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# Generalized Formula for Active Earth Pressure Estimation with Inclined Retaining Wall

## 점착력을 고려한 배면 경사 옹벽에서의 주동토압 산정 공식

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**ABSTRACT :** Active earth pressure formula, which can consider the effects of ground surface inclination, inclination of inside retaining wall face, wall friction, line load, uniform load, soil cohesion and adhesion, was derived based on the force equilibrium principle. In order to verify the accuracy of this proposed formula, the calculated active earth pressures by the proposed formula were compared with those of graphical solutions. Also, the active earth pressures determined by the proposed formula were compared with those by Coulomb's, Rankine's and Mazindrani's solution under specific conditions. The results matched quite well not only with the graphical solutions but also with those by three other methods. Also, the trend of active earth pressures by the proposed formula were corresponded with results of experimental study by Fang, et al. It can be concluded that this generalized formula not only can overcome the limitations of Rankine's, Coulomb's and Mazindrani's active earth pressure formula but also can consider the external loading conditions.

**Keywords :** Active earth pressure, Cohesion, Adhesion, External loading conditions

**요 지 :** 본 논문에서는 지표면 경사각, 벽면 경사각, 벽면 마찰각, 선하중, 등분포하중, 점착력, 부착력의 영향을 고려할 수 있는 주동토압공식을 힘의 평형이론을 근거로 도출하였다. 이 제안식의 정확성을 검증하기 위하여, 도해법에 의해 산정된 토압과 비교하였으며, Coulomb, Rankine, Mazindrani 공식에 의한 산정 결과와도 비교하였다. 산정 결과는 도해법 결과뿐만 아니라 Coulomb, Rankine, Mazindrani 공식의 3가지 방법에 의한 결과와 잘 일치되었다. 또한, 제안식에 의한 주동토압은 Fang 등의 실험연구 결과와 일치하는 경향을 나타내었다. 이 일반화된 공식은 Coulomb, Rankine, Mazindrani 의 주동토압공식의 한계를 극복할 수 있을 뿐만 아니라 외부하중조건을 고려할 수 있는 것으로 평가되었다.

**주요어 :** 주동토압, 점착력, 부착력, 외부하중조건

## 1. Introduction

Coulomb proposed a method for the determination of active earth pressures that included the effect of friction between soil and wall (Coulomb, 1776). In this method, a linear failure surface was assumed and force equilibrium condition was applied. In order to evaluate the maximum active earth pressure, several trial failure surfaces were tried and the one producing the critical force was selected. Especially, Coulomb's solution for active earth pressure refers to the earth pressure due to the soil weight only. However, in many practical problems, the lateral earth pressure was due to not only soil weight but also applied external loads i.e. line load and uniform load. If external loads are applied, the time consuming Culmann's graphical method (1875) was usually applied because Coulomb's method cannot provide analytical solution. With this graphical method, the procedure of estimating active earth pressure is not only complicate and cumbersome but also inaccurate. Another defect of Coulomb's active earth pressure formula is that soil cohesion and adhesion were not considered in its original derivation. As it is well known, the shear strength of soil is measured in terms of two soil parameters i.e. cohesion and soil friction angle. Grain crushing, resistance to rolling and other factors are implicitly included in these two parameters (Bowles, 1988). Therefore, since Coulomb's active earth pressure formula also cannot consider soil cohesion value, the active earth pressure value by Coulomb's formula will be conservative.

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Rankine (1857) suggested a method for the determination of earth pressure applying essentially the same assumptions as Coulomb's but zero wall friction is assumed and soil cohesion is considered. However, according to Sherif et al. (1982), friction between wall and soil is one of the important parameters for active earth pressure evaluation. The other limitations of original Rankine's method are that the slope of ground surface should be horizontal and the inclination of inside wall face should be vertical with cohesive backfilled soil condition. Another formula, which can be applied to the inclined ground surface case with granular backfilled soil, is available but this formula cannot be applied if the soil friction angle is larger than the ground surface inclination. Therefore, since this formula is not practical or cannot be applied to cohesive backfilled soil, the original Rankine formula was used for discussion in this paper. However, in real world, the retaining wall usually has inclined ground surface and inclined inside wall face under cohesive backfilled soil condition. Therefore, in such case, Terzaghi's (1943) graphical approach is usually applied for evaluation of active earth pressure. However, this procedure becomes tiresome for solving practical retaining wall problems because several Mohr circle should be tried to determine the lateral earth pressure. In order to eliminate these inconveniences, Mazindrani and Ganjali (1997) developed a method that can be applied to cohesive soil with inclined surface cases. Since Mazindrani's formula for active earth pressure is developed based on the Rankine theory, this approach has the same limitations with Rankine's such that wall friction was assumed zero and the inclination angle of inside wall face should be vertical. Moreover, if external loads are applied with inclined ground slope condition, Rankine's and Mazindrani's methods cannot be applied for the estimation of active pressure and graphical approach is the only solution up to now.

Significant and valuable studies associated with earth pressure have been carried out by Terzaghi (1932), Schofield (1961), Mackey and Kirk (1967), Matteotti (1970), Bros (1972), Sherif and Mackey (1977), Sherif et al. (1982), Sherif et al. (1984), Duncan et al. (1991) and other researchers and most of the study was concerned with horizontal ground surface. Fortunately, Fang et al. (1997) studied lateral earth pressure of dry sand with inclined ground surface through the experimental research. Based on their experimental data, it has been found that the active and passive earth pressures for various backfill sloping angles are in good agreement with the values determined by Coulomb and Terzaghi's solution. They also found that Rankine's solution tends to overestimate the active earth pressure if inclined ground surface angle is less than 20°. Finally, they concluded that Rankine theory might not be appropriate to determine either active or passive earth pressure against a rigid wall with sloping backfill. Unfortunately, if the geotechnical engineer should design the retaining wall that has inclined ground surface, inclined wall face and external loads under cohesive soil condition, the estimation of active earth pressure was remained problematic.

In this paper, a generalized active earth pressure formula has been developed based on the force equilibrium condition and this formula can consider cohesion, adhesion, wall friction, inclination of inside wall face, ground surface inclination, and external loads. For the verification of this proposed formula, the active earth pressure values are compared with those of graphical and theoretical solutions from the published literatures.

## 2. Analysis of Acting Forces Around Assumed Failure Wedge

The basic assumptions for the active earth pressure estimation are: the soil is homogeneous, the mode of failure plane is linear, all external loads are applied at the inside failure wedge and the length of applied uniform load is long

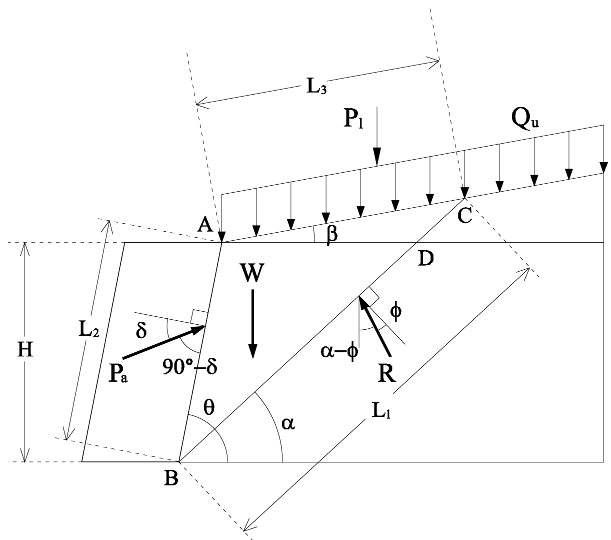


Fig. 1. General Scheme for Active Earth Pressure Estimation.

enough to be intersected by the failure plane. The general scheme for active earth pressure is shown in Fig. 1. For convenience, positive sign was assigned for downward and rightward movement, and negative sign was assigned for upward and leftward movement.

There are seven forces that act around assumed failure wedge ABC and these are included in Fig. 1. As mentioned before, since this formula is driven based on the force equilibrium, these forces should be divided into components of x and y directions in order to apply the force equilibrium principle. Therefore, these seven forces are divided into x and y component as follows.

The soil weight of assumed failure triangle wedge ABC denoted by W acts to the gravitational direction and has no horizontal force. The line load denoted by $P_l$ and uniform load denoted by $Q_u$ also act to the gravitational direction, therefore, have no horizontal forces. In Fig. 1, since the general geometry of retaining wall may have slope, the inclination angle is denoted by $\beta$.

As shown in Fig. 2, the length of uniform load denoted by $L_3$ will be changed with the variation of wedge failure angle denoted by $\alpha$, which is defined the angle between failure plane of wedge and bottom horizontal line. Since $\alpha$ value is function of many other factors, detailed derivation of general equation for $L_3$ will be discussed later.

With the same principle, soil reaction denoted by R can be divided into vertical force denoted by $R_v$ and horizontal force denoted by $R_h$. As shown in Fig. 3, these vertical and horizontal components of soil reaction can be expressed as $R_v = - R \cos(\alpha - \phi)$ and $R_h = - R \sin(\alpha - \phi)$, respectively. The negative signs mean downward movement with vertical component and leftward movement with horizontal component, respectively.

The active earth pressure denoted by $P_a$ can be divided into vertical force denoted by $P_{av}$ and horizontal force denoted by $P_{ah}$. As shown in Fig. 4, $\theta$ is geometrical inclination of inside wall face and $\delta$ is friction angle between wall and soil. Applying these symbols, the vertical and horizontal components of active earth pressure can be expressed as $P_{av} = - P_a \sin(\theta - 90° + \delta)$ and $P_{ah} = P_a \cos(\theta - 90° + \delta)$, respectively. The negative sign means downward movement with vertical component and the positive sign means rightward movement with horizontal component, respectively.

The resistance due to soil cohesion denoted by C can be divided into vertical force denoted by $F_{cv}$ and horizontal force denoted by $F_{ch}$. These vertical and horizontal components

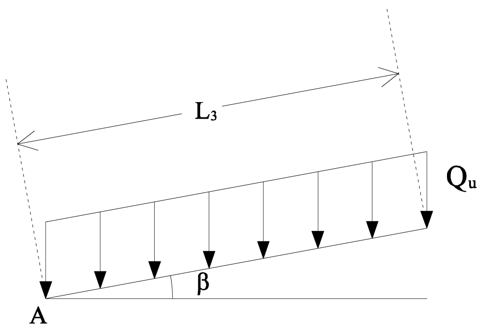


Fig. 2. Reaction of Uniform Load.

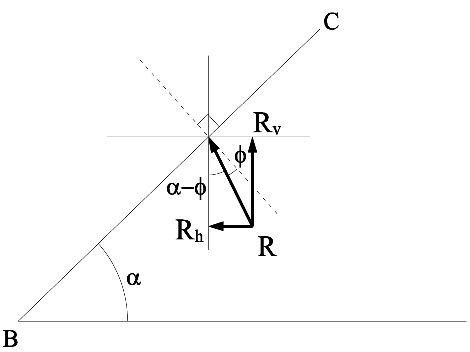


Fig. 3. Reaction of the Soil.

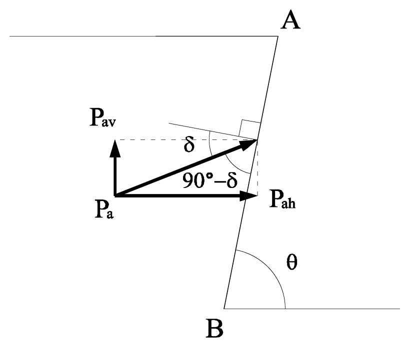


Fig. 4. Reaction of Active Earth Pressure.

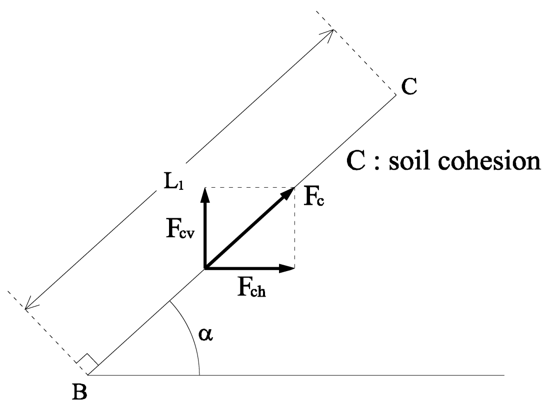


Fig. 5. Reaction of Cohesion Force.

of resistance force by soil cohesion can be expressed as $F_{cv} = - C L_1 \sin \alpha$ and $F_{ch} = C L_1 \cos \alpha$, respectively. As shown in Fig. 5, the length of failure line denoted by $L_1$ is the function of failure angle denoted by $\alpha$. As it is mentioned, $\alpha$ value is function of many other factors, the derivation of general equation for $L_1$ will be discussed later. The negative and positive signs with each formula mean upward movement with vertical component and rightward movement with horizontal component, respectively.

Wall adhesion develops from any cohesion in soil (Bowels, 1988), therefore, adhesion is defined as the adhesive force between the wall and backfilled soil due to soil cohesion only. Resistance due to soil adhesion denoted by $C'$ can be divided into vertical force denoted by $F_{av}$ and horizontal force denoted by $F_{ah}$. These vertical and horizontal components of resistance force by adhesion can be expressed as $F_{av} = - C' L_2 \sin(180° - \theta)$ and $F_{ah} = - C' L_2 \cos(180° - \theta)$, respectively. As shown in Fig. 6, $L_2$ is the length of inside wall face which contacts with soil and $\theta$ is the inclination of inside wall face. The negative sign with each formula means upward movement with vertical component and leftward movement with horizontal component, respectively. The discussed vertical and horizontal forces are summarized in Table 1.

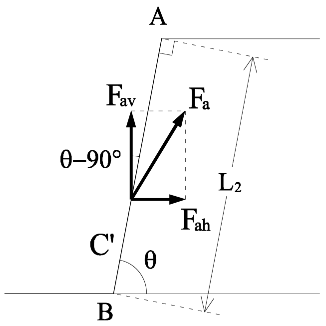


Fig. 6. Reaction of Adhesion Force.

In order to satisfy the force equilibrium principle, $\Sigma F_V = 0$ and $\Sigma F_H = 0$ conditions should be achieved. Therefore, from the $\Sigma F_V = 0$ condition and Table 1, following equation can be driven:

$$0 = W + P_l + Q_u L_3 - R\cos(\alpha - \phi) - P_a \sin(\theta - 90^o + \delta) - CL_1 \sin\alpha - C'L_2 \sin(180^o - \theta) \quad (1)$$

Above formula can be rearranged as following equation,

$$R\cos(\alpha - \phi) = W + P_l + Q_u L_3 - P_a \sin(\theta - 90^o + \delta) - CL_1 \sin\alpha - C'L_2 \sin(180^o - \theta) \quad (2)$$

From the $\Sigma F_H = 0$ condition and Table 1, following equation can be driven:

$$0 = -R\sin(\alpha - \phi) + P_a \cos(\theta - 90^o + \delta) - CL_1 \cos\alpha - C'L_2 \sin(180^o - \theta) \quad (3)$$

Above formula can be rearranged as following equation,

$$R\sin(\alpha - \phi) = P_a \cos(\theta - 90^o + \delta) + CL_1 \cos\alpha - C'L_2 \cos(180^o - \theta) \quad (4)$$

In order to get rid of soil reaction R, Eq. (4) was divided by Eq. (2) and following relationship is driven.

$$\tan(\alpha - \phi) = \frac{P_a \cos(\theta - 90° + \delta) + CL_1 \cos\alpha - C'L_2 \cos(180° - \theta)}{W + P_l + Q_u L_3 - P_a \sin(\theta - 90° + \delta) - CL_1 \sin\alpha - C'L_2 \sin(180° - \theta)} \quad (5)$$

Table 1. Summary of Acting Forces Around Soil Wedge

| | Description | Vertical Forces | Horizontal Forces |
|---|---|---|---|
| 1 | Weight of Wedge<br>W (F/L) | W | 0 |
| 2 | Line Load<br>$P_l$ (F/L) | $P_{lv}$<br>$P_l$ | $P_{lh}$<br>0 |
| 3 | Uniform Load<br>$Q_u$ (F/L$^2$) | $Q_{uv}$<br>$Q_u L_3$ | $Q_{uh}$<br>0 |
| 4 | Soil Reaction<br>R (F/L) | $R_v$<br>$-R\cos(\alpha - \phi)$ | $R_h$<br>$-R\sin(\alpha - \phi)$ |
| 5 | Active Earth Pressure<br>$P_a$ (F/L) | $P_{av}$<br>$-P_a \sin(\theta + \delta - 90°)$ | $P_{ah}$<br>$P_a \cos(\theta + \delta - 90°)$ |
| 6 | Cohesion<br>C (F/L$^2$) | $F_{cv}$<br>$-C L_1 \sin\alpha$ | $F_{ch}$<br>$C L_1 \cos\alpha$ |
| 7 | Adhesion<br>$C'$ (F/L$^2$) | $F_{av}$<br>$-C' L_2 \sin\theta$ | $F_{ah}$<br>$C' L_2 \cos\theta$ |

Eq. (5) can be rearranged as following form,

$$P_a = \left\{ \frac{1}{\sin(\theta - 90^o + \delta)\tan(\alpha - \phi) + \cos(\theta - 90^o + \delta)} \right\} \times$$

$$\left\{ \begin{matrix} W\tan(\alpha - \phi) + P_1\tan(\alpha - \phi) + Q_u L_3 \tan(\alpha - \phi) - CL_1 \sin\alpha \tan(\alpha - \phi) \\ - C'L_2 \sin(180^o - \theta)\tan(\alpha - \phi) - CL_1 \cos\alpha + C'L_2 \cos(180^o - \theta) \end{matrix} \right\} \quad (6)$$

The terms of this equation can be classified into as soil weight, external load, soil cohesion and adhesion. For convenience, let soil weight term is S, external load term is T, soil cohesion term is U and adhesion term is V. Then Eq. (6) can be expressed as follow.

$$P_a = \left\{ \frac{1}{\sin(\theta - 90^o + \delta)\tan(\alpha - \phi) + \cos(\theta - 90^o + \delta)} \right\} (S + T + U + V) \quad (7)$$

Where,

$$S = W\tan(\alpha - \phi)$$
$$T = P_1 \tan(\alpha - \phi) + Q_u L_3 (\alpha - \phi)$$
$$U = -CL_1 \sin\alpha \tan(\alpha - \phi) - CL_1 \cos\alpha$$
$$V = -C'L_2 \sin(180^o - \theta)\tan(\alpha - \phi) + C'L_2 \cos(180^o - \theta)$$

## 3. The Derivation of General Equation for $L_1$, $L_2$ and $L_3$

Above formulas include unknown terms i.e. W, $\alpha$, $L_1$, $L_2$ and $L_3$ and these unknown terms are evaluated as follows. As shown in Fig. 7, $\theta$ and $\beta$ are known values from the geometric condition. Therefore, the intersection points A,

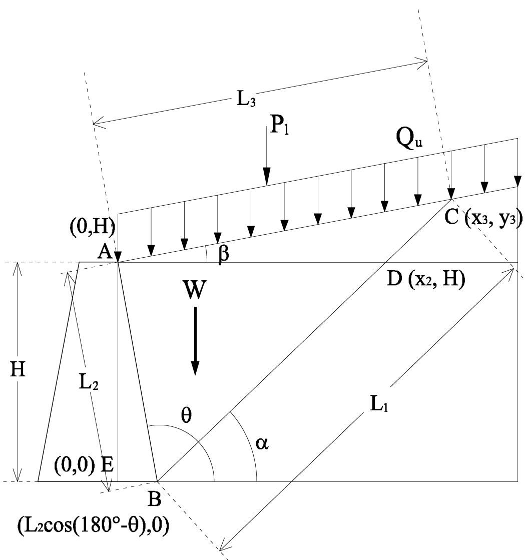


Fig. 7. Coordinates of Point A, B, C, D and E.

B, C, D, and E can be defined as x-y coordinate form i.e. A(0, H), B($L_2$ cos (180° − $\theta$), 0), C($X_3$, $Y_3$), D($X_2$, H) and E(0,0). With these defined coordinates of intersection points, the equations of line BC, AD and AC can be defined as following forms, respectively i.e. F (X, Y)$_1$ = (tan $\alpha$) X + $C_3$, F (X, Y)$_2$ = H and F (X, Y)$_3$ = (tan $\beta$) X + H.

Based on above discussion and Fig. 7, the length of wall $L_2$ can be expressed as following equation.

$$L_2 = \frac{H}{\sin(180° - \theta)} \quad (8)$$

As shown in Fig. 7, since point B is on the Line BC, the values of point B should satisfy following equation F (X, Y)$_1$ = (tan $\alpha$) X + $C_3$. Therefore, substitute the values of point B into the equation of line BC. Then, following relationship is driven i.e. $C_3$ = − (tan $\alpha$) {$L_2$ cos (180° − $\theta$)}. Substitute $C_3$ value into equation of line BC. Then the equation of line BC can be expressed as following form.

$$F(X, Y)_1 = (\tan\alpha)X - (\tan\alpha)\{L_2 \cos(180^o - \theta)\} \quad (9)$$

Since point D is on the line BC, H = (tan $\alpha$) $X_2$ − (tan $\alpha$){$L_2$ cos (180° − $\theta$)} condition should be satisfied. Therefore,

$$X_2 = \frac{H + (\tan\alpha) L_2 \cos(180° - \theta)}{\tan\alpha} \quad (10)$$

From the Fig. 7, the line BC and line AC meet at point C. Therefore, following relationship should be satisfied.

$$(\tan\beta)X_3 + H = (\tan\alpha)X_3 - (\tan\alpha)\{L_2 \cos(180^o - \theta)\} \quad (11)$$

Rearrange Eq. (11) in term of $X_3$ and the value of $X_3$ can be as follow.

$$X_3 = \frac{H + \tan\alpha\, L_2 \cos(180° - \theta)}{\tan\alpha - \tan\beta} \quad (12)$$

Substitute $X_3$ into equation of line AC i.e. F (X, Y)$_3$ = (tan $\beta$) X + H and the value of $Y_3$ can be expressed as follow.

$$Y_3 = \tan\beta \left\{ \frac{H + \tan\alpha L_2 \cos(180° - \theta)}{\tan\alpha - \tan\beta} \right\} + H \quad (13)$$

With applying all above derived relationships, W, $L_1$ and $L_3$ can be expressed in terms of $\alpha$.

$$W = \frac{1}{2}\gamma H L_2 \sin(\theta - 90°) + \frac{1}{2}\gamma H^2 \tan(90° - \alpha) + \frac{1}{2}\gamma \left\{ \frac{H + \tan\alpha L_2 \cos(180° - \theta)}{\tan\alpha} \right\} \left[ \frac{\tan\beta\{H + \tan\alpha L_2 \cos(180° - \theta)\}}{\tan\alpha - \tan\beta} \right] \quad (14)$$

$$L_1 = \frac{Y_3}{\sin\alpha} = \frac{1}{\sin\alpha}\left[\frac{\tan\beta\{H + \tan\alpha L_2 \cos(180° - \theta)\}}{\tan\alpha - \tan\beta} + H\right] \quad (15)$$

$$L_3 = \frac{X_3}{\cos\beta} = \frac{1}{\cos\beta}\left\{\frac{H + \tan\alpha L_2 \cos(180° - \theta)}{\tan\alpha - \tan\beta}\right\} \quad (16)$$

Substitute Eq. (14), (15) & (16) into Eq. (7) and soil weight term S, external load term T, soil cohesion term U and adhesion term V can be written as following forms, respectively.

### Soil Weight Term

$$S = \frac{1}{2}\gamma H^2\left[\frac{\sin(\theta - 90°)}{\sin(180° - \theta)} + \tan(90° - \alpha) + \left\{\frac{1 + \tan\alpha\cot(180° - \theta)}{\tan\alpha}\right\} \times \left\{\frac{\tan\beta\{1 + \tan\alpha\cot(180° - \theta)\}}{\tan\alpha - \tan\beta}\right\}\right]\tan(\alpha - \phi) \quad (17)$$

### External Load Term

$$T = P_l \tan(\alpha - \phi) + Q_u H\left\{\frac{1 + \tan\alpha\cot(180° - \theta)}{\tan\alpha - \tan\beta}\right\}\tan(\alpha - \phi) \quad (18)$$

### Soil Cohesion Term

$$U = -\frac{H}{\sin\alpha}\left[\frac{\tan\beta\{1 + \tan\alpha\cot(180° - \theta)\}}{\tan\alpha - \tan\beta} + 1\right]\{C\sin\alpha\tan(\alpha - \phi) + C\cos\alpha\} \quad (19)$$

### Adhesion Term

$$V = -\frac{H}{\sin(180° - \theta)}\{C'\sin(180° - \theta)\tan(\alpha - \phi) - C'\cos(180° - \theta)\} \quad (20)$$

Substitute Eq. (17), (18), (19) & (20) into Eq. (7) and then the proposed formula can be expressed as following.

$$P_a = \left\{\frac{1}{\sin(\theta - 90° + \delta)\tan(\alpha - \phi) + \cos(\theta - 90° + \delta)}\right\} \times \left[\frac{1}{2}\gamma H^2\left\{\frac{\sin(\theta - 90°)}{\sin(180° - \theta)} + \tan(90° - \alpha) + \left(\frac{1 + \tan\alpha\cot(180° - \theta)}{\tan\alpha}\right) \times \left(\frac{\tan\beta\{1 + \tan\alpha\cot(180° - \theta)\}}{\tan\alpha - \tan\beta}\right)\right\}\tan(\alpha - \phi) + P_l\tan(\alpha - \phi) + Q_u H\left(\frac{1 + \tan\alpha\cot(180° - \theta)}{\tan\alpha - \tan\beta}\right)\tan(\alpha - \phi) - \frac{H}{\sin\alpha}\left(\frac{\tan\beta\{1 + \tan\alpha\cot(180° - \theta)\}}{\tan\alpha - \tan\beta} + 1\right) \times \{C\sin\alpha\tan(\alpha - \phi) + C\cos\alpha\} - \frac{H}{\sin(180° - \theta)}\{C'\sin(180° - \theta)\tan(\alpha - \phi) - C'\cos(180° - \theta)\}\right] \quad (21)$$

The parameters, which are included in Eq. (21), are known values from wall geometry or soil properties except assumed wedge failure angle $\alpha$. Therefore, the maximum active earth pressure $P_a$ can be evaluated with changing $\alpha$ values.

## 4. The Comparison of Active Earth Pressure Among the Original Coulomb, Rankine'S and Proposed Formular

The original Coulomb and Rankine's active earth pressures were expressed as following equations, respectively (Taylor, 1956).

$$P_a = \frac{1}{2}\gamma H^2\left[\frac{\csc\theta\sin(\theta - \phi)}{\sqrt{\sin(\theta + \delta)} + \sqrt{\{\sin(\phi + \delta)\sin(\phi - \beta)\}/\{\sin(\theta - \beta)\}}}\right]^2 \quad (22)$$

$$P_a = \frac{1}{2}\gamma H^2\tan^2\left(45° - \frac{\phi}{2}\right) - 2CH\tan\left(45° - \frac{\phi}{2}\right) \quad (23)$$

The active earth pressure of cohesionless soil with inclined ground surface by Rankine was expressed as following equation (Taylor, 1956).

$$P_a = \frac{1}{2}\gamma H^2\cos\beta\left\{\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}\right\} \quad (24)$$

The direct comparisons of Eq. (21), (22), (23) and (24) are not possible because each formula has its own distinctive form. For comparison purpose, the same conditions, which can be applied to both Coulomb and Rankine's active earth pressure formula, are applied to the proposed formula. Since no cohesion, adhesion, external load are considered for Coulomb's active earth pressure formula, the proposed formula can be reduced as Eq. (25).

$$P_a = \frac{W\tan(\alpha - \phi)}{\sin(\theta - 90° + \delta)\tan(\alpha - \phi) + \cos(\theta - 90° + \delta)} \quad (25)$$

In order to reflect the limitations of Rankine's active earth pressure formula, $\theta$ = 90°, $\beta$ = 0 condition are applied to Eq. (14) and the reduced formula is shown in Eq. (26).

$$W = \frac{1}{2}\gamma H^2\tan(90° - \alpha) \quad (26)$$

Substitute Eq. (26) into Eq. (25), then the proposed active earth pressure formula can be expressed as follow.

$$P_a = \frac{1}{2}\gamma H^2 \left\{ \frac{\tan(90° - \alpha)\tan(\alpha - \phi)}{\sin(\theta - 90° + \delta)\tan(\alpha - \phi) + \cos(\theta - 90° + \delta)} \right\} \quad (27)$$

If the same conditions are applied to Eq. (21), (22), (23), (24) and (27), the active earth pressures values should be same. Because of the limitations of each formula, $\theta$ = 90° was applied and zero values were assigned to C, C′, $\beta$ and $\delta$. When $\gamma$ = 17.4 kN/m$^3$, $\phi$ = 26°, C = 0, H = 6 m, C′ = 0, $\theta$ = 90°, $\beta$ = 0 and $\delta$ = 0° are applied to Eq. (21), (22), (23), (24) and (27), the active earth pressure of all the five equations are exactly same and the value is 122.293 kN/m with failure angle $\alpha$ = 58°. This failure angle $\alpha$ can be determined by proposed formula only.

With original Rankine formula, wall friction $\delta$ = 0° was assumed. However, in reality, most retaining walls are far from frictionless. Generally, one half or two third of soil friction angle $\phi$ is used for design if Coulomb formula is applied. In order to find the effect of wall friction, all the same values but $\delta$ = 17° are applied to Eq. (22), and (27) and the active earth pressures are exactly same value of 108.83 kN/m with failure angle $\alpha$ = 54°. Based on the above comparison, neglecting wall friction value is not only unreasonable but also great loss.

## 5. Verification of Proposed Formular

In order to verify the accuracy of the proposed formula, the active earth pressures by proposed formula were compared with those by graphical approach. These are from Bowles (1988), Das (1984), Dunn et al. (1980), Suton (1975), Prakash (1981), Peck et al. (1974), Taylor (1956), Terzaghi (1943), Terzaghi and Peck (1948), Lamb and Whitman (1979) and Venkatramaiah (1993). These eleven comparison results are summarized in Table 2 and among them, active earth pressure of retaining walls with line load, with no external load and with uniform load under inclined ground surface condition are discussed through case number 2, 7 and 9, respectively. The active earth pressure by proposed formula is compared with result of Culmann's graphical method under cohesive soil condition at case No. 10. At the subtitle of the comparison with Rankine theory, the active earth pressure by proposed formula is compared with that by Mazindrani and Ganjali (1997) method.

### Case No. 2 (Line Load)

A 3.5 m high retaining wall of which ground surface inclination $\beta$ = 0° and 10 kN/m of line load ($P_l$) was applied 2 m behind on the top of wall. From the given conditions, unit weight of soil $\gamma$ = 15.6 kN/m$^3$, friction angle of soil $\phi$ = 32°, inside wall face inclination $\theta$ = 90° and wall friction angle $\delta$ = 20° and soil cohesion C = 0 are used (Dunn et al., 1980). The published $P_a$ value by Culmann's graphical method was $P_a$ = 31 kN/m and the solution obtained by the proposed formula i.e. Eq. (21) is $P_a$ = 30.906 kN/m, with failure angle $\alpha$ = 61°.

### Case No. 7 (Rebhann's Graphical Method)

A 5.0 m high retaining wall of which ground surface slope $\beta$ = 10° and no external loads are applied. From the

Table 2. Comparison of Active Earth Pressures Determined by Graphical, Theoretical Solution and Proposed Formula

| No | H | $\beta$ | $\theta$ | $\delta$ | $\gamma$ | $\phi$ | C | $P_l$ | $Q_u$ | $\alpha$ | $P_{ag}$ | $P_{af}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (m) | (°) | (°) | (°) | (kN/m$^3$) | (°) | (kN/m$^2$) | (kN/m) | (kPa) | (°) | (kN/m) | (kN/m) |
| 1 | 3.5 | 0 | 90 | 20 | 15.6 | 32 | 0 | 0 | 0 | 57 | 26.2 | 26.324 |
| 2 | 3.5 | 0 | 90 | 20 | 15.6 | 32 | 0 | 10 | 0 | 61 | 31 | 30.906 |
| 3 | 3.6 | 10 | 99 | 12 | 18.54 | 30 | 0 | 0 | 0 | 57 | 50.0 | 51.427 |
| 4 | 4.5 | 0 | 90 | 20 | 18.6 | 32 | 0 | 0 | 0 | 57 | 51.5 | 51.883 |
| 5 | 4.6 | 0 | 90 | 15 | 18.85 | 30 | 0 | 29.2 | 0 | 62 | 74.3 | 74.941 |
| 6 | 4.6 | 0 | 90 | 15 | 18.85 | 30 | 0 | 0 | 0 | 57.5 | 60.0 | 60.111 |
| 7 | 5 | 10 | 90 | 20 | 19.0 | 30 | 0 | 0 | 0 | 53 | 80.34 | 80.754 |
| 8 | 6 | 20 | 90 | 20 | 18.0 | 38 | 0 | 0 | 0 | 57 | 88 | 89.719 |
| 9 | 6.1 | 12 | 110 | 0 | 17.3 | 30 | 0 | 0 | 24.54 | 64 | 278.2 | 273.70 |
| 10 | 9 | 20 | 100 | 25 | 15.9 | 30 | 10 | 0 | 0 | 57 | 210 | 208.93 |
| 11 | 10 | 10 | 105 | 18 | 15.0 | 36 | 0 | 90 | 0 | 66 | 360 | 349.417 |

given conditions, unit weight $\gamma$ = 19.0 kN/m$^3$, friction angle of soil $\phi$ = 30°, wall inclination $\theta$ = 90° and wall friction angle $\delta$ = 20° are used (Venkatramaiah, 1993). The published $P_a$ value by Rebhann's graphical method is 80.34 kN/m and the solution obtained by the proposed formula was 80.754 kN/m, with failure angle $\alpha$ = 53°.

## Case No. 9 (Uniform Load)

A 6.1 m high retaining wall, which has a ground surface slope $\beta$ = 12° and applied uniform load $Q_u$ = 24.54 kN/m$^2$, is analyzed. From the given conditions, soil unit weight $\gamma$ = 17.3 kN/m$^3$, soil friction angle $\phi$ = 30°, wall inclination $\theta$ = 110° and wall friction angle $\delta$ = 0° are used (Lamb and Whitman, 1979). The published solution is $P_a$ = 278.22 kN/m, the solution obtained by the proposed formula is $P_a$ = 273.70 kN/m, with failure angle $\alpha$ = 57°. As it was mentioned by Lamb and Whitman (1979), the published solution was approximated value, therefore, there is a little differences between the published value and that determined by the proposed formula.

## Case No.10 (Cohesion)

As mentioned, this proposed formula can consider cohesion (C) and adhesion (C′) values based on the force equilibrium principle. Therefore, the active earth pressure with cohesive backfilled soil was estimated by proposed formula and it was compared with that of graphical solution. From the given conditions, wall height H = 9 m, slope inclination at the top of wall $\beta$ = 20°, unit weight of soil $\gamma$ = 15.9 kN/m$^3$, friction angle of soil $\phi$ = 30°, wall inclination $\theta$ = 100° and wall friction angle $\delta$ = 25°, soil cohesion C = 10 kN/m$^2$ and adhesion between wall and soil C′ = 0 are used (Suton, 1975). The published $P_a$ value by Culmann's graphical method was $P_a$ = 210 kN/m and the solution obtained by the proposed formula is $P_a$ = 208.930 kN/m, with failure angle $\alpha$ = 57°.

## The Comparison with Rankine Theory

The original Rankine's active earth pressure formula cannot be applied if the retaining wall has inclined ground surface and backfilled soil is cohesive. Therefore, Mazindrani and Ganjali (1997) developed a method that can estimate the active earth pressure of cohesive soil with inclined surface. This method can consider the effect of tension crack that can be developed just behind the top of retaining wall. In this example, the solutions developed by Mazindrani and Ganjali (1997) are compared with the solutions by proposed formula. From the given conditions, the wall height H = 6.5 m, slope inclination at the top of wall $\beta$ = 5°, unit weight of soil $\gamma$ = 17.52 kN/m$^3$, friction angle of soil $\phi$ = 15°, inside wall face inclination $\theta$ = 90° and wall friction angle $\delta$ = 0°, soil cohesion C = 10.5 kN/m$^2$ and adhesion between wall and soil C′ = 0 are applied (Mazindrani and Ganjali, 1997). The active earth pressure by the Mazindrani method is 124.7 kN/m with 1.56 m tension crack at the top of retaining wall whereas the active earth pressure by proposed formula is 121.505 kN/m. As it is shown, the active earth pressure by the proposed formula was less than that of Mazindrani's method. Besides this numerical difference, there is another difference i.e. the active earth pressure by proposed formula is under no tension cracked condition. If the tension crack was not developed, the active pressure by Mazindrani method is 164.06 kN/m.

This difference was corresponded with the results of experimental research by Fang et al. (1997). They carried out interesting experimental study with dry sand under $\theta$ = 90, $\beta$ > 0 conditions and the experimental active earth pressure values have good agreement with the values determined by Coulomb and Terzaghi's theory. They also showed that Rankine formula overestimate the active earth pressure if the inclination slope of backfill is smaller than 20° and they concluded that the active earth pressure by Rankine theory may not appropriate for the lateral earth pressure estimation. After Mazindrani and Ganjali (1997), the estimation of active earth pressure with inclined surface is possible but this formula still has limitations i.e. wall friction $\delta$ should be zero, wall inclination $\theta$ should be 90° and external loads cannot be considered. With the same problem, if friction angle $\delta$ between the wall and soil is assumed 7°, the active earth pressure with the proposed formula is $P_a$ = 104.43 kN/m.

The active earth pressures by graphical method and proposed formula are summarized in Table 2 and $P_{ag}$ and $P_{af}$ represent active earth pressure by graphical method and proposed formula, respectively. As shown in Table 2, the case No. 11 (Venkatramaiah, 1993) shows the largest difference and this difference seems due to the inaccuracy by graphical method. Particularly, if same conditions are applied to Coulomb, Rankine and proposed formula, the results are same.

Based on the above comparisons, it can be mentioned that the advantages caused by considering the wall geometries and wall friction values have been neglected for a long time and these are not negligible parameters for the evaluation of active earth pressure.

## 6. Application and Discussion

In this example, general case of retaining wall was provided in order to compare the active earth pressures by Coulomb, Rankine, Mazindrani and propose formula. In these comparisons, wall inclination $\theta$ should be 90° otherwise Rankine's and Mazindrani's formula cannot be applied. Also, for these comparisons, Eq. (21) as proposed formula, Eq. (22) as Coulomb formula and Eq. (23) as Rankine formula were applied for the evaluation of active earth pressures. Mazindrani method was applied under two conditions i.e. without tension crack and with tension crack. It is assumed that the tension crack can be developed between the retaining wall and backfilled soil specifically at the top of the wall.

In order to analyze the effect of soil cohesion, the relationships between active earth pressures and soil cohesion are visualized in Fig. 8. For this comparison, wall height H = 6.5 m, unit weight of soil $\gamma$ = 17.52 kN/m$^3$, soil friction angle $\phi$ = 15°, wall friction angle $\delta$ = 0, slope inclination at the top of wall $\beta$ = 0 and adhesion between wall and soil $C'$ = 0 are applied. Particularly, in order to apply Rankine formula, zero was assigned to $\delta$ and $\beta$ values. As shown in Fig. 8, since soil cohesion cannot be considered with Coulomb's formula, Coulomb's active earth pressures were constant regardless the variation of soil cohesion value. Other two values of active earth pressures decrease as cohesion values increase and these results seem reasonable. Active earth pressures of three methods are exactly same at C = 0 condition and active earth pressures by Rankine's and proposed formula are also exactly same with same soil cohesion values. Based on this comparison, it can be mentioned that soil cohesion value is important parameter for the estimation of active earth pressure.

In order to analyze the effect of wall friction angle, the relationships between active earth pressures and wall friction angle are visualized in Fig. 9. The applied conditions are same as the previous one but C = 0. As shown in Fig. 9, since wall friction angle $\delta$ should be always zero with Rankine's formula, Rankine's active earth pressures are constant regardless the variation of wall friction angle. The active earth pressures by Coulomb's and proposed formula are exactly same and these two active earth pressure values decrease as wall friction values increase. Since Mazindrani's formula can be applied under $\delta$ = 0 condition only, therefore, three cases are compared. As shown in Fig. 9, wall friction is also important parameter for the estimation of active earth pressure.

In order to analyze the effect of ground surface inclination, the relationships of active earth pressures verse slope angle are shown in Fig. 10. The applied conditions are same as the previous ones but C =10.5 kN/m$^2$. In this case, Rankine formula cannot be applied because $\beta$ is not zero and

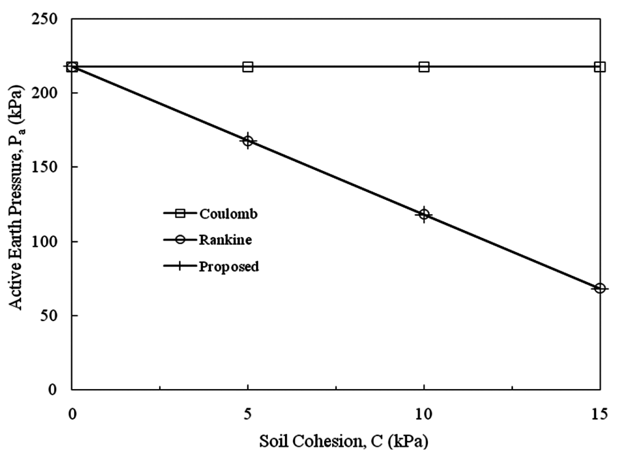


Fig. 8. Variation of Active Earth Pressure regarding Cohesion.

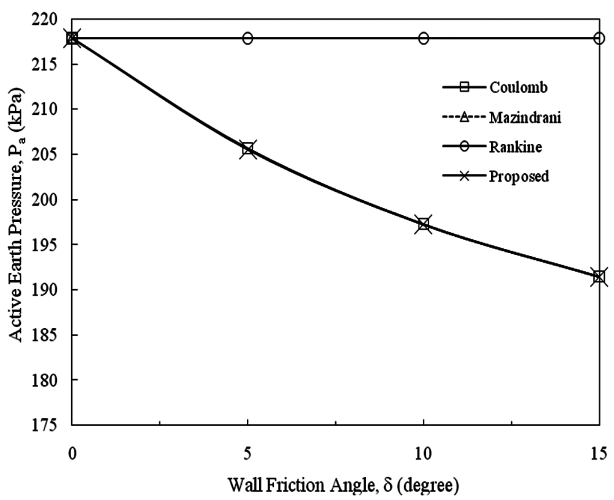


Fig. 9. Variation of Active Earth Pressure regarding Wall Friction Angle.

backfilled soil is cohesive. As shown in Fig. 10, Coulomb's active earth pressure value was much larger than the other three values since cohesion C cannot be considered with Coulomb's formula. The active earth pressure by Mazindrani's under tension cracked condition was slightly larger than the value by proposed formula but Mazindrani's active earth pressure values without tension crack are much larger than the value by proposed formula. The active earth pressure without tension crack and with tension crack is named Mazindrani I and Mazindrani II in Fig. 10, respectively. In this case, the active earth pressure values of Mazindrani's method without tension crack are approximately by 50 ~ 55 kPa larger than those by proposed formula. As already mentioned before, the tensioned cracked case cannot be compared with other three methods because of the different condition. However, the trend of active earth pressure without tension crack corresponds with the results of Fang, et al. (1997). This seems reasonable because Mazindrani's formula was also developed based on the Rankine theory.

If the planned retaining wall includes all parameters that were discussed before, there is no general formula that can evaluate the active earth pressure but graphical approach. However, if all discussed parameters should be considered

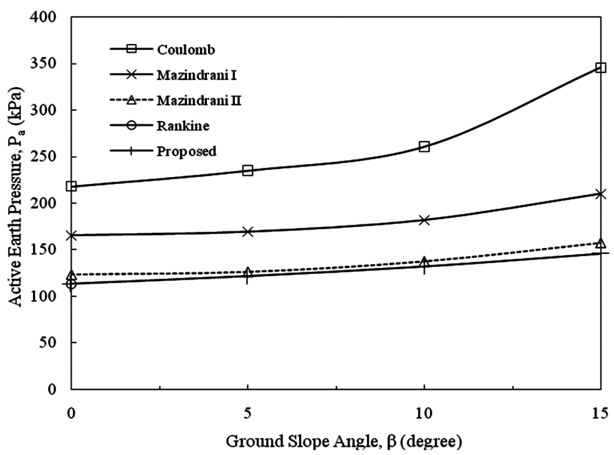


Fig. 10. Variation of Active Earth Pressure regarding Ground Slope Angle.

for the retaining wall design, the graphical approach will be extremely complicate. In such case, this proposed formula will be very convenient and the result will be accurate. As discussed before, this proposed formula can consider the following parameters such that ground surface slope, inclination of inside wall face, wall friction angle, soil cohesion, adhesion, line load and uniform load at the same time. As an example, active earth pressure is calculated by the proposed formula under following conditions i.e. $H = 6.5$ m, $\gamma = 17.52$ kN/m$^3$, $\phi = 15°$, $\delta = 10°$, $\beta = 10°$, $\theta = 100°$, $C = 10.5$ kN/m$^2$, $C' = 5$ kN/m$^2$, $P_l = 10$ kN/m and $Q_u = 24$ kN/m$^2$ and the calculated active earth pressure value by proposed formula was 267.3 kN/m. As shown in Table 3, only proposed formula can consider all parameters but not with other formulas.

## 7. Conclusion

Rankine formula has limitations i.e. wall friction angle $\delta$ should be zero or horizontal with the slope of retaining wall, inside wall face inclination angle $\theta$ should be 90°, ground surface slope angle $\beta$ should be zero and no external loads can be applied. After Mazindrani (1997), the estimation of active earth pressure with inclined surface is possible but this formula still has limitations i.e. wall friction $\delta$ should be zero, inside wall face inclination $\theta$ should be 90° and external loads cannot be considered. Coulomb formula also has limitations i.e. soil cohesion C, adhesion C′ and external loads cannot be considered. Therefore, it can be mentioned, there is no general formula that can consider all discussed parameters for the evaluation of active earth pressure up to now. As it was discussed, wall friction, soil cohesion, soil inclination etc are quite influential parameters and should not be ignored or simplified.

In order to solve all above discussed problems, an active earth pressure formula for general condition is proposed for

Table 3. Available Parameters in Each Method

| | $\beta$ (°) | $\theta$ (°) | $\delta$ (°) | C (kPa) | C′ (kPa) | $P_l$ (kN/m) | $Q_u$ (kPa) |
|---|---|---|---|---|---|---|---|
| Coulomb | A | A | A | NA | NA | NA | NA |
| Rankine | NA | 90 | 0 or $\beta$ | A | NA | NA | NA |
| Mazindrani | A | 90 | 0 or $\beta$ | A | NA | NA | NA |
| Proposed | A | A | A | A | A | A | A |

Note : A (Applicable), NA (Not Applicable) the estimation of maximum active earth pressure based on the force equilibrium. The result matched quite well not only with the graphical solutions but also with those by Coulomb's and Rankine's method.

Conclusively, the advantages from the wall geometries and soil properties should not be neglected and these values are not nominal value any more. This proposed formula can consider slope angle, wall inclination, wall friction angle, soil cohesion, adhesion, two types of external load i.e. line load and uniform load. This proposed formula can overcome the limitations of Coulomb's, Rankine's and Mazindrani's method, therefore, reasonable estimation of active earth pressure is possible with this proposed formula.

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