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ON SECOND ORDER SLOPE ROTATABLE DESIGNS - A REVIEW

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Abstract

In this paper, a review on second order slope rotatable designs (SOSRD) is studied. Further, different methods of constructions of SOSRD like slope rotatable central composite designs (SRCCD), SOSRD using balanced incomplete block designs (BIBD), SOSRD using pairwise balanced designs (PBD), SOSRD using partially balanced incomplete block type designs (PBIBD) and SOSRD using symmetrical unequal block arrangements (SUBA) with two unequal block sizes are examined in detail. A table is provided where for a range of different values of v (v stands for number of factors) the design points needed by different methods are compared. The optimum SOSRD with minimum number of design points for each factor is suggested for $2 \leq v \leq 16$.

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1. Introduction

Response surface methodology is a statistical technique very useful in design and analysis of scientific experiments. In many experimental situations the experimenter is concerned with explaining certain aspects of a functional relationship $Y = f(x_1, x_2, \ldots, x_v) + e$, where Y is the response and $x_1, x_2, \ldots, x_v$ are the levels of v-quantitative variables or factors and e is the random error. The function $f(\cdot)$ is called response surface or response function. Designs, which are used, for the study of response surface methods, are called response surface designs. Response surface methods are useful where several independent variables influence a dependent variable. The independent variables are assumed to be continues and controlled by the experimenter. The response is assumed to be as random

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variable. For example, if a chemical engineer wishes to find the temperature (x_1) and pressure (x_2) that maximizes the yield (response) of his process, the observed response Y may be written as a function of the levels of the temperature (x_1) and pressure (x_2) as $Y = f(x_1 + x_2) + e$.

In many applications of response surface methodology, good estimation of the derivatives of the response function may be as important or perhaps more important than estimation of mean response. Certainly, the computation of a stationary point in a second order analysis or the use of gradient techniques - for example, steepest ascent or ridge analysis - depends heavily on the partial derivatives of the estimated response function with respect to the design variables. Since designs that attain certain properties in Y (estimated response) do not enjoy the same properties for the estimated derivatives (slopes), it is important for the user to consider experimental designs that are constructed with the derivatives in mind.

The concept of rotatability, which is very important in response surface second order designs, was proposed by Box and Hunter (1957). A design is said to be rotatable if the variance of the response estimate is a function only of the distance of the point from the design center. The study of rotatable designs is mainly emphasized on the estimation of differences of yields and its precision. Estimation of differences in responses at two different points in the factor space will often be of great importance. If differences in responses at two points close together are of interest then estimation of local slope (rate of change) of the response is required. Estimation of slopes occurs frequently in practical situations. For instance, there are cases in which we want to estimate rate of reaction in chemical experiment, rate of change in the yield of a crop to various fertilizer doses and rate of disintegration of radioactive material in an animal, *etc.* (Park, 1987).

Hader and Park (1978) introduced slope rotatability for central composite designs. For the central composite designs they modified Box and Hunter (1957) rotatability to slope rotatability simply by adjusting the axial point distance (a), so that the variance of the estimated pure quadratic coefficients is one-fourth the variance of the estimated mixed second order coefficients. Mukerjee and Huda (1985) constructed minimax second- and third-order designs to estimate the slope of a response surface. Park (1987) introduced a class of multifactor designs for estimating the slope of response surfaces and studied second order slope rotatability over all directions and gave necessary and sufficient conditions for a design to have this property. Park and Kim (1988) studied slope rotatable designs for estimating the slope of response surfaces in experiments with mixtures.

Victorbabu and Narasimham (1990) constructed second order slope rotatable designs (SOSRD) through incomplete block designs. Victorbabu and Narasimham (1991a) studied in detail the conditions to be satisfied by a general SOSRD and constructed SOSRD using BIBD. Victorbabu and Narasimham (1991b) constructed SOSRD through a pair of incomplete block designs. Park and Kim (1992) suggested a measure of slope rotatability for second order response experimental designs. Jang and Park (1993) suggested a measure and a graphical method for evaluating slope rotatability in response surface designs. Victorbabu and Narasimham (1993a) constructed SOSRD using pairwise balanced designs (PBD). Victorbabu and Narasimham (1993b) suggested classification and parameter bounds of SOSRD. Victorbabu and Narasimham (1993c) constructed three level SOSRD using BIBD. Anjeneyulu *et al.* (1993) studied embedding in SOSRD.

Victorbabu and Narasimham (1994) suggested a new type of slope rotatable central composite design (SRCCD). Victorbabu *et al.* (1994) constructed SOSRD with equi-spaced levels using incomplete block designs with unequal block sizes. Victorbabu and Narasimham (1994&95) obtained augmented second order rotatable design as second order slope rotatable design. Victorbabu and Narasimham (1995) constructed a new method of construction of SOSRD using a pair of incomplete block designs. Ying *et al.* (1995a) studied slope rotatability over all directions designs. Ying *et al.* (1995b) studied slope rotatability over all directions designs for $k \geq 4$. Victorbabu and Narasimham (1996) suggested construction of SOSRD using partially balanced incomplete block (PBIB) type designs. Victorbabu and Narasimham (1996&97) constructed three level SOSRD using incomplete block designs with unequal block sizes. Victorbabu *et al.* (1996&97) constructed SOSRD over all directions using incomplete block designs. Victorbabu and Narasimham (1997) constructed SOSRD over all directions using partially balanced incomplete block type designs. Anjaneyulu *et al.* (1997) suggested a note on SOSRD over all directions. Anjaneyulu *et al.* (1998) constructed group divisible SOSRD.

Victorbabu (2000) constructed SOSRD over all directions using balanced incomplete block designs with unequal block sizes. Victorbabu and Narasimham (2000-01) suggested a new method of construction of SOSRD using BIBD. Victorbabu (2002a) suggested a note on the construction of four and six level SOSRD. Victorbabu (2002b) constructed SOSRD using symmetrical unequal block arrangements (SUBA) with two unequal block sizes. Victorbabu (2002c) studied augmented second order rotatable design as second order slope rotatable designs. Victorbabu (2002d) studied SOSRD with equi-spaced levels. Anjaneyulu *et al.* (2002) constructed variance-sum group divisible SOSRD. Victorbabu and Vasundharadevi (2003) studied on the efficiency of second order response surface designs for estimation of responses and slopes. Das (2003) studied slope rotatability with correlated errors. Victorbabu (2003) constructed SOSRD using incomplete block designs. Victorbabu and Vasundharadevi (2004a) studied on the efficiency of second order response surface designs for estimation of responses and slopes using BIBD. Victorbabu and Vasundharadevi (2004b) studied performance of second order response surface designs for estimation of responses and slopes using pairwise balanced designs. Anjeneyulu *et al.* (2005) constructed four level SOSRD. Victorbabu and Seshubabu (2005) constructed four level SOSRD using SUBA with two unequal block sizes. Victorbabu (2006) studied a note on variance-sum group divisible SOSRD using BIBD.

In this paper, a review on second order slope rotatable designs is studied. Further, different methods of construction of SOSRD like SRCCD, SOSRD using BIBD, SOSRD using PBD, SOSRD using PBIBD and SOSRD using SUBA with two unequal block sizes are examined in detail. A table is provided where for a range of different values of v the design points needed by different methods are compared. The optimum SOSRD with minimum number of design points for each factor is suggested for $2 \leq v \leq 16$.

2. Conditions for Second Order Slope Rotatable Designs

Suppose we want to use the second order response surface design $D = ((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum\sum_{i<j} b_{ij} x_{iu} x_{ju} + e_u, \tag{2.1}$$

where x_{iu} denotes the level of the i^{th} factor $(i = 1, 2, \ldots, v)$ in the u^{th} run $(u = 1, 2, \ldots, N)$ of the experiment, $e_u's$ are uncorrelated random errors with mean zero and variance σ^2, is said to be SOSRD if the variance of the estimate of first order partial derivative of $Y_u(x_1, x_2, \ldots, x_v)$ with respect to each of independent variables (x_i) is only a function of the distance $(d^2 = \sum_{i=1}^{v} x_{iu}^2)$ of the point $(x_1, x_2, \ldots, x_v)$ from the origin (center) of the design.

Following Box and Hunter (1957), Hader and Park (1978) and Victorbabu and Narasimham (1991a), the general conditions for second order slope rotatability can be obtained as follows. To simplify the fit of the second order polynomial from design points 'D' through the method of least squares, we impose the following simple symmetry conditions on D to facilitate easy solutions of the normal equations:

1. $\sum x_{iu} = 0, \sum x_{iu}x_{ju} = 0, \sum x_{iu}x_{ju}^2 = 0, \sum x_{iu}x_{ju}x_{ku} = 0, \sum x_{iu}^3 = 0,$
$\sum x_{iu}x_{ju}^3 = 0, \sum x_{iu}x_{ju}x_{ku}^2 = 0, \sum x_{iu}x_{ju}x_{ku}x_{lu} = 0,$ for $i \neq j \neq k \neq l$,

2.

(i) $\sum x_{iu}^2 = \text{constant} = N\lambda_2$,
(ii) $\sum x_{iu}^4 = \text{constant} = cN\lambda_4$, for all i,

3.

$$\sum x_{iu}^2 x_{ju}^2 = \text{constant} = N\lambda_4, \text{ for } i \neq j, \tag{2.2}$$

where c, λ_2 and λ_4 are constants and the summation is over the design points.

The variances and covariances of the estimated parameters are

$$\begin{aligned}
V(\widehat{b}_0) &= \frac{\lambda_4(c+v-1)\sigma^2}{N[\lambda_4(c+v-1) - v\lambda_2^2]}, \\
V(\widehat{b}_i) &= \frac{\sigma^2}{N\lambda_2}, \\
V(\widehat{b}_{ij}) &= \frac{\sigma^2}{N\lambda_4}, \\
V(\widehat{b}_{ii}) &= \frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2) - (v-1)\lambda_2^2}{\lambda_4(c+v-1) - v\lambda_2^2}\right], \\
\text{Cov}(\widehat{b}_0, \widehat{b}_{ii}) &= \frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1) - v\lambda_2^2]}, \\
\text{Cov}(\widehat{b}_{ii}, \widehat{b}_{ij}) &= \frac{(\lambda_2^2 - \lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1) - v\lambda_2^2]}
\end{aligned} \tag{2.3}$$

and other covariances vanish.

An inspection of the variance of $\widehat{b}_0$ shows that a necessary condition for the existence of a non-singular second order design is

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)}. \tag{2.4}$$

For the second order model

$$\frac{\partial \widehat{Y}}{\partial x_i} = \widehat{b}_i + 2\widehat{b}_{ii}x_{iu} + \sum_{j\neq i} \widehat{b}_{ij}x_{ju}, \tag{2.5}$$

$$V\left(\frac{\partial \widehat{Y}}{\partial x_i}\right) = V(\widehat{b}_i) + 4x_{iu}^2 V(\widehat{b}_{ii}) + \sum_{j\neq i} x_{ju}^2 V(\widehat{b}_{ij}). \tag{2.6}$$

The condition for right hand side of (2.6) to be a function of $d^2 = \sum_{i=1}^{v} x_{iu}^2$ alone (for slope rotatability) is

$$V(\widehat{b}_{ii}) = \frac{1}{4}V(\widehat{b}_{ij}). \tag{2.7}$$

Therefore, conditions (2.2), (2.3) and (2.7) lead to the condition

$$[v(5-c)-(c-3)^2]\lambda_4 + [v(c-5)+4]\lambda_2^2 = 0. \tag{2.8}$$

3. SLOPE ROTATABLE CENTRAL COMPOSITE DESIGNS

(*cf.* Hader and Park, 1978)

The widely used design for fitting a second order model is the central composite design. Central composite designs are constructed by adding suitable factorial combinations to those obtained from $(1/2^p)2^v$ fractional factorial design (here $(1/2^p)2^v$ denotes a suitable fractional replicate of 2^v), in which no interaction with less than five factors is confounded. In coded form, the points of $2^v(2^{t(v)})$ factorial have coordinates $(\pm 1, \pm 1, \ldots, \pm 1)$ and $2v$ axial points have coordinates of the form $(\pm a, 0, \ldots, 0), (0, \pm a, \ldots, 0), \ldots, (0, 0, \ldots, \pm a)$, *etc.* and if necessary, n_0 central points may be replicated sometimes. The method of construction of SRCCD is given in the following theorem.

THEOREM 3.1. *A central composite design will be a v-dimensional SRCCD in $N = 2^{t(v)} + 2v + n_0$ design points were a^2 is a positive real root of the fourth degree polynomial equation,*

$$(8v-4N)a^8 + 2^{t(v)+3}va^6 + \left[2^{t(v)+1}(4-v)N + 2^{2t(v)+1}v + (1-v)2^{t(v)+4}\right]a^4$$
$$+(1-v)2^{2t(v)+4}a^2 + \left[2^{2t(v)+2}(v-1)N + (1-v)2^{3t(v)+2}\right] = 0.$$

4. SOSRD USING BALANCED INCOMPLETE BLOCK DESIGNS

(*cf.* Victorbabu and Narasimham, 1991a)

Let (v, b, r, k, λ) denote a BIBD, $2^{t(k)}$ denote a fractional replicate of 2^k in ± 1 levels, in which no interaction with less than five factors is confounded. $[1 - (v, b, r, k, \lambda)]$ denote the design points generated from the transpose of incidence matrix of BIBD. $[1 - (v, b, r, k, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from BIBD by 'multiplication' (*cf.* Raghavarao, 1971, pp. 298–300), $(a, 0, 0, \ldots, 0)2^1$ denote the design points generated from $(a, 0, 0, \ldots, 0)$ point set and $\cup$ denotes combination of the design points generated from different sets of points. n_0 denote the number of central points. The method of construction of SOSRD using BIBD is given in the following theorem.

THEOREM 4.1. *The design points,* $[1-(v, b, r, k, \lambda)]2^{t(k)} \cup (a, 0, \ldots, 0)2^1 \cup (n_0)$ *will give a* v*-dimensional SOSRD in* $N = b2^{t(k)} + 2v + n_0$ *design points, where* a^2 *is positive real root of the fourth degree polynomial equation,*

$$\begin{aligned}&(8v - 4N)a^8 + 8vr2^{t(k)}a^6\\&+ \left[2vr^2 2^{2t(k)} + \{((12 - 2v)\lambda - 4r)N + (16\lambda - 20v\lambda + 4vr)\}2^{t(k)}\right] a^4\\&+ \left[4vr^2 + (16 - 20v)r\lambda\right] 2^{2t(k)}a^2 + \left[(5v - 9)\lambda^2 + (6 - v)r\lambda - r^2\right] N2^{2t(k)}\\&+(vr + 4\lambda - 5v\lambda)r^2 2^{3t(k)} = 0.\end{aligned}$$

5. SOSRD USING PAIRWISE BALANCED DESIGNS

(*cf.* Victorbabu and Narasimham, 1993a)

Let $(v, b, r, k_1, k_2, \ldots, k_p, \lambda)$ is an equi-replicated pairwise balanced design (PBD), $k = \sup(k_1, k_2, \ldots, k_p)$ and $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ± 1 levels such that no interaction with less than five factors is confounded. The method of construction of SOSRD using PBD is given in the following theorem.

THEOREM 5.1. *The design points,* $[1 - (v, b, r, k_1, k_2, \ldots, k_p, \lambda)]2^{t(k)} \cup (a, 0, \ldots, 0)2^1 \cup (n_0)$ *give a* v*-dimensional SOSRD in* $N = b2^{t(k)} + 2v + n_0$ *design points, where* a^2 *is a positive real root of the fourth degree polynomial equation,*

$$\begin{aligned}&(8v - 4N)a^8 + 8vr2^{t(k)}a^6\\&+ \left[2vr^2 2^{2t(k)} + \{((12 - 2v)\lambda - 4r)N + (16\lambda - 20v\lambda + 4vr)\}2^{t(k)}\right] a^4\end{aligned}$$

$$+\left[4vr^2+(16-20v)r\lambda\right]2^{2t(k)}a^2+\left[(5v-9)\lambda^2+(6-v)r\lambda-r^2\right]N2^{2t(k)}$$
$$+(vr+4\lambda-5v\lambda)r^2 2^{3t(k)}=0.$$

6. SOSRD USING PARTIALLY BALANCED INCOMPLETE BLOCK TYPE DESIGNS

(*cf.* Victorbabu and Narasimham, 1996)

Take an incomplete block arrangement with constant block size and replication in which some pairs of treatments occur λ_1 times each ($\lambda_1 \neq 0$) and some other pairs do not occur at all ($\lambda_2 = 0$) (the design need not be PBIBD). Take this as the first design. For the second design take the incomplete block design with all missing pairs (in the first design) once each with $k = 2, \lambda'_1 = 0$ and $\lambda'_2 = 1$. Such pairs of PBIB type designs can be constructed in a straightforward manner in particular using existing two-associate PBIB designs with one of the λ's equals to zero.

Let $D_1 = (v, b_1, r_1, k_1, \lambda_1 \neq 0, \lambda_2 = 0)$ be an incomplete block design with constant replication in which only some pair of treatments occur a constant number of times $\lambda_1 (\lambda_2 = 0)$. $[1-(v, b_1, r_1, k_1, \lambda_1, \lambda_2 = 0)]$ denote the design points generated from the transpose of the incidence matrix of incomplete block design.

$[1-(v, b_1, r_1, k_1, \lambda_1, \lambda_2 = 0)]2^{k_1}$ are the $b_1 2^{k_1}$ design points generated from D_1 by 'multiplication'. Let $D_2 = (v, b_2, r_2, k_2 = 2, \lambda'_1 = 0, \lambda'_2 = 1)$ be the associated second design containing only the missing pairs of treatments of above design D_1. Let $\lfloor a_1 - (v, b_2, r_2, k_2, \lambda'_1 = 0, \lambda'_2 = 1)\rfloor 2^2$ are the $b_2 2^2$ design points generated from D_2 by multiplication. $(a, 0, \ldots, 0)2^1$ denote the design points generated from $(a, 0, \ldots, 0)$ point set. The method of construction of SOSRD using PBIB type designs is given in the following theorem.

THEOREM 6.1. *The design points,*

$$[1-(v, b_1, r_1, k_1, \lambda_1; \lambda_2 = 0)]2^{k_1}$$
$$\cup\lfloor a_1 - (v, b_2, r_2, k_2, \lambda'_1 = 0; \lambda'_2 = 1)\rfloor 2^2 \cup (a, 0, \ldots, 0)2^1 \cup n_0$$

give a v-dimensional SOSRD in $N = b_1 2^{k_1} + b_2 2^2 + 2v + n_0$ *design points.*

7. SOSRD USING SYMMETRICAL UNEQUAL BLOCK ARRANGEMENTS WITH TWO UNEQUAL BLOCK SIZES

(*cf.* Victorbabu, 2002b)

SUBA with two unequal block sizes is studied here (*cf.* Raghavarao, 1962).

The arrangement of v-treatments in b blocks where b_1 blocks of size k_1 and b_2 blocks of size k_2 is said to be a symmetrical unequal block arrangement with two unequal block sizes, if

(i) every treatment occurs $b_i k_i/v$ blocks of size $k_i (i = 1, 2)$ and

(ii) every pair of first associate treatments occurs together in u blocks of size k_1 and in $(\lambda - u)$ blocks of size k_2 while every pair of second associate treatments occurs together in λ blocks of size k_2.

From (i) each treatment occurs in $(b_1k_1/v)+(b_2k_2/v) = r$ blocks in all. $(v, b, r, k_1, k_2, b_1, b_2, \lambda)$ are known as the parameters of the SUBA with two unequal block sizes.

Let $(v, b, r, k_1, k_2, b_1, b_2, \lambda), k = \sup(k_1, k_2)$ and $b_1 + b_2 = b$ be a SUBA with two unequal block sizes. $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ± 1 levels, such that no interaction with less than five factors is confounded. $[1 - (v, b, r, k_1, k_2, b_1, b_2, \lambda)]$ denote the design points generated from the transpose of incidence matrix of SUBA with two unequal block sizes, $[1 - (v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from SUBA with two unequal block sizes by 'multiplication'. $(a, 0, 0, \ldots, 0)2^1$ denote the design points generated from $(a, 0, 0, \ldots, 0)$ point set. The method of construction of SOSRD using SUBA with two unequal block sizes is given in the following theorem.

THEOREM 7.1. *The design points,* $[1 - (v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)} \cup (a, 0, \ldots, 0)2^1 \cup (n_0)$ *give a v-dimensional SOSRD in* $N = b2^{t(k)} + 2v + n_0$ *design points, where a^2 is a positive real root of the fourth degree polynomial equation,*

$$\begin{aligned}
&(8v - 4N)a^8 + 8vr2^{t(k)}a^6 \\
&+ \left[2vr^2 2^{2t(k)} + \{((12 - 2v)\lambda - 4r)N + (16\lambda - 20v\lambda + 4vr)\}2^{t(k)}\right] a^4 \\
&+ \left[4vr^2 + (16 - 20v)r\lambda\right] 2^{2t(k)}a^2 + \left[(5v - 9)\lambda^2 + (6 - v)r\lambda - r^2\right] N2^{2t(k)} \\
&+(vr + 4\lambda - 5v\lambda)r^2 2^{3t(k)} = 0.
\end{aligned}$$

8. CONCLUDING REMARKS

Different methods of constructions of SOSRD were examined in detail. These methods are useful in deciding a proper design for second order response surface

TABLE 8.1 *Comparison of different methods of construction of SOSRD (N denotes total number of design points with one central point)*

No. of factors	*SRCCD (1978)*	*SOSRD using BIBD (1991a)*	*SOSRD using PBD (1993a)*	*SOSRD using PBIBD (1996)*	*SOSRD using SUBA with two unequal block sizes (2002b)*
(v)	*N*	*N*	*N*	*N*	*N*
2	9	–	–	–	–
3	15	19 (3,3,2,2,1)	–	–	–
4	25	33 (4,6,3,2,1)	–	–	–
5	27	51 (5,10,4,2,1)	–	–	–
6	45	73 (6,15,5,2,1)	69 (6,7,3,3,2,1)	57	69 (6,7,3,2,3,3,4,1)
7	79	71 (7,7,3,3,1)	–	–	–
8	81	129 (8,28,7,2,1)	257 (8,15,6,4,3,2,2)	97	113 (8,12,4,2,3,4,8,1)
9	147	115 (9,12,4,3,1)	195 (9,11,5,5,4,3,2)	–	163 (9,18,5,2,3,9,9,1)
10	149	201 (10,45,9,2,1)	197 (10,11,5,5,4,2)	169	197 (10,11,5,4,5,5,6,2)
11	151	199 (11,11,5,5,2)	–	–	–
12	281	377 (12,44,11,3,2)	537 (12,16,6,6,5,4,3,2)	–	233 (12,13,4,3,4,4,9,1)
13	283	235 (13,13,4,4,1)	538 (13,16,6,6,5,4,3,2)	–	–
14	285	–	541 (14,16,6,6,5,4,2)	–	309 (14,35,7,2,3,7,28,1)
15	287	311 (15,35,7,3,1)	543 (15,16,6,6,5,2)	–	351 (15,20,5,3,4,5,15,1)
16	289	353 (16,20,5,4,1)	–	–	481 (16,28,6,4,3,12,16,1)

polynomial models for the construction of SOSRD with desired properties or minimum number of design points. It is often necessary to choose a response surface design in which the number of levels of factors are unequal and in such a case second order asymmetric slope rotatable designs are useful.

Another area in which one may be interested is to study slope rotatability

TABLE 8.2 *Optimum SOSRD with minimum number of design points for* $2 \leq v \leq 16$

S. No.	*No. of factors*	1^{st} *best SOSRD*	2^{nd} *best SOSRD*
1	2	SRCCD, $N = 9$	–
2	3	SRCCD, $N = 15$	BIBD, $N = 19$ (3,3,2,2,1)
3	4	SRCCD, $N = 25$	BIBD, $N = 33$ (4,6,3,2,1)
4	5	SRCCD, $N = 27$	BIBD, $N = 51$ (5,10,4,2,1)
5	6	SRCCD, $N = 45$	PBIBD, $N = 57$
6	7	BIBD, $N = 71$ (7,7,3,3,1)	SRCCD, $N = 79$
7	8	SRCCD, $N = 81$	PBIBD, $N = 97$
8	9	BIBD, $N = 115$ (9,12,4,3,1)	SRCCD, $N = 147$
9	10	SRCCD, $N = 149$	PBIBD, $N = 169$
10	11	SRCCD, $N = 151$	BIBD, $N = 199$ (11,11,5,5,2)
11	12	SUBA, $N = 233$ (12,13,4,3,4,4,9,1)	SRCCD, $N = 281$
12	13	BIBD, $N = 235$ (13,13,4,4,1)	SRCCD, $N = 283$
13	14	SRCCD, $N = 285$	SUBA, $N = 309$ (14,35,7,2,3,7,28,1)
14	15	SRCCD, $N = 287$	BIBD, $N = 311$ (15,35,7,3,1)
15	16	SRCCD, $N = 289$	BIBD, $N = 353$ (16,20,5,4,1)

with correlated errors. Not much work is available with regard to construction of designs in this area. It may be interesting to study some new methods of constructions of slope rotatability with correlated errors using central composite designs, balanced incomplete block designs, pairwise balanced designs, *etc.*

There is scope for further research to evolve new methods of constructions of SOSRD which will lead to designs with lesser number of design points for different 'v' compared with the existing methods of construction, vide Table 8.1.

There is also scope for further work on measure of slope rotatability and to study it in detail with reference to different designs/different methods of constructions.

A comparison of different methods of constructions of SOSRD for $2 \leq v \leq 16$ is given in Table 8.1.

A list of optimum SOSRD constructed using different methods is given in Table 8.2.

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