Jour. of the KGS, Vol. 23, No. 9. September 2007, pp. 39~49

Effect of Pile Cap Flexibility on the Response of Pile Group Supported Column

교대를 지지하는 군말뚝의 캡강성효과

Jeong, Sang-Seom[1] 정 상 섭
Won, Jin-Oh[2] 원 진 오
Kim, Young-Ho[3] 김 영 호

요 지

본 연구에서는 수치해석을 통해 말뚝캡의 강성에 따른 거동차이를 분석하였다. 따라서 강성캡과 연성캡을 선정하여 상부교각과 하부군말뚝기초를 일체화하여 일련의 해석을 수행하였다. 해석 시 캡강성에 영향을 주는 캡의 탄성계수와 두께, 상부 교각길이, 지반반력, 그리고 말뚝직경 등을 고려하였다. 본 연구결과 군말뚝내 개별말뚝에 작용하는 하중은 말뚝캡의 강성에 크게 좌우되었으며 그 결과, 연성캡 해석결과는 강성캡에 비하여 수평하중과 휨모멘트가 상대적으로 크게 발생하므로 좀 더 보수적인 설계가 가능하리라 판단된다.

Abstract

The load deformation behavior of the cap-pile-soil system is investigated, based on numerical analysis. Special attention is given to consideration of pile cap flexibility. Rigid pile cap analysis and flexible cap analysis were conducted for comparison. A numerical method that takes into account the coupling between the rigidities of the piles, the cap, and the column has been introduced to analyze the response of pile group supported columns. The prediction of the lateral loads and bending moments in the pile cap is much more conservative for a flexible cap than for a rigid cap.

Keywords : Bending moment, Cap-pile-soil, Cap flexibility, Cap thickness, Numerical analysis

1. Introduction

Piles are often used in groups for the support of bridge structures. There are numerous analytical and numerical methods for designing pile groups. These methods can generally be classified into three different types: (1) the equivalent single pile method (Bogard and Matlock, 1983; Brown et al., 1988; Ooi et al., 2004), (2) the elasticity method (Poulos, 1971; Banerjee and Driscoll, 1976; Randolph, 1980), and (3) the general three-dimensional load transfer method (Reese et al., 1970; Chow 1987; Hoit et al., 1996; Kitiyodom and Matsumoto, 2002).

An essential element of pile foundation designs is the effort required to define the stiffness of the cap-soil-pile system confidently. According to the US Army Manual (1991), knowing the correct relationship between the

---

[1] Member, Dept. of Civil Engrg., Yonsei Univ., Seoul, 120-749, Korea, soj9081@yonsei.ac.kr, Corresponding Author
[2] Member, Civil and Environmental Engrg., Cornell Univ., Newyork 14850, USA
[3] Dept. of Civil Engrg., Yonsei Univ., Seoul, 120-749, Korea

stiffness of the pile and cap is extremely important for accurately designing pile groups for use in flexible base structures. Two kinds of methods are available for calculating the pile–cap interaction: the stiffness method and the finite element method.

The stiffness method was suggested by Hrennikoff (1950), and has been extended into a general three-dimensional method by Saul (1968) and Reese et al. (1970). Group 6.0 (Reese and Wang, 2004), a commercial package which has been used for practical design, is based on this stiffness method. This method can consider the pile–cap interaction, the nonlinear behavior of individual piles, and the pile–soil–pile interaction, but has limitations in that it cannot consider the coupling of the rigidities of the pile cap to each other and to the column, since this method assumes a pile cap to be a rigid body. To consider these coupled pile cap rigidities, a pile cap needs to be modeled by finite elements such as beams, frames, plates, and flat-shell elements.

The overall objective of the present study is to investigate the effect of changes in the pile stiffness. A numerical method that considers coupled cap rigidities has been introduced to overcome the restrictions associated with the conventional stiffness method. Rigid pile cap analysis and flexible cap analysis are conducted to examine the effect of changes in the pile stiffness. An example analysis is carried out and the results of a numerical analysis are highlighted.

2. Method of Analysis for Structure-Pile-Soil System

2.1 Modelling of a Pile Group

Fig. 1 illustrates a schematic diagram of a typical pile group supported column (a piled pier) and the numerical model of it used in the present study. The pile group consists of four piles and a flexible pile cap, which is not in direct contact with the soil. Here, the individual piles are modeled using beam–column elements. The soil around the individual piles is represented by a set of load transfer curves, and the interaction between piles is represented by a p-multiplier. For the modeling of the flexible pile cap and the column, three-dimensional finite elements, such as four-node flat shell elements for the pile cap and three-dimensional beam elements for the column, were used.

The equation for equilibrium at the pile head in the local coordinate system (u, v, w) is as follows:

$$\{F\}_i = \begin{Bmatrix} F_u \\ F_v \\ F_w \\ M_u \\ M_v \\ M_w \end{Bmatrix}_i = \begin{bmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{22} & 0 & 0 & 0 & k_{26} \\ 0 & 0 & k_{33} & 0 & -k_{35} & 0 \\ 0 & 0 & 0 & k_{44} & 0 & 0 \\ 0 & 0 & -k_{53} & 0 & k_{55} & 0 \\ 0 & k_{62} & 0 & 0 & 0 & k_{66} \end{bmatrix}_i \begin{Bmatrix} \delta_u \\ \delta_v \\ \delta_w \\ \alpha_u \\ \alpha_v \\ \alpha_w \end{Bmatrix}_i$$

$$= \left[K_E^P\right]_i \{\delta\}_i \qquad (1)$$

where $\left[K_E^P\right]_i$ is the stiffness matrix of the pile head, $\{\delta\}_i$ is the displacement vector, and $\{F\}_i$ is the vector of the force at the i^{th} pile head. The pile head stiffness matrix $\left[K_E^P\right]_i$ is of order 6 × 6, representing three displacement constants, three rotational constraints, and four couplings between the displacement and rotational constraints.

In this study, the individual piles were analyzed one by one to keep all the load–displacement relationships for each pile head in the first instance. The stiffness matrices $\left[K_E^P\right]$ of the individual piles which were incorporated into the structural analysis were derived from the load–displacement curves of the individual piles.

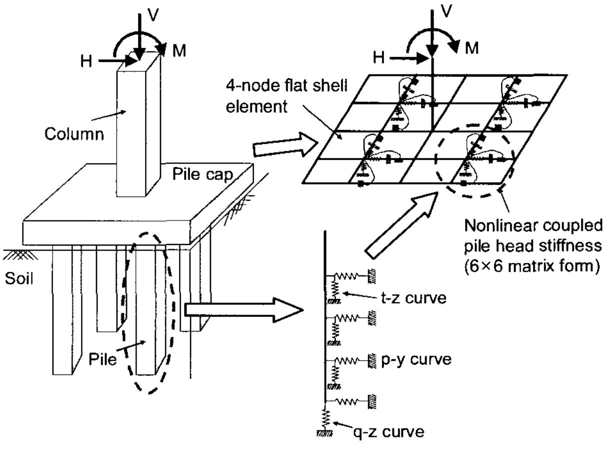


Fig. 1. Modelling of a pile group

2.2 Modelling of a Flexible Pile Cap

The stiffness method (Hrennikoff, 1950) considers neither coupling between the pile group and the column nor flexibility of the pile cap because the pile cap is assumed to be a rigid body. Much work has been done to analyze this coupling and the effect of pile cap flexibility. Typically, a plate element has been used as a pile cap, in several numerical methods (Clancy and Randolph, 1993; Zhang and Small, 2000; Kitiyodom and Matsumoto, 2002). These methods, however, have some limitations in that the horizontal behavior of the pile cap cannot be considered, because horizontal degrees of freedom (in the *x*- and *y*-directions) are excluded. These limitations can be overcome by using a flat-shell element (Choi and Lee, 1996). In the present study, a four node flat-shell element was adopted; this was developed by combining a Mindlin plate element and a membrane element with torsional degrees of freedom, as shown in Fig. 2. This element, having six degrees of freedom per node, permits an easy connection to other elements such as beams and folded elements. The stiffness matrix of a flat-shell element ($\left[K_E^s\right]$), which is of order 24 × 24 per element, is constructed by combination of the stiffness matrices of a plate element and of a membrane element in a local coordinate system as follows:

$$\left[K_E^s\right]=\begin{bmatrix} K_p^e & 0 \\ 0 & K_m^e \end{bmatrix} \tag{2}$$

where, K_p^e is the stiffness matrix of a plate element and K_m^e is the stiffness matrix of a membrane element.

2.3 Analysis Process

In this study, the deformations and the forces on members (the stress in the cap, and the bending moments and shear forces in the piles and column) is calculated for given conditions, such as the geometry, the load, and the properties of the structures (piles, cap, and column) and of the soil layers. The properties of the structures here include the pile cap flexibility. Before constructing the stiffness matrices of the flat-shell elements (for the cap) and of the beam elements (for the column), present method calculates 10 load-displacement curves (axial direction=1; lateral direction=8; torsional direction=1) for each pile head by repeated load transfer analyses using *t-z*/*q-z* and *p-y* curves and saves them. In this step, reduction factors, typically *p*-multipliers for the lateral responses, are incorporated into the load transfer curves. These initially calculated load-displacement curves are not changed during the iterative procedure. For a nonlinear analysis, all external forces are first divided by the increment number (*N*), and then the current stiffness matrices of the individual piles $\left[K_E^p\right]$ are calculated from the stored load-deflection curves. Combining $\left[K_E^p\right]$ with the stiffness matrix of the flat-shell element $\left[K_E^s\right]$ and beam element $\left[K_E^b\right]$, the coupled stiffness matrix of the pile group supported column $\left[K_E\right]$ is calculated as follows:

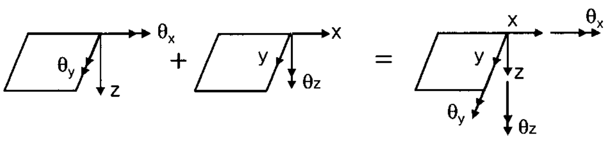


Plate element

Membrane element with a torsional d.o.f

Flat shell element

Fig. 2. Flat shell element

$$\left[K_E\right]=\left[K_E^s\right]+\left[K_E^b\right]+\left[K_E^p\right] \tag{3}$$

From the global stiffness matrix $\left[K_G\right]$ and the global load increment vector $\left[\Delta F_G\right]$ obtained by combining $\left[K_E\right]$ and $\left[\Delta F_E\right]$, the total displacement increment vector $\left[\Delta D_G\right]$ is given by

$$\left[\Delta D_G\right]=\left[K_G\right]^{-1}\left[\Delta F_G\right] \tag{4}$$

The procedure described above is iterated until the error between the assumed and calculated displacements falls within a tolerance limit. The displacement increments at the target step are obtained by adding the calculated results to those from the previous step. The internal forces in the pile cap and columns and in the individual piles are then calculated.

3. Example Analysis

An example analysis was performed based on the proposed numerical approach. A pile group supported column (a piled pier) with large diameter piles (D = 1.5 m), as shown in Fig. 3, was analyzed. The material properties are described in Table 1. The dimension, material properties, and the load magnitude of the analyzed pile groups were set up based on a real bridge pier which is located in Nak-Dong River in Korea. To investigate the effect of pile cap flexibility, the results predicted by the flexible cap analysis have been compared with results obtained by the stiffness method, which assumes that the pile cap is a rigid body.

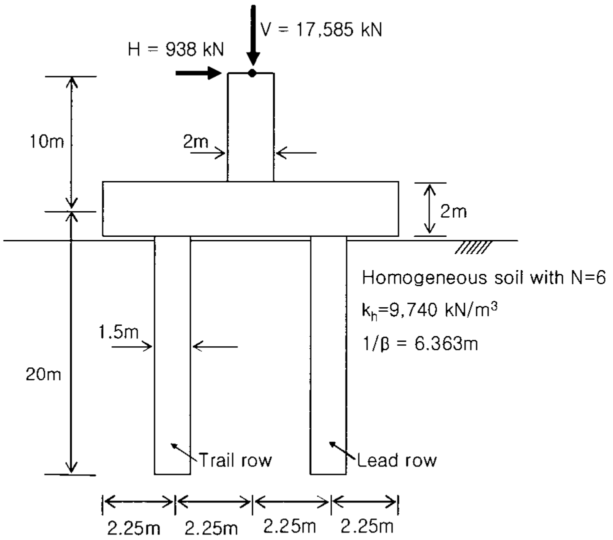


Fig. 3. Schematic diagram of a piled pier

Table 1. Material properties for pile groups

Elements	Properties	Values
Pier	Elastic modulus (E) (MN/m^2)	25,000
	Moment of inertia (I_x, I_y) (m^4)	0.79
	Area (A) (m^2)	3.14
	Polar moment of inertia (J) (m^4)	1.58
Pile cap	Elastic modulus (E) (MN/m^2)	25,000
	Poisson's ratio (ν)	0.18
	Thickness (t) (m)	2.00
Pile	Elastic modulus (E) (MN/m^2)	24,000
	Moment of inertia (I_x, I_y) (m^4)	0.25
	Area (A) (m^2)	1.77

3.1 Problem Description

A total of four piles are arranged in a 2 × 2 pattern and the center-to-center spacing of the piles is three times the pile diameter D (= 1.5 m). One column (pier) is located at the center of the pile cap. External forces are applied at the top of the pier, namely a vertical load V_0 = 17585 kN and a lateral load H_0 = 938 kN. The thickness of the pile cap is 2.0 m and the pile cap is located above the ground surface, so that the soil reaction beneath the cap can be ignored. The connection between the pile and the cap is a fixed condition. The SPT-N value of the homogeneous soil is 6, and the modulus of the lateral subgrade reaction k_h is 9740 kN/m^3. The p-multiplier is set to one to allow comparison with the results of the rigid cap analysis and to investigate the pile-cap interaction rather than the interaction between piles, although in this configuration (s/D = 3) a reduction of the soil reaction is expected.

3.2 Analysis Results

3.2.1 Deformation Shape and Stress of Pile Cap

Fig. 4 shows the deformed shape of a pile cap calculated by the present approach. It is drawn with a 200 times enlarged scale of the displacement for better visibility of the deformed shape. The center point, at which the pier is located, has sunk, not only in the x-z plane but also in the y-z plane. The displacement added by the deformation of the pile cap is smaller than that caused by the movement of the pile cap. However, it is important to consider the effects of the flexibility of the pile cap for correct estimation of the individual forces on the pile heads.

Fig. 5 shows the horizontal membrane stress and the

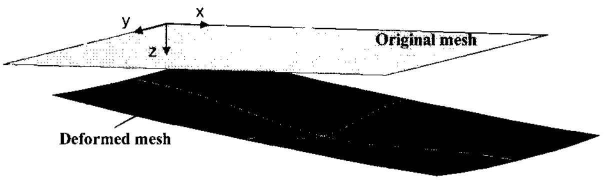
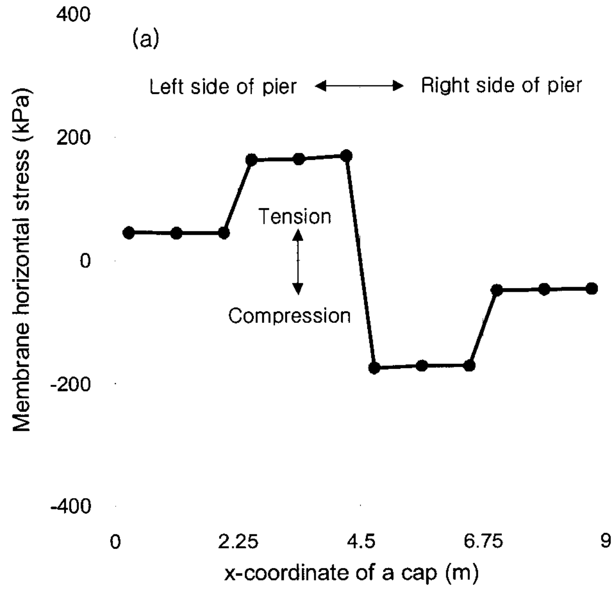
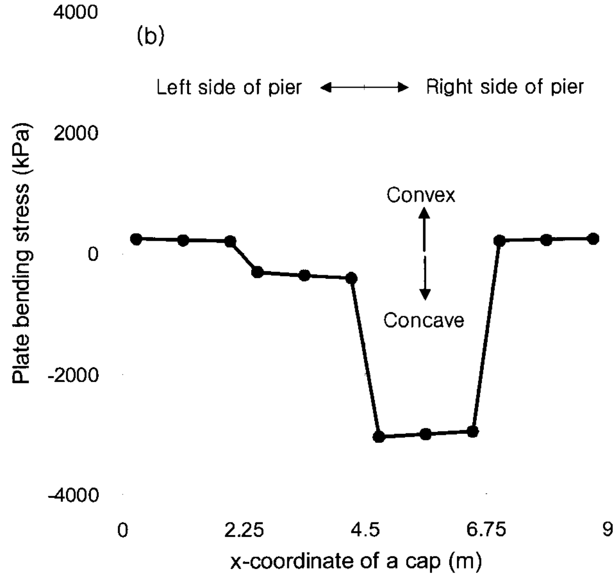


Fig. 4. Deformed shape of a pile cap

plate-bending stress in the pile cap along the x-direction. Fig. 5 (a) shows that a horizontal tensile stress occurs in the pile cap elements to the left of the pier, and a horizontal compressive stress occurs to the right. From the result for the plate-bending stress in Fig. 5 (b), it is found that the cap shows a slightly convex curvature on the outside of the piles and a concave curvature between the piles. In the pile cap elements on the right side of the pier, a significant bending stress is estimated, which is caused by the combined effect of the external axial and lateral loads.



(a) membrane horizontal stress

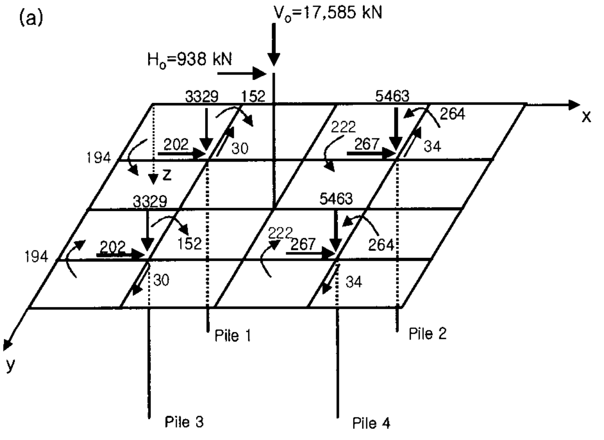


(b) plate bending stress

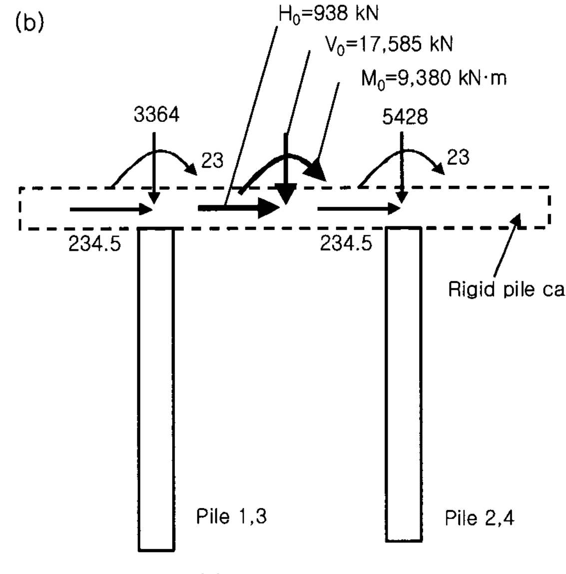
Fig. 5. Stress distribution of a pile cap

3.2.2 Pile Head Reactions

The axial and lateral loads and bending moments in the individual pile heads were estimated and compared with those obtained from the analyses considering rigid cap (Fig. 6). Both cases satisfied equilibrium conditions, but the load distributions into each pile head were somewhat different in the two analyses. Somewhat larger forces were distributed to the leading row (piles 2 and 4) and somewhat smaller axial forces to the trailing row (piles 1 and 3) in the flexible cap analysis, as compared with the results obtained from the rigid cap analysis. It is noted that the compressive horizontal stress in the pile cap on the right side of the pier increases the lateral forces in the leading row and the tensile horizontal stress decreases



(a) flexible cap analysis



(b) rigid cap analysis

Fig. 6. Distributed forces and bending moments at the pile head

Table 2. Pile head forces on cap flexibility

		Flexible cap		Rigid cap	
Plane	Pile head force	Trail row	Lead row	Trail row	Lead row
x–z plane	Lateral load	202	267	234.5	234.5
	Bending moment	−152	264	23	23
y–z plane	Lateral load	30	34	–	–
	Bending moment	±194	±222	–	–
	Axial load	3329	5463	3364	5428

the lateral forces in the trailing row. The bending moments at the pile heads calculated by the flexible cap analysis were -152 in the trailing row and 264 kN m in the leading row. The values obtained by the rigid cap analysis were 23 kN m regardless of the pile location. Significantly larger bending moments are generated at each pile head in the flexible analysis compared with the results obtained by the rigid cap analysis. Therefore, it is concluded that the loads in the individual pile heads are highly influenced by the flexibility of the pile cap. Table 2 summarizes the results of pile head forces by flexible cap analysis and rigid cap analysis.

3.2.3 Shear Forces and Bending Moments in Pile Members

Fig. 7 shows the bending moments and shear forces along the depth of the piles. The bending moments obtained by the flexible cap analysis show different shapes along the pile depth between the trailing row (piles 1 and 3) and the leading row (piles 2 and 4): there is a negative bending moment at the pile heads in the leading row and a positive bending moment in the trailing row. The dotted line, showing the results calculated by the flexible cap analysis, which are identical for the two rows, follows almost the mean of the values for the trailing and leading rows. Similarly, the shear force in rigid cap analysis also has the mean of the values for the trailing row and leading row by the flexible cap analysis. From this comparative study, it is found that the bending moments in individual piles are changed, as well as the pile head forces, when the flexibility of the pile cap is considered.

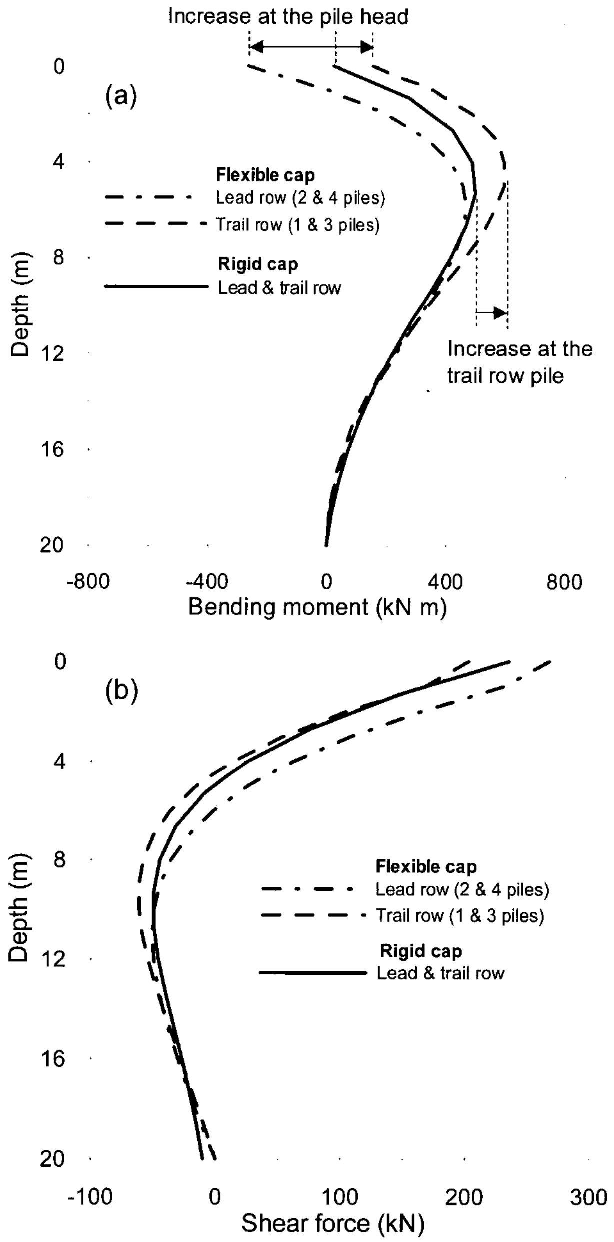
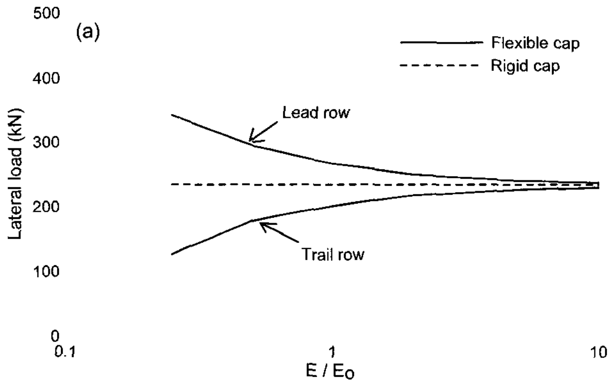


Fig. 7. Bending moment and shear force along the pile depth

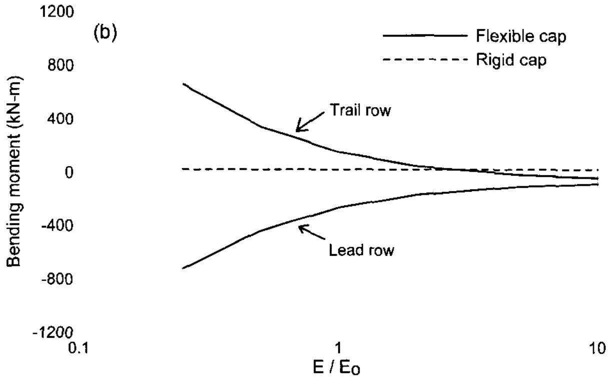
3.2.4 Effects of Elastic Modulus and Thickness of a Pile Cap

The lateral loads and the bending moments at the individual pile heads predicted by the flexible cap analysis have been compared with results from rigid cap analysis, for various values of the elastic modulus (E/E_0) of the pile cap (Fig. 8). Here, E_0 is basic elastic modulus. Except for the elastic modulus of the pile cap, all other conditions were identical to those in Fig. 3. The basic elastic modulus E_0 is 2.5×10^7 kN/m^2.

Figure 8 (a) shows well the significant effect of the elastic modulus on the lateral load and bending moment in the individual pile heads. In flexible cap analysis, a very large lateral load is distributed to the leading row when the elastic modulus of the pile cap is low, but when a very high elastic modulus ($E/E_0 = 10$) is used, identical lateral loads are generated in the two rows, and these loads are almost equal to the results from the rigid cap analysis. As shown in Fig. 8 (b), the bending moment in the leading row has a large negative value (-264 kN m), and that in the trailing row has a large positive value (152 kN m), for $E/E_0 = 1$ with the flexible cap analysis. The absolute magnitudes of these two bending moments are larger than that calculated by the rigid cap analysis, which was very small, only 23 kN m. However, it should be noted that these bending moments at the pile head are significantly smaller than the maximum bending moments developed at a depth of $3D$ (= 4.5 m) depth in this piled pier, as shown in Fig. 7. The maximum bending moment calculated by the rigid cap analysis is 519 kN·m, and the bending moments in the leading and trailing rows calculated by the flexible cap analysis are 464 kN m and 593 kN m, respectively.

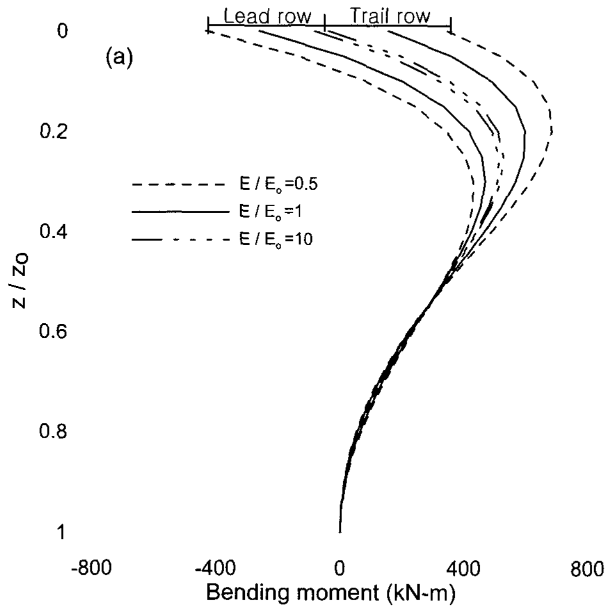


(a) lateral load

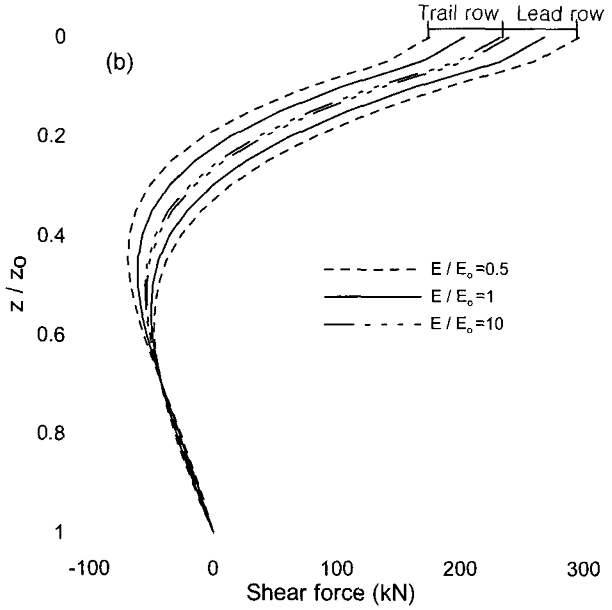


(b) bending moment at the pile head

Fig. 8. Comparison of pile cap modulus



(a) bending moment

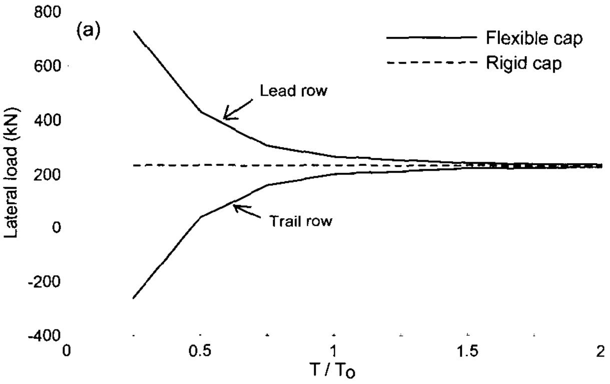


(b) shear force

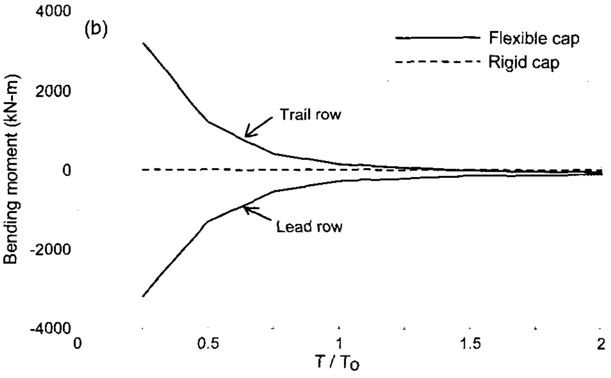
Fig. 9. Effects of elastic modulus of a pile cap

Fig. 9 shows profiles of the bending moment and the shear force along the pile depth for various values of the elastic modulus *E* of the pile cap. Three different elastic modulus ratios (E/E_0 = 0.5, 1, and 10) were investigated. Except for the elastic modulus of the pile cap, all other conditions were identical to the conditions in Fig. 3. As shown in Fig. 9, the difference in the bending moment and shear force between the trailing row and the leading row decreases with increasing elastic modulus of the pile cap. Therefore, there is a definite effect of pile cap flexibility on the bending moments and shear forces in individual piles, which represents profiles of bending moment and shear force different from those obtained by assuming a rigid pile cap.

Similar to the cases for various elastic moduli, the flexible cap analysis has been compared with rigid cap analysis, for various thicknesses (T/T_0) of the pile cap, as shown in Fig. 10. Here, T_0 is original thickness of a cap. The lateral load and bending moment caused by the changes of pile cap's thickness are similar to those caused by the changes of elastic modulus of a pile cap. In a half pile cap thickness, the discrepancy between lateral loads and bending moments of trailing row and those of leading row is very large. However, this discrepancy significantly decreases as the thickness of the pile cap increases. Even though the thickness of pile cap is doubled, the discrepancy diminishes and becomes so small that it can be ignored. From this study, it is found that the pile cap flexibility affects individual pile forces distributed from external forces, even though the displacement of a pile cap varies not much.



(a) lateral load



(b) bending moment at the pile head

Fig. 10. Comparison of the pile cap thickness

3.2.5 Effects of the Length of a Pier (Column)

Fig. 11 shows the bending moment and the shear force

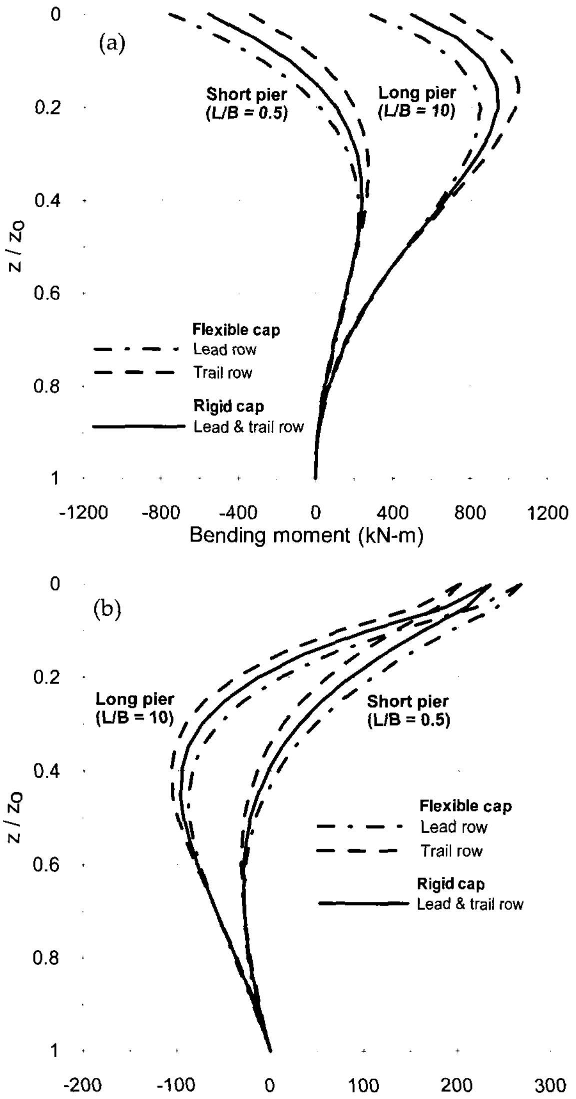


Fig. 11. Effects of the pier length

profiles for various values of the pier length (L). Three different pier lengths (L/B = 0.5 and 10, B: pier diameter) are investigated. Except for the pier length, all other conditions were identical to the conditions in Fig. 3. The condition of L/B = 10 and L/B = 0.5 represents a large and a small M/H ratio (M: bending moment, H: lateral load), respectively. This figure shows that the sign of bending moment at the pile head significantly depends on the M/H ratio. For a ratio of 10, the bending moment at the pile head was developed as a positive value and maximum one at a depth of $z/z_o = 0.15$, whereas for a ratio of 0.5, at the pile head a maximum and negative bending moment was developed. In this figure, the dotted line representing a rigid cap was obtained by using 1,000 larger elastic modulus of a pile cap in the flexible cap analysis.

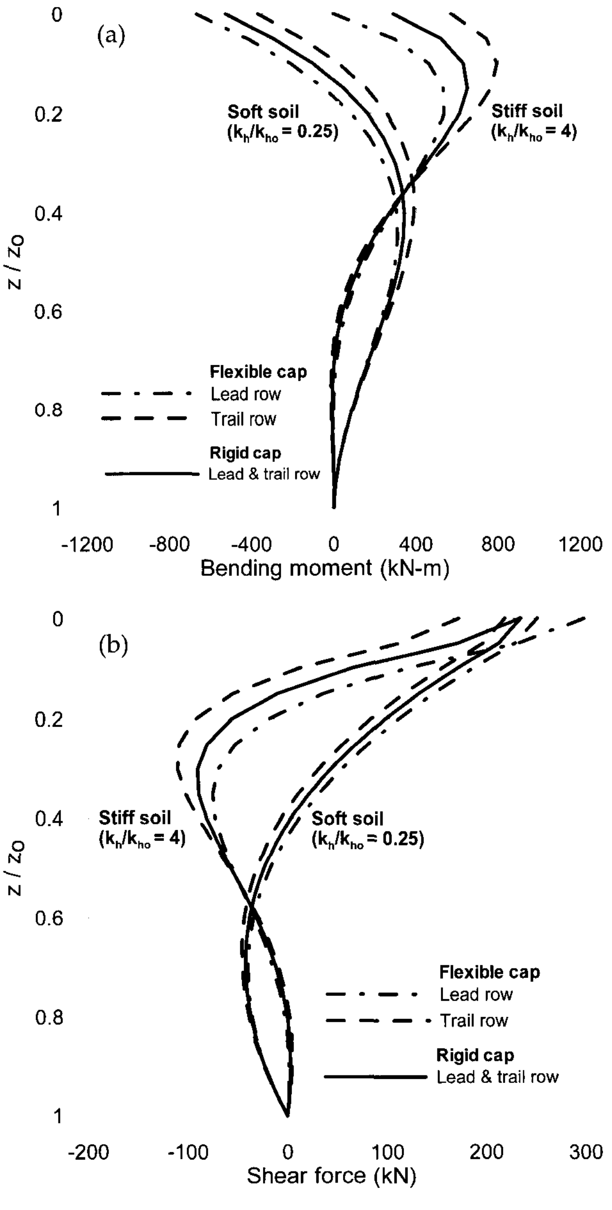


Fig. 12. Effects of the modulus of soil subgrade reaction

3.2.6 Effects of Soil Reaction

Figure 12 shows the bending moment and shear force profiles for different values of the modulus of subgrade reaction (k_h). Two different moduli of lateral subgrade reactions (k_h/k_{h0} = 0.25 and 4) are investigated. Here, k_{ho} is the lateral subgrade reaction of 1-foot diameter plate. This figure shows that there is a significant effect of the soil modulus on the bending moment and shear force of individual piles. Positive bending moment was developed at the pile head for a stiff soil (k_h/k_{h0} = 4) and negative bending moment for a soft soil (k_h/k_{h0} = 0.25). However, unlike the results with varying pier length, the general trend of the bending moment and shear force profiles was changed by the modulus of the lateral subgrade reaction for the stiff soil (k_h/k_{h0} = 4). It is important to mention that for a stiff soil (k_h/k_{h0} = 4), the bending moment and shear force obtained by considering the flexible pile cap show large discrepancy to those by rigid cap rather than for a soft soil. This is because the pile head stiffness of the former is greater than that of the latter and therefore the relative stiffness between the pile and the cap is small.

3.2.7 Effects of Pile Diameter

Figure 13 shows well the effect of pile diameter (D) on the profiles of the bending moment and shear force. Two different pile diameters (D/D_0 = 0.5 and 2) were investigated. Here, D_0 is basic pile diameter. For a small pile diameter (D/D_0 = 0.5), the bending moment and shear force obtained by considering the flexible pile cap are identical to those by rigid cap. However, for a large pile diameter (D/D_0 = 2), significantly large bending moment with a positive value at the pile head was developed for the trail row and also significantly large bending moment with negative value for the lead row. The shear force profile for a large pile diameter shows similar results. Here, it is found that for a large diameter (D/D_0 = 2), the bending moment and shear force obtained by con-

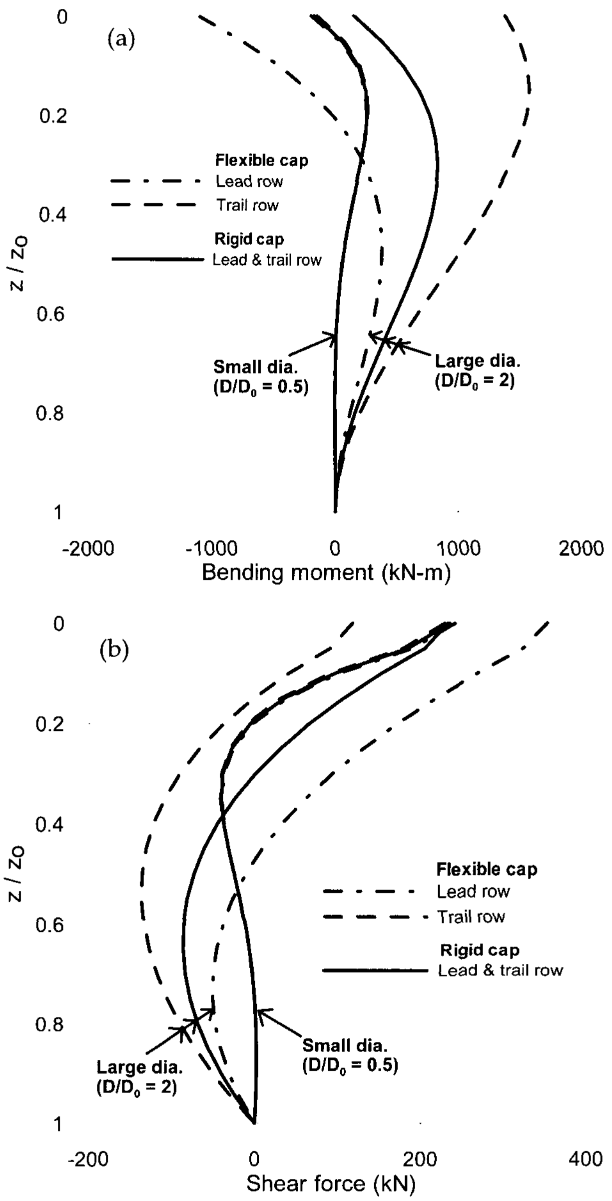


Fig. 13. Effects of the pile diameter

sidering the flexible pile cap shows significantly large discrepancy to those by rigid cap rather than for a small pile diameter. As previously described, the reason for this is that the pile head stiffness of the former is greater than that of the latter, and therefore large stress is developed in the pile cap with its deformation.

4. Summary and Conclusions

The behavior of pile group supported column (piled pier) has been investigated by using a numerical study. The emphasis was quantifying the pile cap flexibility. A numerical method was introduced to overcome the restrictions associated with the conventional stiffness method. The example analysis has clearly demonstrated the important effect of cap flexibility in the pile system. From the findings of this study, the following conclusions are drawn:

(1) The present approach, using a flat-shell element can be applied to clarifying the difference between the flexible cap analysis and rigid cap analysis. The rigid cap analysis by using the stiffness method can consider neither coupling between the pile group and the column nor flexibility of the pile cap because the pile cap is assumed to be rigid.

(2) The flexibility of the pile cap affects individual pile head forces significantly and affects the bending moments and shear forces in individual piles as well, even though the displacement of the pile cap becomes small.

(3) The prediction of the lateral loads and bending moments in the pile cap is much more conservative for a flexible cap than for a rigid cap and thus, represents a definitely larger lateral load and bending moment for various cap thicknesses.

(4) The major parameters highly influencing the cap flexibility are the cap modulus and thickness, the pier length, the soil reactions and the pile diameter. Particularly the effect of flexibility is more significant for groups of large pile diameter, having large subgrade soil reactions.

References

1. Banerjee, P. K., and Driscoll, P. M. (1976), "Three-Dimensional Analysis of Raked Pile Groups", *Proc. of Institution of Civil Engineers*, Vol.61, pp.653-671.
2. Bogard, D. and Matlock, H. (1983), "Procedures for Analysis of Laterally Loaded Pile Groups in Soft Clay", *Proc. Spec. Conf. of Geotech. Engrg. in Offshore Practice*, ASCE, pp.499-535.
3. Brown, D., Morrison, C., and Reese L. C. (1988), "Lateral Load Behavior of Pile Group in Sand", *J. of Geotech. Engrg.*, ASCE, Vol.114, No.11, pp.1261-1276.
4. Choi, C. K., and Lee W, H. (1996), "Versatile variable-node flat-shell element", *J. of Engrg. Mech.*, ASCE, Vol.122, No.5, pp.432-441.
5. Clancy, P., and Randolph, M. F. (1993), "An Approximate Analysis Procedure of Piled Raft Foundations", *Int. J. for Num. and Anal.*

Meth. in Geomech., Vol.17, pp.849-869.

6. Hrennikoff, A. (1950), "Analysis of pile foundations with batter piles", *Transactions* ASCE, Paper No. 2401.
7. Kitiyodom, P., and Matsumoto, T. (2002), "A Simplified Analysis Method for Piled Raft and Pile Group Foundations with Batter Piles", *Int. J. for Num. and Anal. Meth. in Geomech.*, Vol.26, pp.1349-1369.
8. Ooi, P. S. K., Chang, B. K. F., and Wang S. (2004), "Simplified Lateral Analyses of Fixed-Head Piles and Pile Groups", *J. of Geotech. and Geoenviron. Engrg.*, ASCE, Vol.130, No.11, pp.1140-1151.
9. Poulos, H. G. (1971), "Behavior of Laterally Loaded Piles. II: Pile Groups", *J. of Soil Mech. and Found. Div.*, ASCE, Vol.97, No.5, pp.733-751.
10. Randolph, M. F. (1980), "PIGLET: A Computer Program for the Analysis and Design of Pile Groups under General Loading Conditions", *Soil Rep. TR91, CUED/D*, Cambridge Univ., England.
11. Reese, L. C., and Wang, S. T. (2004), "Group 6.0 for Windows, Analysis of a group of piles subjected to axial and lateral loading", Ensoft, Inc., Austin, Tex.
12. Reese, L. C., O'Neill, M. W., and Smith, R. E. (1970), "Generalized analysis of pile foundations", *J. of Soil Mech. and Found. Div.*, ASCE, Vol.96, No.1, pp.235-250.
13. Saul, W. E. (1968), "Static and Dynamic Analysis of Pile Foundation", *J. of Str. Div.*, ASCE, Vol.94, No.5, pp.1077-1100.
14. US Army Corps of Engineers (1991), Engineering and Design: Design of Pile Foundations. *Engineer Manual*, No.1110-2-2906.
15. Zhang, H. H., and Small, J. C. (2000), "Analysis of Capped Pile Groups subjected to Horizontal and Vertical Loads", *Comp. and Geotech.*, Vol.26, pp.1-21.

(received on Aug. 7, 2007, accepted on Sep. 30, 2007)