

Effect of Temperature on Frequency and Damping Properties of Polymer Matrix Composites

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Abstract

The effect of temperature on natural frequency and damping is investigated in two different composite materials, Kevlar 29 fiber woven and polyethylene cloth, used especially to design ballistic armor. A damping monitoring method is used experimentally to measure the frequency response curve and it is also modeled numerically using a finite element program. The natural frequencies of a material, or a system, are a function of its elastic properties, dimensions and mass. This concept is used to calculate theoretical vibration modes of the composites. The damping properties in terms of the damping factor are determined by the half-power bandwidth technique. Numerically analyzed and experimentally measured time response curves are compared. It is seen that polymer matrix composites have temperature dependent mechanical properties. This relationship is functional and they have different effects against temperature.

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Keywords

Polymer matrix composites, damping, frequency, temperature effects, Kevlar, polyethylene

1. Introduction

Advanced laminated composite materials such as Kevlar, polyethylene, and carbon fiber are commonly used in weight sensitive structures due to their high stiffness/weight ratio. It is especially significant in aircraft, aerospace and military applications. Materials used to design ballistic armor require high strength for stiffness, lightness so that they may be carried easily, and high damping capacity to absorb impact energy of a projectile. Another important point for polymer matrix composites is that they have temperature dependent mechanical properties. If elastic modulus increases with increasing temperature, natural frequencies also increase. Therefore, their damping and vibration properties must be investigated under varied temperature before using them in different seasons, climates and regions.

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Some studies were carried out for the analysis of polymer matrix composites with regard to their vibration properties. Adams and Maheri [1] investigated the damping capacity of fiber reinforced plastic and developed a damping energy equation for analysis. Using finite element, Rayleigh-Ritz and experimental methods, damping capacity and frequency of the fiber reinforced plastic composite plate with (0, 90, 0, 90)_s cross-ply were compared at room temperature. Thermal and morphological characteristics of E-glass/Kevlar 49 reinforced siliconized epoxy composites were studied in [2]. Characterization of the impact behavior as well as damage tolerance properties of Kevlar multiaxial warp knit fabric composites based on the energy approach was the objective of Kang and Kim [3]. The multiaxial warp knit fabric composite structures were compared with those of Kevlar woven laminates in this study to characterize and evaluate the impact damage mechanisms of Kevlar multiaxial warp knit fabric composites. Variations of tangent and storage modulus with temperature were investigated by Tjong *et al.* [4] for maleic anhydride compatibilized short glass fiber/SEBS/polypropylene hybrid composites. The study of Kim and Hwang [5] examined the effect of debonding on the natural frequency and flexural rigidity, and on the changes in frequency response functions of sandwich beams. The elastic modulus of resin-based materials was determined as a function of resonance frequency during polymerization by Meredith [6]. Using also an experimental method, dynamic Young's modulus and damping factors for a Kevlar 49 fabric-reinforced polyester composite material were investigated in [7]. Vinylester-resin-matrix composites reinforced with untreated and 5% NaOH treated jute fibers with different fiber loading were subjected to dynamic mechanical and thermal analysis to determine their dynamic properties as a function of temperature [8]. The storage modulus for all the examined composites decreased with increasing temperature, with a significant fall in the temperature range 110–170°C. The damping of glass and Kevlar composites was analyzed experimentally as a function of frequency and fiber orientation using a cantilever beam test specimen and an impulse technique at the room temperature by Berthelot and Sefrani [9]. The damping parameters were derived by fitting the experimental Fourier responses with the analytical motion responses. The same authors also studied the temperature effect on damping and bending modulus of unidirectional glass fiber composite [10]. They used the Ritz method and a measurement technique to define the damping. Characterizing the damping properties of interleaved carbon-fiber/epoxy laminates such as polyurethane elastomers, polyamide elastomers and polyethylene-based ionomers was the main objective of Kishi *et al.* [11]. Wei and Kukureka [12] evaluated the damping and elastic properties of composites and composite structures experimentally by the resonance technique. The dynamic behaviour of the composite supporter was investigated by Zhang and Chen [13] numerically and experimentally. The numerical and experimental results show that the proposed finite element modeling and analysis procedure could be used effectively to characterize the vibration behaviour of the composite supporter. Dynamic mechanical behavior of natural rubber and its composites reinforced with short coir fibers was studied by Geethamma

et al. [14]. Elastic and tangent modulus against temperature was examined. It was found that there is a nonlinear relation between loss factor and temperature. Finally, Lopez-Manchado and Arroyo [15] studied thermal and dynamic mechanical properties of polypropylene and short organic fiber composites. Elastic modulus and loss factor against temperature were also investigated.

As explained above in the literature review, only a few of the studies analyzed the damping properties of laminated composites as a function of temperature for different composite materials. Therefore, the objective of this study was chosen as the effect of temperature on the damping and frequency in two different polymer matrix composites, Kevlar 29 and polyethylene cloths, used to design ballistic armor. First, a tension test was applied to the specimens to get mechanical properties of both composites. Then, temperature dependent frequency responses were measured experimentally using a damping monitoring technique [16] and modeled numerically by a finite element method using Ansys software. Numerical and experimental results were comparable in the study.

2. Experimental and Numerical Methods

2.1. Composite Materials

Two different composite materials, Kevlar 29/polyvinyl butyral and polyethylene (UHMW-PE UD-HB2) are used in the experiment. Kevlar 29 manufactured by Du-Pont Company is a woven fabric composite but every lamina of polyethylene manufactured by Dyneema Company has 0 and 90 deg. fiber layers on it. Therefore, the elastic modulus of the lamina is the same in the principal directions for these materials ($E_1 = E_2$). Composite laminates are produced using Kevlar 29/polyvinyl butyral and polyethylene laminas. The composite armor manufacturing process is shown in Fig. 1 as a flow chart. Kevlar 29/polyvinyl butyral composite specimens are manufactured at 160°C under 6.5 MPa pressures for a total pressing time of 15 min. Polyethylene fiber composite specimens, on the other hand, are manufactured at 125°C under 20 MPa pressures for a total pressing time of 30 min.

Firstly, the specimens are applied tension test to get mechanical properties. The results are shown in Table 1. Although Kevlar 29 fibers are almost perfectly linear elastic materials, laminate specimens have non-linear elastic structure and the elastic modulus quite decreases from 85 to 7 GPa because polymer matrix resin has 2 GPa elastic modulus and the volume fraction of the woven Kevlar 29 fibers is 15% in the composite. The pressing temperature and pressure during the fabrication are also significant in its manufacture. The elastic modulus is calculated using tensile strength and elongation and it is accepted as linear up to this point. Measured, calculated and numerically determined frequencies prove that the accuracy of this elastic modulus is almost perfect (Table 2). In addition, polyethylene fibers and lamina have similar linear elastic characteristics to Kevlar 29 fibers. On the other hand, polyethylene composite is also non-linear elastic structure and its elastic modulus is approximately 25.5 MPa.

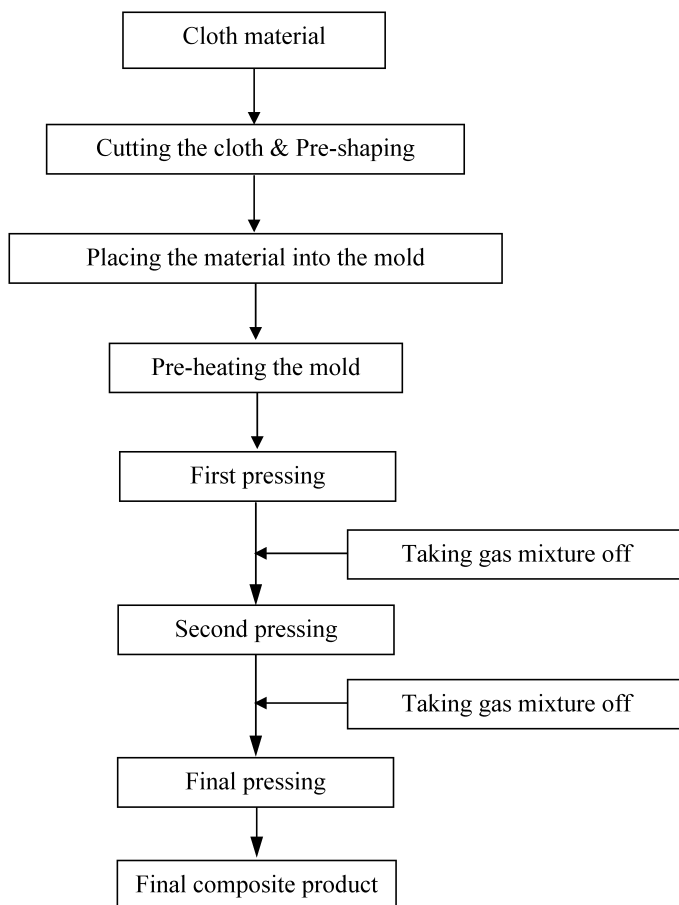


Figure 1. Manufacturing process of polymer matrix composites.

Table 1.

Mechanical properties of composites used in the experiment

Materials	Type	Modulus E (GPa)	Density ρ (kg/m ³)	Tensile strength (MPa)	Failure strain (%)
Aramid	Fiber	$E_1 = 85$	1440	$\sigma_1 = 3160$	$\varepsilon_1 = 3.7$
	Laminate	$E_1 = 7$	1160	$\sigma_1 = 450$	$\varepsilon_1 = 10$
		$E_2 = 7$		$\sigma_2 = 450$	$\varepsilon_2 = 10$
		$E_3 = 2.4$			
Polyethylene	Fiber	$E_1 = 115$	970	$\sigma_1 = 3500$	$\varepsilon_1 = 3.5$
	Laminate	$E_1 = 25.5$	900	$\sigma_1 = 860$	$\varepsilon_1 = 8$
		$E_2 = 25.5$		$\sigma_2 = 860$	$\varepsilon_2 = 8$
		$E_3 = 3.4$			

Table 2.

The temperature dependent first natural frequency of Kevlar 29 fiber composite beam (the size of the specimen used in this measurement is $4 \times 30 \times 305 \text{ mm}^3$)

Temperature (C°)	Measured f_1 (Hz)	Analytical f_1 (Hz)	Numerical f_1 (Hz)
60	97.51	97.3	97.16
55	100.35	99.42	99.28
50	101.72	101.4	101.25
45	102.46	103.2	103.04
40	103.92	104.81	104.65
35	105.3	106.21	106.05
30	107.6	107.38	107.21
25	110.53	108.29	108.12
20	110.16	108.91	108.75
15	109.07	109.23	109.06
10	109.8	109.21	109.05
5	108.7	108.91	108.75
0	107.6	108.05	107.89
-5	106.12	106.84	106.68
-10	104.66	105.14	104.98
-15	104.3	102.9	102.75

Ten-layered beam specimens are used in the experiment. The length (L) and width (w) of the beams are 300 mm and 30 mm, respectively. The thickness (t) is varied from 4 to 4.3 mm for Kevlar 29/polyvinyl butyral specimens and from 2.8 to 2.9 mm for polyethylene specimens on these measurements. To measure magnitude of the impact load, a strain gage, strain gage conditioner and power supply, and voltmeter are used. The strain gage is stuck on the mid-point of the specimen where the deflection is maximum for the first natural frequency. Then, an impact load is applied to the specimen by hand to induce vibration on it using a sphere steel ball hammer as seen in Fig. 2. The voltage measured by a voltmeter is recorded. The strain-gage is calibrated using a force-voltage graph to get the magnitude of the impact load. Its magnitude is generally measured from 3.5 to 4 N. The reason for this range is that the hammer is used by hand. This load level is also used in numerical investigation to get the frequency response and both results are compared. The impact load is taken as 3.65 N in the numeric analysis for both composites because the magnitude of the impact load is usually measured between 3.6 and 3.7 N experimentally. Although the magnitude of the impact load does not make any difference in the natural frequency and damping factor in the frequency response, it is effective in the amplitude of the natural frequency peaks.

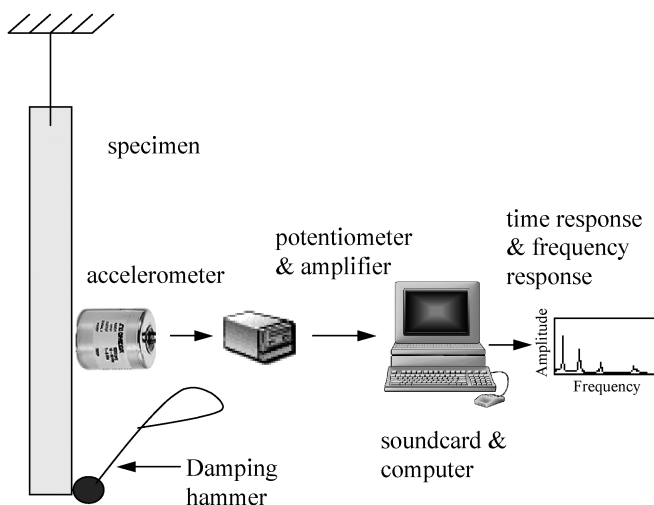


Figure 2. Diagrammatic view of the damping monitoring method.

2.2. Measurement of Damping and Natural Frequency

The damping factor and natural frequency are measured experimentally using a damping monitoring technique [17] from the frequency response. In this process, the specimen is firstly attached to a platform using a thin wire to measure the damping factor and natural frequency at a free–free boundary condition as seen Fig. 2. The accelerometer is placed just below the mid-point to get also the peak of the second vibration mode. A vibration is induced in the specimen using a steel ball hammer because an impulse or impact load must be apply to the specimen to get the time response. The accelerometer measures the vibration and produces an electrical signal that is amplified by the amplifier and then input to the computer. The amplified signal is measured by the soundcard for data acquisition. A Fast Fourier Transform (FFT) is performed by the software for the signal providing a measurement of the lateral natural vibration modes. This damping monitoring device has two different programs: the first one controls hardware and the second one is used to obtain the frequency response from the time response using the FFT. The computer also determines the damping factor from the frequency response by the half-power bandwidth method using a curve fitting technique.

2.3. Numerical Model and Investigation

Two different analyses, modal and harmonic, are made in ANSYS 9.0 finite element software. A linear elastic orthotropic model is used to investigate the cloth composite laminate beams. Only the elastic modulus of the materials is necessary for the analysis. Mechanical properties of the laminate beams are taken from Table 1. For this problem, the BEAM3 (Beam 2D elastic) element is used. This element has 3 degrees of freedom (translation along the X and Y axes, and rotation about the Z axis). To determine the natural frequencies, modal analysis is carried out using

a subspace method in Ansys 9.0 at free–free boundary conditions. Harmonic analysis is also investigated to get the frequency response numerically. A stepped loads method is used to describe the response with 3.65 N load which is also applied to the specimen experimentally. At the same time, the number of sub-steps is 300 in the analysis for 0–750 Hz frequency range. Finally, the measured damping factor is used to compare both frequency responses.

3. Theory

The lateral vibration of a beam can be derived from the Euler–Bernoulli equation [18]:

$$\frac{\partial^4 y}{\partial x^4} = -\frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}, \tag{1}$$

where $c = \sqrt{E_1 I / \mu}$. Using the method of separation of variables to solve equation (1):

$$y(x, t) = W(x)T(t).$$

To satisfy the differential equation:

$$L(w) = \frac{1}{c^2} \frac{\partial^2 w}{\partial t^2} \tag{2}$$

and (changing y with w) $w(x, t) = W(x)T(t)$. Then equation (2) yields:

$$L(W) = \frac{1}{c^2} \frac{T''}{T}. \tag{3}$$

It is observed that the left side of equation (3) is a function of x although the right-hand side is a function of t only. If equation (1) is rewritten again in the form

$$W^{(IV)} - \left(\frac{\omega}{c}\right)^2 W = 0, \tag{4}$$

where ω is a constant, the substitution of $W(x) = e^{kx}$ gives the general solution:

$$W(x) = A \cosh kx + B \sinh kx + C \cos kx + D \sin kx, \tag{5}$$

where $k = \sqrt{\omega/c}$ and A, B, C, D are constants. Four boundary conditions are necessary to determine A, B, C and D . In addition, to compare the theoretical and experimental natural frequency modes, equation (5) must be solved for free–free boundary conditions which are:

$$\frac{\partial^2 y}{\partial x^2} \Big|_{x=0} = 0, \quad \frac{\partial^2 y}{\partial x^2} \Big|_{x=L} = 0, \quad \frac{\partial^3 y}{\partial x^3} \Big|_{x=0} = 0, \quad \frac{\partial^3 y}{\partial x^3} \Big|_{x=L} = 0, \tag{6}$$

for the lateral vibration of the beam. L is the length of the beam. Finally, the solution gives frequency equation:

$$f_i = \frac{\beta_i^2}{2\pi L^2} \sqrt{\frac{E_1 I}{\rho A}}, \tag{7}$$

where E_1 is the elastic modulus in x direction, I is moment of inertia, A is cross sectional area, β_i is a constant and $i = 1, 2, \dots, n$ for natural frequencies. At the free–free boundary condition, $\beta_1 = 4.73$, $\beta_2 = 7.853$ and $\beta_3 = 10.996$ for the first, second and third vibration modes, respectively. To calculate the elastic modulus from the measured frequency response curve and to compare it with the result from the tension test:

$$E_1 = \frac{\rho AL^4}{I\beta_i^4} (2\pi f_i)^2. \quad (8)$$

4. Results and Discussions

Using equation (7) the analytically calculated, measured, and numerically found first natural frequencies are shown in Table 2 for Kevlar 29/polyvinyl butyral composite laminate. It is seen that Kevlar 29 fiber composite laminate is a temperature dependent material. Its first natural frequency decreases with not only increasing but also decreasing temperature from the room temperature. The reason for this is that the transition temperature is close to the room temperature, 25°C, for Kevlar 29/polyvinyl butyral composite. This approach is reasonable because short glass fiber polypropylene hybrid composites [4] have their transition temperature around 20°C. In addition, the transition temperature of polypropylene and short organic fiber composites [15] approximately equals 15°C.

In addition to experimental results, Table 2 also shows analytical and numerical results, which are in good agreement. To get analytical and numerical frequencies, the elastic modulus is calculated using equation (8) and those results are normalized using a polynomial curve-fit equation. In addition, the size of the beam specimens is measured under different temperatures from –15 to 60°C because it may affect the natural frequencies but no change is observed in them. The natural frequencies of polyethylene laminate beam are also temperature dependent but these decrease with increasing temperature only, as seen Table 3 because any transition temperature is not covered in the range of –15 and 60°C for polyethylene fiber composite.

The damping factor of both composite beam specimens is measured using the damping monitoring method for the first natural frequency with free–free boundary conditions under varied temperature. There is a functional relationship opposite to the frequency between temperature and the damping factor as seen in Fig. 3. The damping factor of polyethylene varies approximately 100% from –10 to 60°C. On the other hand, the damping factor of Kevlar 29 beam increases from 25°C to –15 and 60°C due to the fact that the transition temperature is close to room temperature. The maximum increase of approximately 50% occurred at 60°C in this experiment.

The mean of the first natural frequency of Kevlar 29 fiber composite $f_1 = 110.53$ Hz and the damping factor $\zeta_1 = 0.0148$ at the room temperature, 25°C. In addition, $f_1 = 175.17$ Hz and $\zeta_1 = 0.0124$ for polyethylene fiber composite beam. Those values for 1018 hot rolled carbon steel beam with different size are measured as $f_1 = 65$ Hz and $\zeta_1 = 0.0044$ [17]. $f_1\zeta_1$ gives a constant independent of

Table 3.

The temperature dependent first natural frequency of polyethylene composite beam (the size of the specimen used in this measurement is $2.9 \times 30 \times 301 \text{ mm}^3$)

Temperature (C°)	Measured f_1 (Hz)	Analytical f_1 (Hz)	Numerical f_1 (Hz)
60	168.81	169.29	169.06
55	169.46	169.65	169.41
50	170.44	170.22	169.98
45	171.26	170.99	170.76
40	171.5	171.98	171.74
35	172.7	173.17	172.93
30	174.42	174.55	174.31
25	175.17	175.34	175.1
20	177.73	177.87	177.63
15	179.04	179.83	179.58
10	182.37	181.96	181.71
5	185.42	184.26	184.01
0	187.08	186.73	186.48
-5	190.18	189.36	189.1
-10	190.74	192.11	191.85

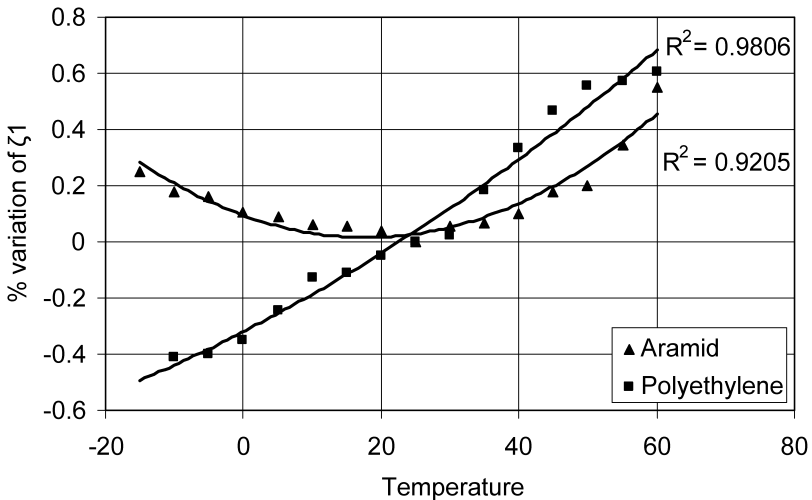


Figure 3. Percentage variation on the damping factor of Kevlar 29 and polyethylene fiber composite specimens under varied temperatures.

beam size to compare the damping capacities with each other. This constant or the damping capacity for Kevlar 29 and polyethylene fiber composites is about 5.7 and 7.6 times, respectively, more than that of 1018 hot rolled carbon steel beam.

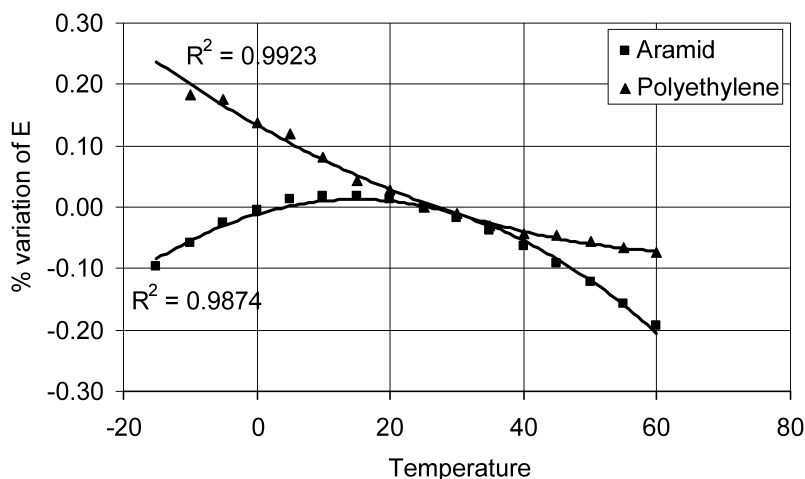


Figure 4. Percentage variation in E_1 of Kevlar 29 and polyethylene composites under varied temperatures.

Another investigation in this study is the elastic modulus (E_1), which is experimentally found by tension test as 6.98–7 GPa for Kevlar 29 composite and as 25.6 GPa for polyethylene composite at the room temperature. Using equation (8), E_1 is also calculated from the frequency response. It is determined as $E_1 = 6.6$ GPa for Kevlar 29 and $E_1 = 25.5$ GPa for polyethylene fiber composites, which are in perfect agreement with experiment. Temperature dependent percentage variation of E_1 , which is calculated using the measured first natural frequency is shown in Fig. 4. It is seen that E_1 of both materials have different functional relationships against temperature because Kevlar 29/polyvinyl butyral composite has a transition temperature close to room temperature. On the other hand, any transition temperature is not covered in the range of -15 to 60°C for polyethylene fiber composite. Similar to the natural frequency, the elastic modulus decreases with increasing temperature, but this rule is not applicable to Kevlar 29/polyvinyl butyral composite due to the transition temperature.

The output of the measured frequency response is shown in Fig. 5. It is measured more than 10 times and one of the best signals for each material is used as a figure in this study. The mean of the measured frequencies and damping factors are recorded as data, which are shown or used in Tables 2 and 3, and in Figs 3 and 4 for both composites.

Figure 6 shows how to measure the damping factor using the curve-fitting technique which is controlled by the software of the damping monitoring method. The results of the damping factor and first natural frequency are also shown in Fig. 6 for polyethylene fiber composite at room temperature. They are recorded as 0.0128 and 173.69 Hz in this measurement.

Numerically modeled frequency response is also investigated using the harmonic analysis in Ansys. Figure 7 shows the response of Kevlar 29/polyvinyl butyral com-

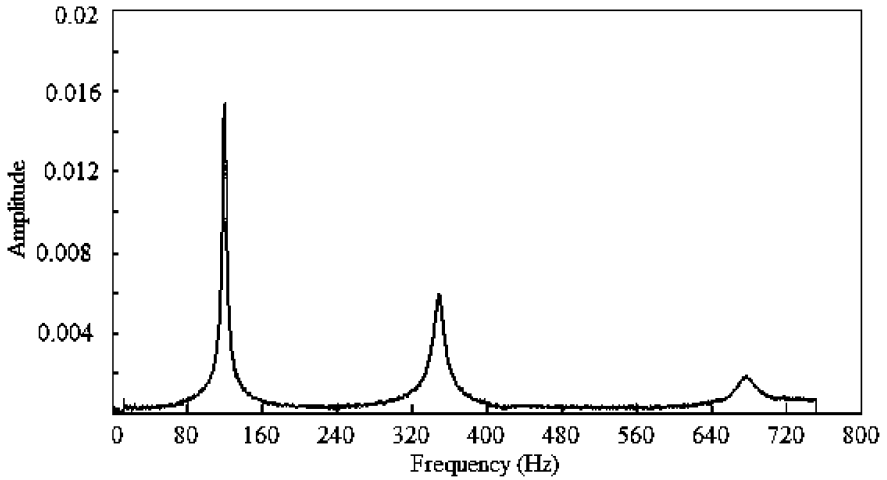


Figure 5. Measured frequency response for Kevlar 29 fiber composite at room temperature.

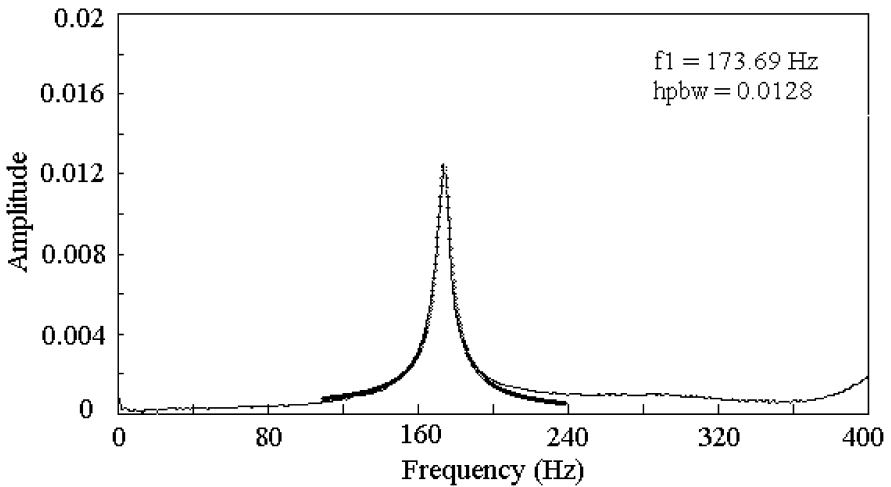


Figure 6. Measurement of the damping factor using the damping monitoring method.

posite with the damping factor using the impact load, 3.65 N. The peaks of the experimental and numerical first three natural frequencies are seen clearly in Figs 5 and 7. Both responses are shown in Fig. 8 for better comparison. The peaks of the first natural frequency are almost in good agreement. However, the experimentally measured second and third peaks have not only 6 and 5% respectively bigger natural frequencies but also bigger amplitudes if they are compared with numerical peaks. These results show that the numerical method can be used to analyze the dynamic behavior of Kevlar 29/polyvinyl butyral composite for a preliminary result in complex structures in addition to experimental techniques.

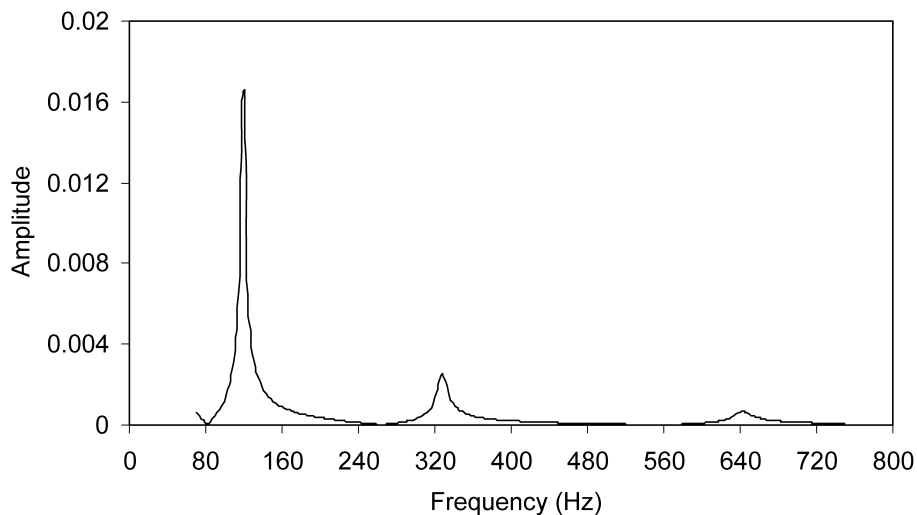


Figure 7. Numerical frequency response for Kevlar 29 fiber composite at the room temperature.

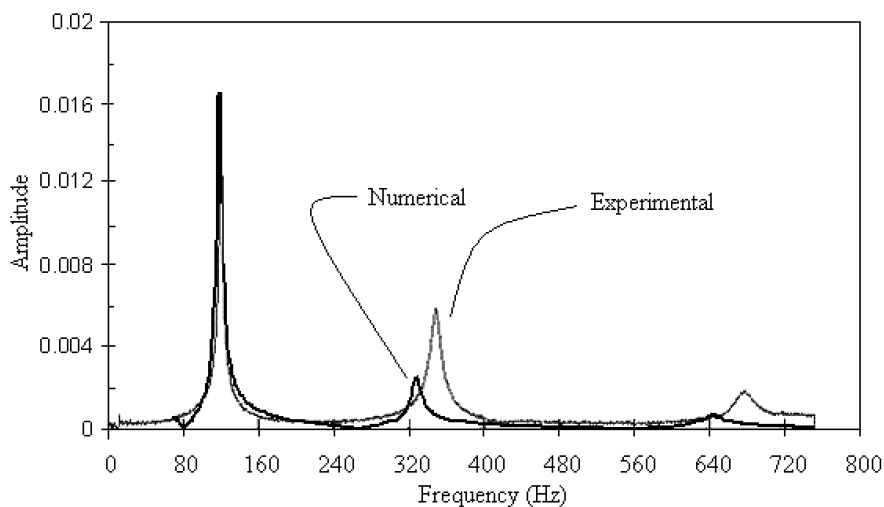


Figure 8. Comparison of the frequency responses for Kevlar 29/Polyvinyl butyral composite at room temperature.

The same analysis is also done for polyethylene fiber composite beam. Only the first and second natural modes are seen in Fig. 9 because the third mode is out of scale in this analysis. When the experimental and numerical frequency responses are compared, their amplitudes are almost same for the first and second peaks. Similar to Kevlar 29/polyvinyl butyral composite, the first natural frequencies are nearly in good agreement. On the other hand, the numerically found second peak has 15% larger natural frequency than the experimental result. Finally, the numerical results for the first natural vibration mode at -10 , 25 and 60°C are compared in Fig. 10.

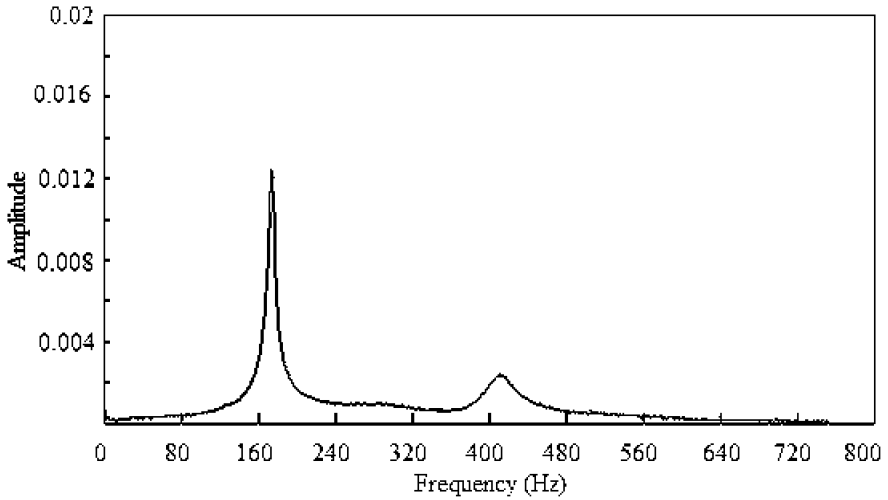


Figure 9. Measured frequency response of polyethylene fiber composite at room temperature.

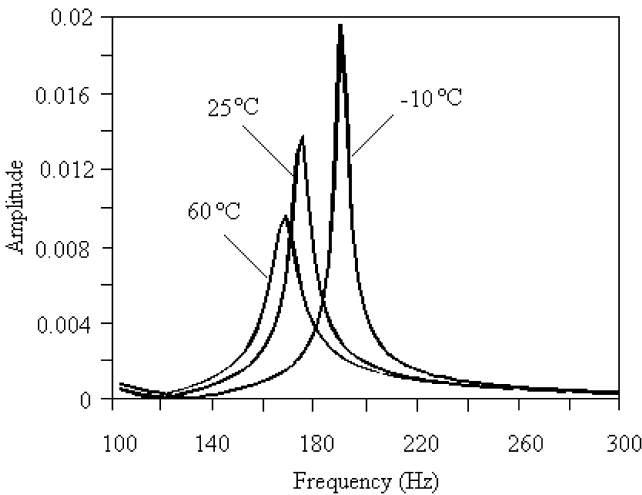


Figure 10. Temperature effect on the numerical frequency response for the first vibration mode of polyethylene fiber composite at -10 , 25 and 60°C .

The natural frequency decreases with increasing temperature but the damping factor increases. In addition, similar to the natural frequency, the peak amplitude decreases with increasing temperature under the constant induced load.

5. Conclusions

Kevlar 29 and polyethylene fiber composites, which are mostly used in weight sensitive structures such as light armor design, are experimentally and numerically investigated in this study. The effect of temperature on the frequency, the damp-

ing factor, and the elastic modulus are analyzed. It is observed that these properties are temperature dependent for both composites but functional relations are different. The natural frequency decreases with not only increasing but also decreasing temperature from the room temperature for Kevlar 29 fiber composite because its transition temperature is observed at 25°C. However, it decreases with increasing temperature only for polyethylene fiber laminate beam. Similar effects are seen for variation of the elastic modulus. In the range of –15 to 60°C, the elastic modulus changes by a maximum of 20% for Kevlar 29 and 25% for polyethylene composite beams. The damping factor also varies with temperature but if it is compared to frequency, it has the opposite relationship. Finally, experimentally measured and numerically modeled frequency response curves are compared. It is seen that they are almost in good agreement in the first mode. As a result, having found the material properties of Kevlar 29 and polyethylene fiber composite experimentally, the numerical method could be used to determine the dynamic behavior of the materials for a preliminary result.

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