

# Comparative Study on the Selection Algorithm of CLINAID using Fuzzy Relational Products

Chan Sook Noe

Department of Technical Management Information System Woosong University

## Abstract

The Diagnostic Unit of CLINAID can infer working diagnoses for general diseases from the information provided by a user. This user-provided information in the form of signs and symptoms, however, is usually not sufficient to make a final decision on a working diagnosis. In order for the Diagnostic Unit to reach a diagnostic conclusion, it needs to select suitable clinical investigations for the patients. Because different investigations can be selected for the same patient, we need a process that can *optimize* the selection procedure employed by the Diagnostic Unit. This process, called a selection algorithm, must work with the fuzzy relational method because CLINAID uses fuzzy relational structures extensively for its knowledge bases and inference mechanism. In this paper we present steps of the selection algorithm along with simulation results on this algorithm using fuzzy relational products, both harsh product and mean product. The computation results of applying several different fuzzy implication operators are compared and analyzed.

Key Words : fuzzy relational products, fuzzy logic, fuzzy implication operators, harsh relational products, mean relational products, CLINAID

## 1. Introduction

The Diagnostic Unit of CLINAID can infer working diagnoses for general diseases from the information provided by a user [1]. A general disease represents a group of specific diseases with *similar* signs and symptoms. This user-provided information, however, is usually not sufficient to make a final decision on a working diagnosis. Accordingly, the Diagnostic Unit needs more information about the patient's illness to reach a diagnostic conclusion. There must be a way to obtain more information about the patient, at the specific disease level. A set of available medical investigations are first inferred by the relation from general diseases to medical investigations. The Diagnostic Unit must then choose one or more investigations from those that have been inferred, and request they be performed, in order to acquire more information about the patient's illness.

In order for the Diagnostic Unit to reach a diagnostic conclusion, it needs to select suitable clinical investigations for the patients. Because different investigations can be selected for the same patient, we need an algorithm that can *optimize* the selection procedure employed by the Diagnostic Unit. This algorithm is called a selection algorithm. Since each of these investigations has a different cost related to it which is stored in the relational knowledge base, we can use this information, called the generalized cost, as the selection criteria [2].

Finding the partial order of the generalized cost of

each investigation is completed by means of the pre-order of the fuzzy relations computed by the triangle fuzzy relational products and fast fuzzy relational algorithms [3]. The triangle products and other fuzzy relational products are further described in the next section.

This paper presents steps of the selection algorithm along with simulation results on this algorithm using fuzzy relational products, both harsh product and mean product. The computation results of applying several different fuzzy implication operators are compared and analyzed.

## 2. Theoretical Background

### 2.1 General Overview of CLINAID

CLINAID is a medical knowledge-based system that is designed to capture the complete semantics of medical activity in a hospital environment using fuzzy relational method as its main computational engine. It is intended to assist in not only the diagnosis of a patient's illness, but also other types of hospital activities such as consultation, the prescription of medication, and update of patient records, etc.

The conceptual structures as well as the basic architecture of CLINAID have been described in [1][4]. To give complete cover for all hospital activities, the basic architecture of CLINAID consists of four main cooperating units, each unit comprising a complex autonomous subsystem. The four main units of CLINAID are:

a) *Diagnostic Unit*, which infers a working diagnosis based on the information given by a user, or alternatively sends an indication that the information provided is not sufficient to produce a working

diagnosis.

- b) *Patient Clinical Record Unit*, which maintains and updates patients medical records. It also allows the information to be retrieved as required in order to ensure that all the relevant patient information can be provided at critical moments.
- c) *Treatment Recommendation Unit*, which advises appropriate treatment and medication for the patient based on the working diagnosis provided by the Diagnostic Unit. These recommendations take account of both the personal history of the patient and any possible adverse effects from the available treatments.
- d) *Planning and Co-ordination Unit*, which controls communication and interaction with the other units. It also ensures that the information necessary to the current activity of each unit is always provided.

## 2.2 Fuzzy Relational Method

The fuzzy relational method provides a means of analyzing real-world scientific data as an effective mathematical tool for structural analysis. It helps to identify meaningful structures implicit in real-world data. This method can also be used in Knowledge Engineering. A fuzzy knowledge-based system, such as CLNAID, which utilizes the fuzzy relational method to capture, represent, and infer knowledge, can be operated and manipulated in a unified computational framework [5][6].

Practically, the fuzzy relational method was established using fuzzy relational products, and related computational procedures such as fast fuzzy relational algorithms. A brief explanation of the fuzzy relational method is given below [7].

Given two relations, the relation  $R$ , which is an element of the lattice of relations from set  $A$  to set  $B$ , i.e.,  $R \in \mathcal{R}(A \rightarrow B)$  and the relation  $S \in \mathcal{R}(B \rightarrow C)$ , a product relation  $(R * S)$  is a relation from  $A$  to  $C$ , determined by  $R$  and  $S$ . There are several types of product used to yield product relations, each having distinctive mathematical properties and applicability. Of those four definitions will be presented by the scheme of  $(R * S) \in \mathcal{R}(A \rightarrow C)$ , where  $*$   $\in$   $\{ \circ, \triangleleft, \triangleright, \square \}$ .

Given relations  $R \in \mathcal{R}(A \rightarrow B)$  and  $S \in \mathcal{R}(B \rightarrow C)$ ,

1. The circle product is defined as:

$$a(R \circ S)c \Leftrightarrow (aR \cap Sc) \neq \emptyset.$$

In this product relation, an element  $a$  is related to an element  $c$  by the composed relation  $(R \circ S)$  if and only if the intersection of the afterset of  $a$  and the foreset of  $c$  is non-empty. That is, there exists at least one element common to the afterset of  $a$  and the foreset of  $c$ .

2. The triangle subproduct is defined as:

$$a(R \triangleleft S)c \Leftrightarrow aR \subseteq Sc.$$

In this product relation, an element  $a$  is related to an element  $c$  by the composed relation  $(R \triangleleft S)$  if and only

if the afterset of  $a$  is a subset of the foreset of  $c$ .

3. The triangle superproduct is defined as:

$$a(R \triangleright S)c \Leftrightarrow aR \supseteq Sc.$$

In this product relation, an element  $a$  is related to an element  $c$  by the composed relation  $(R \triangleright S)$  if and only if the afterset of  $a$  is a superset of the foreset of  $c$ .

4. The square product is defined as:

$$a(R \square S)c \Leftrightarrow aR = Sc.$$

In this product relation, an element  $a$  is related to an element  $c$  by the composed relation  $(R \square S)$  if and only if the afterset of  $a$  is exactly equal to the foreset of  $c$ . Mathematically, the square product is the intersection of two triangle products, that is  $(R \triangleleft S) \cap (R \triangleright S)$ .

## 2.3 Fuzzy Implication Operators

The definitions described in the previous section work for both Boolean logic and fuzzy logic. For Boolean logic computations, the logical connectives used in definitions ( $\wedge, \vee, \rightarrow$ ) are Boolean logic operators; such as, *and* for  $\wedge$ , *or* for  $\vee$ , and a material implication operator for  $\rightarrow$ .

However, to be of any real use, above product relations must be fuzzified so that the meanings of the product relations can have varying degrees of truth instead of absolutely true or false values. Thus, for the fuzzy logical computations discussed in this work, the following logical connectives are used: *min* for  $\wedge$ , *max* for  $\vee$ , and fuzzy implication operators for  $\rightarrow$ .

Various fuzzy implication operators exist which use different fuzzy logics. The standard condition, commonly accepted in literature which requires the compliance of each fuzzy implication operator, is that it must agree with Boolean logic values at the corners of its own implication table. That is, fuzzy variables must take the crisp value of 0 or 1 at the corners. In this paper, we will consider 10 fuzzy implication operators listed in Table 1. For detailed explanations, see [8].

There exist two versions of computation for triangle and square products: *harsh and mean*. The harsh criterion takes the minimum value over all the elements, whereas the mean criterion takes the arithmetic mean. For instance, the triangle product can be computed either using the harsh criterion as  $(R \triangleleft S)_{ik} = \wedge_j (R_{ij} \rightarrow S_{jk})$ , or the mean criterion as  $(R \triangleleft S)_{ik} = \frac{1}{N} \sum_{j=1}^N (R_{ij} \rightarrow S_{jk})$ , where  $N$  is the number of elements which are involved in the computation.

Thus the computation of fuzzy relational products depends on the choice of the fuzzy implication operator and the version of the two computational criteria used in computing it. As soon as these two are selected, the computation of each element in the matrix of the composed relation becomes a well-determined task.

Table 1. Fuzzy Implication Operators

1.	$S^\#$	Standard Sharp $a \rightarrow_1 b = \begin{cases} 1 & \text{if } a \neq 1 \text{ or } b = 1 \\ 0 & \text{otherwise} \end{cases}$
2.	$S$	Standard Strict $a \rightarrow_2 b = \begin{cases} 1 & a \leq b \\ 0 & \text{otherwise} \end{cases}$
3.	$S^*$	Standard Star $a \rightarrow_3 b = \begin{cases} 1 & a \leq b \\ 0 & \text{otherwise} \end{cases}$
4.	$G43$	Gaines 43 $a \rightarrow_4 b = \min\left(1, \frac{b}{a}\right)$
4'.	$G43'$	Modified Gaines 43 $a \rightarrow_{4'} b = \min\left(1, \frac{b}{a}, \frac{1-a}{1-b}\right)$
5.	$\text{Ł}$	Łukasiewicz $a \rightarrow_5 b = \min(1, 1 - a + b)$
5.5.	$KD\text{Ł}$	Kleene-Dienes-Łukasiewicz $a \rightarrow_{5.5} b = \min(1, 1 - a + ab)$
6.	$KD$	Kleene-Dienes $a \rightarrow_6 b = (1 - a) \vee b$
7.	$EZ$	Early Zadeh $a \rightarrow_7 b = (a \wedge b) \vee (1 - a)$ $= (a \rightarrow_6 b) \wedge ka,$ where $ka = (1 - a) \vee a$
8.	$W$	Willmott $a \rightarrow_8 b = ((1 - a) \vee b) \wedge (a \vee (1 - b) \vee (b \wedge (1 - a)))$ $= (a \rightarrow_7 b) \wedge kb$ $= (a \rightarrow_6 b) \wedge ka \wedge kb$

### 3. Diagnostic process of CLINAID

In this section, the diagnostic process of CLINAID is presented in terms of the fuzzy relational products. The diagnostic process consists of the four levels: body system level, syndrome level, general disease level, and specific disease level.

When each level is activated, only the fuzzy relations that are relevant for reaching the conclusion of that particular level are utilized [9]. So, the knowledge represented at each level is captured in a fuzzy relational structure and the inference is performed by means of the fuzzy relational products. As fuzzy methods work with linguistic labels, we also have to define the clinical meaning of all relational components that enter into fuzzy relational computations. This is captured in the form of semiotic descriptors of basic sets and relations [10]. Hence, each set and relation are assigned non-mathematical meaning carried by their semiotic descriptors. The semiotic descriptors of all the sets entering into the fuzzy relations of the diagnostic process are shown in Table 2.

Each level of the diagnostic process in terms of the fuzzy relational products is presented below.

Table 2. Basic sets

Name	A set of
$B$	body systems
$C$	generalized costs
$D$	specific diseases
$G$	general diseases
$I$	investigations
$J$	investigation results
$P$	patients
$S$	signs and symptoms
$Y$	syndromes

- Body system level:  $SB \in R(S \sim B)$   
The purpose of the fuzzy relation  $SB$  is to identify the body systems relevant to the signs & symptoms presented by the user.

- Syndrome level:  $SY \in R(S \sim Y)$   
The purpose of the fuzzy relation  $SY$  is to identify the possible syndromes that are related to the body systems inferred at the body system level.

- General disease level:  $YG \in R(Y \sim G)$   
The purpose of the fuzzy relation  $YG$  is to identify the general diseases of the patient related to the syndromes.

- Specific disease level:  $GI \in R(G \sim I)$   
The purpose of the fuzzy relation  $GI$  is to identify the possible investigations that can be performed for a particular general disease.

$$- JD \in R(J \sim D)$$

The purpose of the fuzzy relation  $JD$  is to identify the possible specific diseases that might be currently in the working diagnoses set.

### 4. Steps of the Selection Algorithm

STEP 1 is to obtain the preorder of investigations according to their monetary cost, the efficiency of the test, and the convenience to the patient. The fuzzy relational triangle product  $(IC \triangleleft IC^T)$  is formed first by composing two fuzzy relational input matrices using a fuzzy implication operator. The implication operator can be either specified by the user or chosen by the system. Also, a harsh or mean criterion is selected by either the user or the system. Then the local preorder closure of the fuzzy relational triangle product  $(IC \triangleleft IC^T)$  is computed by the fast fuzzy relational algorithm [3]. This is followed by computing the degree of local preorderness and the degree of reflexivity of the product  $(IC \triangleleft IC^T)$ .

The system also computes four different levels of a

-cuts: namely height, half-upper, mean, half-lower value. Cut at height takes the highest value in the matrix and half-upper cut is given the medium value between height and mean. Similarly, cut at mean takes the mean value over the whole matrix and half-lower cut is given the medium value between mean and lowest value in the matrix called a plinth. The technique of  $\alpha$ -cuts makes it possible to examine a fuzzy relation in crisp perspective and to analyze it in a more realistic view.

At each  $\alpha$ -cut, the index of transitivity and the index of local reflexivity are computed in order to determine to what degree each  $\alpha$ -cut of the fuzzy relational triangle product possesses the properties of transitivity and local reflexivity.

STEP 2 is to obtain the number of diseases applicable for each investigation, using the relation  $ID$ . In this step, the applicability of each investigation is computed. Here, applicability means how many diseases an investigation can identify if performed. When two or more investigations are available, the algorithm chooses the most applicable one among them. Of course, the most applicable investigation is the one that can cover the largest number of diseases from the available investigations.

STEP 3 is to obtain the available investigation(s) for the working diagnoses. Starting from the relation  $PG$  which represents the working diagnoses computed by CLINAID, the algorithm combines two relations  $PG$  and  $GI$  to get the relation  $PI$ : ( $PI = PG \triangleleft GI$ ). From the relation  $PI$ , the algorithm acquires the available investigation(s) for the patient. If only one investigation is available, the algorithm selects that investigation and the procedure is terminated. Otherwise, it proceeds to the next step. STEP 1 through STEP 3 can be performed in any order or simultaneously if the hardware supports parallel processing.

STEP 4 is to find the best investigation. In this step, the algorithm tries to match the investigations computed in STEP 3 with those at the top level of the highest  $\alpha$ -cut available. If it can not find matching investigations, it continues to the next highest  $\alpha$ -cut and tries again to find matching investigations. The reason why the algorithm initially considers investigations at the top level is that the top-level investigations have the least generalized costs related to them.

If more than one investigation are found, the algorithm compares the number of diseases applicable for each investigation. If more than one investigation still have the same number of applicable diseases, the total-weight values of each investigation are compared and the one with the largest total-weight is chosen. The total-weight for each investigation represents the sum of each component in the set of the generalized costs.

The reason why the algorithm chooses the investigation with the largest total-weight is that our data

are set up to represent the relationship: "the larger the total-weight, the less the generalized cost." For example, the most efficient investigation is set to 1 while the least efficient investigation to 0. Other investigations are given values between 0 and 1 after being fuzzified. On the contrary, the most expensive investigation is given the value of 0, and the least expensive 1.

### 5. Simulation Results

This section reports on the results of experimentation on the selection algorithm using various implication operators. Since the computation results of operators  $S\#$  (Standard Sharp) and  $S$  (Standard Strict) did not show any useful information, the outcome of implication operators  $S^*$  to  $W$  is presented and the outcome of the application of two different types of relational products, namely, harsh and mean is analyzed.

First, we report the results of harsh relational product type with the various implication operators, and then those of mean relational product type. Then, we proceed with comparison of the results of harsh and mean product types.

The input data used in this experiment are presented in the Tables 3 and 4. The signs & symptoms of two cardiovascular patients are listed in the tables with their weights. See [2] for detailed input data and patient information used in this experiment.

Table 3. data set 1

Signs & symptoms	Weights
BP high	0.9000
Dizziness	0.7500
Haemorrhages in eye fundus	0.8000
Heart sounds loud	0.9000
Morning headache	0.9000
Pulse difficult to compress	0.8500
Weakness	0.7000

Table 4. data set 2

Signs & symptoms	Weights
Apex beat tapping	0.8000
Apical diastolic murmur	0.7500
Atrial fibrillation	0.9000
Dyspnoea	0.8000
Fatigue	0.6000
Malar flush	0.9000
Oedema of legs	0.6000
Palpitations	0.7000

5.1 Harsh Results

The degree of local preorderliness for operator  $S^*$  was 1.0, and the rest of the operators were within the range of 0.79 and 0.93. This outcome for operator  $S^*$  was expected because according to the theorem,  $S^*$  was supposed to exhibit the degree of 1.0 for a local pre-orderliness if there indeed existed such an order. As for the degree of local reflexivity, operators  $S^*$ ,  $G43$ ,  $G43'$ ,  $L$ , and  $W$  produced 1.0, and 0.91 for  $KDL$ , 0.81 for  $KD$ , and 0.86 for  $EZ$ . Table 5 shows each value corresponding to 4  $\alpha$ -cuts for the given operators.

Table 5.  $\alpha$ -values for the Harsh Case

Operators	Height	HU	Mean	HL
$S^*$	1.0	0.79	0.57	0.39
$G43$	1.0	0.91	0.81	0.66
$G43'$	1.0	0.85	0.71	0.60
$L$	1.0	0.95	0.90	0.85
$KDL$	0.88	0.80	0.72	0.68
$KD$	0.80	0.67	0.54	0.47
$EZ$	0.80	0.67	0.53	0.47
$W$	0.80	0.65	0.51	0.45

For the experiment with data set 1, the selection algorithm found a matching investigation at the height  $\alpha$ -cut with operators  $S^*$ ,  $G43$ ,  $G43'$ ,  $L$ ,  $KD$ , and  $EZ$ . With operator  $KDL$ , it selected the same investigation at the half-upper  $\alpha$ -cut. With operator  $W$ , it found the matching choice at the mean  $\alpha$ -cut.

In summary, the algorithm selected the same investigation, namely #22, with all implication operators, even though the  $\alpha$ -cut levels where the matching choice was found were different with operators  $KDL$  and  $W$ . Also, it was very encouraging to discover that the algorithm was able to select the same investigation with all implication operators, notwithstanding the distinctive structures generated by each operator. For example, the structures of hierarchy at the height  $\alpha$ -cut with operators  $KD$  and  $EZ$  were extremely different from those of other implication operators. Table 6 shows a summary of the harsh results for the experiment with data set 1.

For the experiment with data set 2, the algorithm found a matching investigation at the half-lower  $\alpha$ -cut with operators  $S^*$ ,  $G43$ ,  $L$ ,  $KDL$ ,  $KD$ , and  $W$ . With operator  $EZ$ , it selected the same investigation at the half-upper  $\alpha$ -cut, yet it could not find any matching investigation with operator  $G43'$ .

Thus, the algorithm was able to identify the same investigation, which was #1, with all implication operators except with operator  $G43'$ , even though the number of steps required to find its matching choice was different with operator  $EZ$ . It should be noted that the experiment with operator  $EZ$  found a matching inves-

tigation at a higher  $\alpha$ -cut level than those using other operators. See Table 7 for a summary of the harsh results for data set 2.

Table 6. The Harsh Results with Data Set 1

Operators	Investigation Found	$\alpha$ -cut
$S^*$	#22 (Electrolyte (plasma))	Height
$G43$	#22 (Electrolyte (plasma))	Height
$G43'$	#22 (Electrolyte (plasma))	Height
$L$	#22 (Electrolyte (plasma))	Height
$KDL$	#22 (Electrolyte (plasma))	HU
$KD$	#22 (Electrolyte (plasma))	Height
$EZ$	#22 (Electrolyte (plasma))	Height
$W$	#22 (Electrolyte (plasma))	Mean

Table 7. The Harsh Results with Data set 2

Operators	Investigation Found	$\alpha$ -cut
$S^*$	#1 (Electrocardiogram)	HL
$G43$	#1 (Electrocardiogram)	HL
$G43'$	None	None
$L$	#1 (Electrocardiogram)	HL
$KDL$	#1 (Electrocardiogram)	HL
$KD$	#1 (Electrocardiogram)	HL
$EZ$	#1 (Electrocardiogram)	HU
$W$	#1 (Electrocardiogram)	HL

5.2 Mean Results

No operator achieved the degree 1.0 of local pre-orderliness, yet all operators' degrees were greater than 0.90. The degrees of local reflexivity were ranged between 0.89 and 1.0. Table 8 shows the  $\alpha$ -cut values in the mean case.

Table 8.  $\alpha$ -values for the Mean Case

Operators	Height	HU	Mean	HL
$S^*$	1.0	0.91	0.82	0.69
$G43$	1.0	0.96	0.92	0.84
$G43'$	1.0	0.94	0.88	0.81
$L$	1.0	0.98	0.95	0.91
$KDL$	0.92	0.87	0.81	0.75
$KD$	0.86	0.79	0.72	0.62
$EZ$	0.83	0.75	0.67	0.59
$W$	0.83	0.74	0.66	0.58

For the experiment with data set 1, the selection algorithm found a matching investigation, #22 at the height  $\alpha$ -cut with operators  $S^*$ ,  $G43$ ,  $G43'$ ,  $L$ ,  $KD$ , and  $EZ$ . The algorithm found the same investigation at the half-upper  $\alpha$ -cut with operators  $KDL$  and  $W$ . Thus,

just as with the harsh product, the selection algorithm was able to find the same investigation, #22 with all implication operators, even though the  $\alpha$ -cut level where the matching choice was found was different with two operators. See Table 9 for a summary of the mean computation results of data set 1.

Table 9. The Mean Results with Data Set 1

Operators	Investigation Found	$\alpha$ -cut
$S^*$	#22 (Electrolyte (plasma))	Height
$G43$	#22 (Electrolyte (plasma))	Height
$G43'$	#22 (Electrolyte (plasma))	Height
$L$	#22 (Electrolyte (plasma))	Height
$KDL$	#22 (Electrolyte (plasma))	HU
$KD$	#22 (Electrolyte (plasma))	Height
$EZ$	#22 (Electrolyte (plasma))	Height
$W$	#22 (Electrolyte (plasma))	HU

For the experiment with data set 2, the algorithm found a matching investigation, #1 at the half-lower  $\alpha$ -cut with operators  $S^*$  and  $G43'$ . With operators  $G43$ ,  $L$ ,  $KDL$ , and  $KD$ , the algorithm found the matching investigation, #1 at the mean  $\alpha$ -cut, while with operators  $EZ$  and  $W$ , it was able to find the same investigation at the half-upper  $\alpha$ -cut.

Thus, the algorithm was able to find the same investigation, #1 with all implication operators, even though its matching choice was found at three different  $\alpha$ -cuts; half-upper cut ( $EZ$ ,  $W$ ), mean cut ( $G43$ ,  $L$ ,  $KDL$ ,  $KD$ ), and half-lower cut ( $S^*$ ,  $G43'$ ). See Table 10 for a summary of the mean computation results of data set 2.

Table 10. The Mean Results with Data Set 2

Operators	Investigation Found	$\alpha$ -cut
$S^*$	#1 (Electrocardiogram)	HL
$G43$	#1 (Electrocardiogram)	Mean
$G43'$	#1 (Electrocardiogram)	HL
$L$	#1 (Electrocardiogram)	Mean
$KDL$	#1 (Electrocardiogram)	Mean
$KD$	#1 (Electrocardiogram)	Mean
$EZ$	#1 (Electrocardiogram)	HU
$W$	#1 (Electrocardiogram)	HU

### 5.3 Comparison of Harsh and Mean Results

In the experiment with data set 1, the algorithm was able to find the same investigation, which is #22 with all implication operators using for both harsh and mean products.

The results of the computation with operators  $S^*$ ,  $G43$ ,  $G43'$ ,  $L$ ,  $KD$ , and  $EZ$  showed that the algorithm chose the matching investigation, #22 at the height cut *regardless* of the product, harsh or mean.

With operator  $KDL$ , the algorithm found the matching investigation, #22 at the half-upper  $\alpha$ -cut in both

harsh and mean products. With operator  $W$ , the algorithm found the matching investigation, #22 at two different  $\alpha$ -cut levels, namely the mean cut in the harsh product, and the half-upper cut in the mean product.

Similarly, in the experiment with data set 2, the algorithm was able to identify the same investigation, which is #1, with all implication operators in both harsh and mean products with one exception operator  $G43'$  in the harsh operation. In this case, the algorithm could not find any matching investigation.

The harsh products with operators  $S^*$ ,  $G43$ ,  $L$ ,  $KDL$ ,  $KD$ ,  $W$  and the mean products with operators  $S^*$ ,  $G43'$  found investigation #1 at the half-lower  $\alpha$ -cut. The mean products with operators  $G43$ ,  $L$ ,  $KDL$ ,  $KD$  on the other hand found investigation, #1 at the mean  $\alpha$ -cut. Thus, operator  $S^*$  was the only operator that obtained its result using the same computational steps in both harsh and mean products.

With operator  $EZ$  in both harsh and mean products, the algorithm found the matching investigation at the half-upper  $\alpha$ -cut. With operator  $W$  in the mean product, the algorithm found the matching investigation at the half-upper  $\alpha$ -cut.

## 6. Conclusion

We have found the following results from the comparative study:

1. Despite the fact that the hierarchical structures generated by each implication operator were different, the selection algorithm was able to identify the same investigation (Electrolyte (plasma)) with all implication operators in the experiment with data set 1.
2. For the experiment with data set 2, the selection algorithm identified the same investigation (Electrocardiogram) with all implication operators using both mean and harsh computations except with operator  $G43'$  using a harsh computation. The  $G43'$  harsh product did not produce any outcome.
3. A mean computation always identified the same investigation either at the same  $\alpha$ -cut level or at a higher level than the computation using a harsh product type.

This study shows that the selection algorithm is reliable because the outcome is consistent among all implication operators using both harsh and mean computations.

Empirical studies in the application of CLINAID show that the computations using mean product are computationally more efficient than the computations of harsh product, in most cases requiring fewer computational steps. This study also shows that the outcome of our experiment is consistent with the findings from other research on CLINAID in that respect.

Important distinction between harsh and mean product of relation is following: On crisp data, the harsh product yields crisp result (a special instance of fuzzy). The mean product, however, may produce fuzzy result.

The fact that mean product tends to yield results in fewer computational steps shows that fuzzy computations can be more efficient than non-fuzzy (crisp) ones. Thus, the contribution to computer science offered by the results of this study shows that contrary to the popular belief, fuzzy computations may produce more efficient computation without sacrificing the accuracy of the outcome.

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## 저 자 소 개

노찬숙

2003년 13권 2호 참조