

Thus an interval-valued fuzzy soft set (F, E) can be viewed as a collection of interval-valued fuzzy approximations like as: $(F, E) = \{p_1 = v_1, p_2 = v_2, \cdots, p_5 = v_5\}$. For the purpose of storing an interval-valued fuzzy soft set in computer, we could represent the interval-valued fuzzy soft set in the above example as follows. In this table, the entries are h_{ij} = interval-valued membership of h_i in $F(e_j)$ corresponding the house h_i and parameter e_j.

	e_1	e_2	e_3	e_4	e_5
h_1	[0.5,0.8]	[1,1]	[0.2,0.5]	[1,1]	[1,1]
h_2	[1,1]	[0.4,0.7]	[0.3,0.6]	[0,0.6]	[0.1,0.6]
h_3	[0.4,0.7]	[1,1]	[1,1]	[1,1]	[0.5,0.8]
h_4	[1,1]	[0.4,0.8]	[1,1]	[0.4,0.7]	[0.3,0.8]
h_5	[0.3,0.6]	[0.6,0.8]	[1,1]	[0.6,0.8]	[0.2,0.4]
h_6	[0,0.7]	[0.8,0.9]	[0,0.6]	[0.8,0.9]	[0.3,0.6]

Definition 3.3. Let (F, A) and (G, B) be interval-valued fuzzy soft sets over a common universe set U. Then

(a) (F, A) is a subset of (G, B), denoted by $(F, A) \sqsubseteq (G, B)$, if

(i) $A \subset B$,

(ii) for any $e \in A$, $F(e)$ is an interval-valued fuzzy subset of $G(e)$.

(b) (F, A) equal to (G, B), denoted by $(F, A) = (G, B)$, if $(F, A) \sqsubseteq (G, B)$ and $(G, B) \sqsubseteq (F, A)$.

(c) the complement of (F, A), denoted by $(F, A)^c$, is defined by

$$(F, A)^c = (F^c, \neg A),$$

where $F^c : \neg A \to P(U)$ is a mapping given by $F^c(\alpha)$= interval-valued fuzzy complement of $F(\neg\alpha)$, for any $\alpha \in \neg A$.

(d) (F, A) is called null interval-valued fuzzy soft set, denoted by Φ, if for any $e \in A$, $F(e)$ = null interval-valued fuzzy set of U.

(e) (F, A) is called absolute interval-valued fuzzy soft set, denoted by $\widetilde{A}$, if for any $e \in A$, $F(e) = U$.

Remark 3.4. (a) $(F^c)^c = F$ and $((F, A)^c)^c = (F, A)$.

(b) $\widetilde{A}^c = \Phi$ and $\Phi^c = \widetilde{A}$.

Definition 3.5. Let (F, A) and $G, B)$ be two interval-valued fuzzy soft sets. Then

(a) $(F, A) \wedge (G, B)$ is an interval-valued fuzzy soft set defined by

$$(F, A) \wedge (G, B) = (H, A \times B),$$

where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for any $\alpha \in A$ and $\beta \in B$, where $\cap$ is the intersection operation of interval-valued fuzzy sets.

(b) $(F, A) \vee (G, B)$ is an interval-valued fuzzy soft set defined by

$$(F, A) \vee (G, B) = (K, A \times B),$$

where $K(\alpha, \beta) = F(\alpha) \cup G(\beta)$ for any $\alpha \in A$ and $\beta \in B$, where $\cup$ is the union operation of interval-valued fuzzy sets.

Example 3.6. Consider the interval-valued fuzzy soft set (F, A) which describes the "cost of the houses" and the interval-valued fuzzy soft set (G, B) which describes the "attractiveness of the houses". Suppose that $U = \{ h_1, h_2, h_3, h_4, h_5, h_6 \}$, $A = \{$ very costly, costly, cheap $\}$ and $B = \{$ beautiful, in the green surroundings, cheap $\}$. Let

F(very costly) $= \{ h_1/[0.7, 0.7], h_2/[1, 1], h_3/[0.8, 1], h_4/[1, 1], h_5/[0.9, 1], h_6/[0.3, 0.9] \}$,
F(costly) $= \{ h_1/[1, 1], h_2/[1, 1], h_3/[1, 1], h_4/[1, 1], h_5/[1, 1], h_6/[0.4, 1] \}$,
F(cheap) $= \{ h_1/[0.4, 1], h_2/[0.2, 0.8], h_3/[0.5, 1], h_4/[0.4, 1], h_5/[0.3, 1], h_6/[1, 1] \}$,
G(beautiful) $= \{ h_1/[0.6, 0.9], h_2/[1, 1], h_3/[1, 1], h_4/[0.8, 0.9], h_5/[0.6, 1], h_6/[0.8, 0.8]\}$,
G(in the green surroundings) $= \{h_1/[0.5, 0.7], h_2/[0.7, 0.9], h_3/[0.8, 0.9], h_4/[0.6, 0.9], h_5/[1, 1], h_6/[1, 1] \}$,
G(cheap) $= \{ h_1/[0.4, 1], h_2/[0.2, 0.9], h_3/[0.5, 0.8], h_4/[0.4, 0.9], h_5/[0.3, 1], h_6/[1, 1] \}$.

Then $(F, A) \wedge (G, B) = (H, A \times B)$, where
H(very costly, beautiful) $= \{h_1/[0.6, 0.9], h_2/[1, 1], h_3/[0.8, 1], h_4/[0.8, 0.9], h_5/[0.6, 1], h_6/[0.3, 1] \}$,
H(very costly, in the green surroundings) $= \{h_1/[0.5, 0.7], h_2/[0.7, 0.9], h_3/[0.8, 0.9], h_4/[0.6, 0.8], h_5/[0.9, 0.9], h_6/[0.3, 0.9] \}$,
H(very costly, cheap) $= \{h_1/[0.4, 0.7], h_2/[0.2, 0.9], h_3/[0.5, 0.7], h_4/[0.4, 0.9], h_5/[0.3, 0.9], h_6/[0.3, 0.9] \}$,
H(costly, beautiful) $= \{h_1/[0.6, 0.9], h_2/[1, 1], h_3/[1, 1], h_4/[0.8, 0.9], h_5/[0.6, 1], h_6/[0.4, 0.8]\}$,
H(costly, in the green surrounding) $= \{h_1/[0.5, 0.7], h_2/[0.7, 0.9], h_3/[0.8, 0.9], h_4/[0.6, 0.8], h_5/[1, 1], h_6/[0.4, 1]\}$,
H(costly, cheap) $= \{h_1/[0.4, 1], h_2/[0.2, 0.9], h_3/[0.5, 0.8], h_4/[0.4, 0.9], h_5/[0.3, 1], h_6/[0.4, 1]\}$,
H(cheap, beautiful) $= \{h_1/[0.4, 0.9], h_2/[0.2, 0.8], h_3/[0.5, 1], h_4/[0.4, 1], h_5/[0.3, 1], h_6/[0.8, 0.8]\}$,
H(cheap, in the green surroundings) $= \{h_1/[0.4, 0.8], h_2/[0.2, 0.8], h_3/[0.5, 0.9], h_4/[0.4, 0.8], h_5/[0.3, 1], h_6/[1, 1]\}$ and
H(cheap, cheap) $= \{h_1/[0.4, 1], h_2/[0.2, 0.8], h_3/[0.5, 0.8], h_4/[0.4, 0.9], h_5/[0.3, 1], h_6/[1, 1]\}$.

We see that the following De Morgan's types of results are true.

Proposition 3.7. Let (F, A) and $G, B)$ be two interval-valued fuzzy soft sets. Then

(a) $((F, A) \vee (G, B))^c = (F, A)^c \wedge (G, B)^c$.

(b) $((F, A) \wedge (G, B))^c = (F, A)^c \vee (G, B)^c$.

Proof. (a) Let $(F, A) \vee (G, B) = (H, A \times B)$, where H is a mapping given by

$$H(x, y) = \{(h, [\max(\inf F(x)(h), \inf G(y)(h)), \max(\sup F(x)(h), \sup G(y)(h))]) : h \in U\}$$

for any $x \in A$ and $y \in B$. Hence we have

$$\begin{aligned} ((F,\ A) \vee (G,\ B))^c &= (H,\ A \times B)^c \\ &= \{\{(h, [\min(\inf F^c(x)(h), \inf G^c(y)(h)), \\ &\quad \min(\sup F^c(x)(h), \sup G^c(y)(h))]) : h \in U\} \\ &\quad : x \in \neg A, y \in \neg B\} \\ &= \{\{(h, [1 - \sup F(x)(h), 1 - \inf F(x)(h)]) : \\ &\quad h \in U\} : x \in \neg A\} \\ &\quad \wedge \{\{(h, [1 - \sup G(y)(h), 1 - \inf G(y)(h)]) : \\ &\quad h \in U\} : y \in \neg B\} \\ &= (F, A)^c \wedge (G, B)^c. \end{aligned}$$

(b) Similar to (a). □

Definition 3.8. Let (F, A) and (G, B) be two interval-valued fuzzy soft sets over the common universe set U. Then

(a) union of (F, A) and (G, B) is the interval-valued fuzzy soft set $(H, A \cup B)$ defined by

$$H(e) = \begin{cases} F(e), & e \in A - B, \\ G(e), & e \in B - A, \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$$

and denoted by $(F, A) \sqcup (G, B)$.

(b) intersection of (F, A) and (G, B) is the interval-valued fuzzy soft set $(H, A \cap B)$ given by $H(e) = F(e)$ or $G(e)$ for all $e \in A \cap B$ and denoted by $(F, A) \sqcap (G, B)$.

Example 3.9. Consider the Example 3.6. Then $(F, A) \sqcup (G, B) = (H, A \cup B)$, where $A \cup B = \{$very costly, costly, cheap, beautiful, in the green surroundings$\}$ and H is given by $H(\text{very costly}) = \{h_1/[0.7, 0.7], h_2/[1, 1], h_3/0.8, 1], h_4/[1, 1]h_5/[0.9, 0.9], h_6/[0.3, 0.9]\}$, $H(\text{costly}) = \{h_1/[1, 1], h_2/[1, 1], h_3/[1, 1], h_4/[1, 1], h_5/[1, 1], h_6/[0.4, 1]\}$, $H(\text{cheap}) = \{h_1/[0.4, 1], h_2/[0.2, 0.9], h_3/0.5, 1], h_4/[0.4, 0.9], h_5/[0.3, 1], h_6/[1, 1]\}$, $H(\text{beautiful}) = \{h_1/[0.6, 0.9], h_2/[1, 1], h_3/[1, 1], h_4/[0.8, 0.9], h_5/[0.6, 1], h_6/[0.8, 0.8]\}$, H(in the green surroundings= $\{h_1/[0.5, 0.8], h_2/[0.7, 0.9], h_3/[0.8, 0.9], h_4/[0.6, 0.8], h_5/[1, 1], h_6/[1, 1]\}$. $(F, A) \sqcap (G, B) = (H, A \cap B)$, where $A \cap B = \{\text{cheap}\}$ and H is given by $H(\text{cheap}) = \{h_1/[0.4, 1]$, $h_2/[0.8, 0.8]$, $h_3/[0.5, 0.8]$, $h_4/[0.4, 0.9]$, $h_5[0.3, 1]$, $h_6/[1, 1]\}$.

The following results are straightforward.

Proposition 3.10. Let (F, A) be an interval-valued fuzzy soft set over an universe set U. Then

(a) $(F, A) \sqcup (F, A) = (F, A)$ and $(F, A) \sqcap (F, A) = (F, A)$.

(b) $(F, A) \sqcup \Phi = \Phi$ and $(F, A) \sqcap \Phi = \Phi$, where Φ is the null soft set.

(c) $(F, A) \sqcup \tilde{A} = \tilde{A}$ and $(F, A) \sqcap \tilde{A} = \tilde{A}$, where $\tilde{A}$ is the absolute soft set.

4. An application of interval-valued fuzzy soft sets in a decision making problem

In this section, we present an application of interval-valued fuzzy soft set in a decision making problem.

Consider the problem of selecting the most suitable house which one is going to buy on the basis of his five parameters out of six houses. Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be the unverse set of houses and $E = \{$expensive, beautiful, wooden, cheap, in the green surroundings$\}$ be the set of parameters. Consider the interval-valued fuzzy soft set $(F.E)$ which describes the attractiveness of the houses given by

$(F, E) = \{$expensive houses $= \{h_1/[0.5, 0.8], h_2/[1, 1], h_3/[0.4, 0.7], h_4/[1, 1], h_5/[0.3, 0.6], h_6/[0, 0.7]\}$, beautiful houses $= \{h_1/[1, 1], h_2/[0.4, 0.7], h_3/[1, 1], h_4/[0.4, 0.8], h_5/[0.6, 0.8], h_6/[0.8, 0.9]\}$, wooden houses $= \{h_1/[0.2, 0.5], h_2/[0.3, 0.6], h_3/[1, 1], h_4/[1, 1], h_5/[1, 1], h_6/[0, 0.6]\}$, cheap houses $= \{h_1/[1, 1], h_2/[0, 0.6], h_3/[1, 1], h_4/[0.4, 0.7], h_5/[0.6, 0.8], h_6/[0.8, 0.9]\}$, houses in the green surroundings $= \{h_1/[1, 1], h_2/[0.1, 0.6], h_3/[0.5, 0.8], h_4/[0.3, 0.8], h_5/[0.2, 0.4], h_6/[0.3, 0.6]\}\}$.

Suppose that Mr. X is interested to buy a house on the basis of his choice parameters $A = \{$beautiful, wooden, cheap, in the green surrounding$\}$ from the set E. This means, out of available houses in U, that he is to select the house which qualifies with all parameters of the set A.

Suppose that another customers Mr. Y and Mr. Z want to buy a house on the basis of their choice parameters B and C, respectively. The problem is to select the house which is most suitable with the choice parameters of Mr. X. The house which is most suitable for Mr. X need not be most suitable for Mr. Y or Mr. Z because the selection is dependent on the set of choice parameters of each customer.

To find the house which is most suitable with the choice parameters of Mr. X, we define 'Comparison Table' of interval-valued fuzzy soft set (F, A) by considering its tabular representation.

4.1 Comparison Table of (F, A)

The Comparison Table of interval-valued fuzzy soft set (F, A) is a square table in which number of rows and number of columns are equal, rows and columns are labelled by the object names h_1, h_2, h_3, h_4, h_5 and h_6 of the universe set U, and the entries c_{ij} $(i, j = 1, 2, 3, 4, 5, 6)$ is the number of parameters satisfying

$$\frac{\inf \mu_{ik} + \sup \mu_{ik}}{2} \geq \frac{\inf \mu_{jk} + \sup \mu_{jk}}{2},$$

where μ_{ik} and μ_{jk} are, respectively, the interval-valued membership values of h_i and h_j in $F(e_k)$ for $k = 1, 2, 3, 4$.

Clearly, $0 \leq c_{ij} \leq k$ for any i, j, where k is the number of parameters in A. Thus, c_{ij} indicates a numerical measure which h_i dominates h_j in c_{ij} number of parameters out of k parameters.

4.2 Row sum, column sum and score of an object h_j

- The row sum r_i of an object h_i is calculated by $r_i = \sum_{j=1}^{6} c_{ij}$.
- The column sum t_j of an object h_j is calculated by $t_j = \sum_{i=1}^{6} c_{ij}$.
- The score s_i of an object h_i is calculated by $s_i = r_i - t_i$.

4.3 Algorithm for selection of the house

We present an algorithm for most appropriate selection of house which Mr. X wish to buy as follows:

1. Input the interval-valued fuzzy soft set (F, E).
2. Input the set A of choice parameters of Mr. X which is subset of E .
3. Consider the interval-valued fuzzy soft set (F, A) and write it in tabular form.
4. Calculate the Comparison Table of the interval-valued fuzzy soft set (F, A).
5. Calculate row sum r_i and column sum t_i for h_i for all i.
6. Calculate score s_i of h_i for all i.
7. Find k, for which $s_k = \max_i s_i$. Then Mr. X will buy the house h_k. If k is not unique, then Mr. X will buy any one of h_k by using his options.

Now, we use the algorithm to solve our original problem. Consider the above-mentioned interval-valued fuzzy soft set (F, E) and the set $A = \{$beautiful, wooden, cheap, in the green surroundings$\}$ of choice parameters of Mr. X on the basis of which he wants to buy a house from the unverse set U. Then the tabular representation of the interval-valued fuzzy soft set (F, A) is as follows:

	beautiful	wooden	cheap	in the green surroundings
h_1	[1,1]	[0.2,0.5]	[1,1]	[1,1]
h_2	[0.4,0.7]	[0.3,0.6]	[0,0.6]	[0.1,0.6]
h_3	[1,1]	[1,1]	[1,1]	[0.5,0.8]
h_4	[0.4,0.8]	[1,1]	[0.4,0.7]	[0.3,0.8]
h_5	[0.6,0.8]	[1,1]	[0.6,0.8]	[0.2,0.4]
h_6	[0.8,0.9]	[0,0.6]	[0.8,0.9]	[0.3,0.6]

Then the Comparison Table of the above interval-valued fuzzy soft set (F, A) is calculated in Table below.

	h_1	h_2	h_3	h_4	h_5	h_6
h_1	4	3	3	3	3	4
h_2	1	4	0	0	1	1
h_3	3	4	4	4	4	4
h_4	1	4	1	4	1	2
h_5	1	3	1	3	4	1
h_6	1	3	0	3	3	4

Next, we calculate row sum, column sum and score of each h_i as follows:

	row sum (r_i)	column sum (t_i)	score(s_i)
h_1	20	11	9
h_2	7	21	-14
h_3	23	9	14
h_4	13	17	-4
h_5	13	16	-3
h_6	14	16	-2

Thus, the maximum score is 14, scored by h_3, and hence Mr. X will buy the house h_3. In the case that he does want to buy h_3 due to certain reasons, his second choice will be h_1.

5. Conclusion

We observe that the elements of parameter set are not always precise rather vague and interval-valued fuzzy. From this point in view, we define interval-valued fuzzy soft set, study some properties of this interval-valued fuzzy soft sets and present an application of this new have mathematical tool.

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