Effect of Side Friction on Consolidation Test of Normally Consolidated Kaolinite Slurry

정규압밀된 연약점토의 압밀시험시 측면 마찰의 영향

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ABSTRACT: Side friction is often neglected in the analysis of the results of a consolidation test when the specimen has a high ratio of diameter to height. As the height of a specimen increases, however, side friction becomes important. This paper presents an investigation of the effect of side friction on consolidation test results for normally consolidated kaolinite slurry. Consolidation tests were performed to obtain settlement, pore pressure, compressibility, and hydraulic conductivity of kaolinite slurry. The compressibility relationship is corrected for side friction using a modified form of Taylor (1942) analytical solution. Numerical simulations using the CS2 piecewise-linear model are compared with settlement and excess pore pressure measurements. Experimental results show improved agreement with a modified CS2 model in which the effect of side friction is considered. The numerical and experimental results indicate that the side friction is an important factor affecting the rate of consolidation as well as the compressibility relationship for the specimen.

Keywords: Normally consolidated, Side friction, Consolidation test, Compressibility, Numerical simulation

요지: 압밀시험시 시료의 두께에 대한 적절한 비율이 크면 일반적으로 측면마찰이 없다고 간주된다. 그러나, 시료의 두께가 커질수록 측면 마찰의 영향은 무시할 수 없게 된다. 본 연구에서는 정규압밀된 카올라니트 슬리퍼리의 압밀시험시 측면마찰의 영향을 조사하였다. 압밀시험을 통해 압밀시험곡선과 과밀견수압, 압밀-간극비, 투수계수-간극비 관계를 측정하였고, 압밀-간극비 관계에 있어 측면마찰의 영향을 고려하기 위해 Taylor(1942)가 제안한 식을 적용하였다. 측정된 압밀시험곡선과 과밀견수압에 대해 수치해석 결과와 비교 분석을 통해 측면 마찰이 압밀시험에 미치는 영향을 검토하였다. 그 결과, 측면 마찰의 영향을 고려한 수지해석 결과는 측면 마찰을 무시한 결과보다 측정값에 근사한 것으로 나타났으며, 측면 마찰은 압밀-간극비 관계 뿐만 아니라 압밀시간에도 영향을 주는 요인이라는 결과를 얻어졌다.

주요어: 정규압밀, 측면 마찰, 압밀시험, 압밀-간극비, 수치해석

1. Introduction

When a laboratory consolidation test is performed, care must be taken to remove the effect of side friction from the test results because side friction does not exist for vertical consolidation in situ. Side friction can be minimized using consolidometer ring coated with a non-water soluble lubricant; however, side friction still occurs and increases as the height of tested specimen increases (Leonards and Girault, 1961). The American Society for Testing and Materials (ASTM) specifies that a consolidation test specimen should be prepared with a minimum diameter to height ratio of 2:1. Taller specimens are required in some cases, such as slurry consolidation tests, because initially soft clayey soils experience large settlement even under small applied loads. In such cases, the effects of side friction should be considered in the analysis of test results.

Several investigations have been conducted to assess the effects of side friction. In general, side friction acts to reduce the applied vertical stress inside the specimen and this effect becomes greater with depth. Taylor (1942) measured side friction forces during consolidation and concluded that the magnitude of side friction can be significant. He provided a simple analysis to account for this effect on the compressibility curve (i.e., void ratio vs. vertical effective stress). Olson (1986) summarized side friction researches, which include experimental measurements and analysis using Taylor’s method, and concluded that side friction decreases

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with greased rings of typical geometry. Tan et al. (1988) conducted finite strain consolidation tests to obtain constitutive relationships of very soft clay. They installed load cells in the loading piston and bottom of chamber to measure side friction throughout the tests. They observed that, initially, side friction was negligible due to low vertical effective stress (i.e., slurry specimen) and concluded that the final side friction force was approximately 4% of applied load. Sivrikaya and Togrol (2006) measured side friction forces using modified consolidometer consisting of a rigid confining ring placed on a load cell, and compared measured values with estimates based on Taylor’s solution. They concluded that side friction generally increased with time with continued specimen loading and side friction forces obtained from Taylor’s solution are close to measured forces. Most of previous studies focus on side friction at the end of primary consolidation and there are few studies that have investigated the effect of side friction during consolidation. Leonardis and Girault (1961) computed pore pressure dissipation with a modified infinitesimal strain (Terzaghi) consolidation theory and found that the corrected theoretical curves were in better agreement with the measured rates of pore pressure dissipation.

This paper presents an experimental and numerical investigation on the effect of side friction during large strain consolidation tests. Consolidation tests were performed on kaolinite slurry to measure settlement, pore pressure, and constitutive relationships. The measured compressibility curve was corrected using Taylor’s method, which is modified to accommodate side friction caused by specimen self-weight. Measured settlement and pore pressures are compared with numerical simulations with and without side friction using CS2 piecewise linear model (Fox and Berles, 1997). Conclusions are reached with regard to the effect of side friction on the rate of consolidation.

2. Side Friction Analysis

Taylor (1942) presented a method to correct soil compressibility for the effect of side friction. In the absence of side friction, the vertical effective stress ($\sigma'_v$) will be constant over depth and equal to vertical applied stress ($q$). Knowing that the consolidometer walls are not frictionless, however, side friction forces act in the upward direction and reduce the vertical effective stress for lower portions of the specimen. It is assumed that side friction is proportional to vertical effective stress, and then vertical effective stress at depth $z$ is

$$\sigma'_v = q - \frac{2f}{r} \int_0^z \sigma_z dz$$

(1)

where $f$ is a side friction coefficient and $r$ is the radius of a specimen. The solution of Eq. (1) is

$$\sigma'_v = qe^{-\frac{2f z}{r}}, \quad 0 \leq z \leq H_f$$

(2)

where $H_f$ is the final height of the specimen. Eq. (2) indicates that vertical effective stress decreases exponentially with depth inside the specimen. The vertical effective stress increment caused by self-weight may not be negligible for a specimen with low ratio of diameter to height. Eq. (2) can be thus modified to accommodate effective stress due to self-weight of the specimen ($\sigma'_{\text{self}}$) as,

$$\sigma'_v = (q + \sigma'_{\text{self}})e^{-\frac{2f z}{r}}, \quad 0 \leq z \leq H_f$$

(3)

Effective stress due to specimen self-weight at the end of primary consolidation is linearly distributed with depth and can be expressed as,

$$\sigma'_{\text{self}} = \frac{G_s - 1}{1 + e_o} \frac{H_o}{H_f} z$$

(4)

where $G_s$ is specific gravity of solids, $e_o$ is initial void ratio, $\gamma_o$ is unit weight of water, and $H_o$ is initial height of the specimen. Eq. (4) is derived by assuming that $e_o$ is constant. Due to variable effective stress over the depth of the specimen, the average vertical effective stress ($\sigma'_{\text{avg}}$) is calculated by integrating Eq. (3) and divided by the final height,

$$\sigma'_{\text{avg}} = \frac{q - \frac{2f}{r} \left(1 - e^{-\frac{2f H_f}{r}}\right)}{2H_f}$$

$$+ \frac{1}{H_f} \frac{G_s - 1}{1 + e_o} \frac{H_o}{H_f} \left[-\frac{r}{2f} e^{-\frac{2f H_f}{r}} \left(1 - e^{-\frac{2f H_f}{r}}\right) \left(e^{-\frac{2f z}{r}} \left(1 - e^{-\frac{2f H_f}{r}}\right)\right)^2 \right]$$

(5)

The effect of self weight is removed by specifying $G_s = 1$, in which case Eq. (5) reduces to Taylor’s solution.
The side friction coefficient can be defined as (Olson, 1986),

\[ f = \mu K_c \]  

(6)

where \( \mu \) (\( = \tan \delta \)) is the interface friction coefficient between the consolidometer cell and the specimen, \( K_c \) is the coefficient of earth pressure at rest, and \( \delta \) is the angle of interface friction. The friction coefficient \( \mu \) is the ratio between shear strength and normal effective stress when full frictional resistance is mobilized. Using direct shear tests, Littleton (1976) measured the angle of interface friction between cohesive soils and a smooth steel surface and found that the ratio of \( \delta \) to the effective stress friction angle (\( \phi' \)) for clay ranged from 0.89 to 0.92. Tsubakihara and Kishida (1993) studied friction between normally consolidated clay and steel using direct shear apparatus. The experimental results demonstrate that \( \delta \) is similar to \( \phi' \). In this paper, therefore, it is reasonable to assume \( \delta = \phi' \).

Mesri and Hayat (1993) reported measured values of \( K_c \) from a large number of consolidation tests on clay soils. The magnitude of \( K_c \) for sediments, normally consolidated young clay deposits is

\[ K_c = 1 - \sin \phi' \]  

(7)

Combining (6) and (7), and assuming \( \delta = \phi' \),

\[ f = \tan \phi'(1 - \sin \phi') \]  

(8)

3. Laboratory Tests

3.1 Material Properties

The experimental program was conducted using specimens of kaolinite slurry. The kaolinite was purchased from the Unimin Corporation in powdered form. Soil property tests followed ASTM standard procedures. Index properties are liquid limit (LL) = 47.6, plastic limit (PL) = 21.8, and \( G_s = 2.61 \). The material is classified according to the Unified Soil Classification System as CL - lean clay.

3.2 Consolidation and Hydraulic Conductivity Measurements

Consolidation test specimens were prepared to a slurry condition with a target moisture content of 2 LL (= 95.2%). Consolidation test apparatus is shown in Fig. 1. A consolidometer cell (dia. = 10.2 cm) was coated with a lubricant oil to minimize side friction and entrapped air was removed from the porous disks, fittings, and tubing. The specimens were then placed in the cell by a combination of spooning and careful pouring. Due to the high initial void ratio of each specimen, a very small initial stress was applied using only the porous disk and piston. Two consolidation tests were performed on the kaolinite slurry. Test J0 was conducted to obtain the hydraulic conductivity constitutive relationship, which for clay depends on void ratio rather than effective stress (Al-Tabbaa and Wood, 1987; Nagaraj et al., 1994; Fox, 2007b) and thus would not be affected by side friction. Vertical stress levels were 2.9, 5.3, 10.1, 19.8, 39.2, 78.2, 39.2, and 10.1 kPa. Hydraulic conductivity was measured at the end of each load increment using a syringe flow pump connected to the base of the consolidometer. At the end of each load increment, a valve at the base of the cell was opened to the pump, which flowed water upward through the specimen at a constant rate and thus minimized possible additional large volume change due to seepage forces. Dial gauge readings during the hydraulic conductivity measurements indicated that the specimen void ratio remained constant during permeation. Each hydraulic conductivity test lasted approximately 24 hours until steady state pore pressures were measured at the base of the specimen. At the end of each hydraulic conductivity test, the valve was closed and the next load increment was applied.

Fig. 1, Diagram of large strain consolidation apparatus
Test J1 was conducted to obtain settlement, excess pore pressure, and compressibility data for a second essentially identical specimen of kaolinite slurry. The J1 specimen was consolidated using six load increments (3.1, 5.6, 10.4, 20.1, 39.5, and 78.4 kPa), single drainage at the top boundary, and pore pressure measurements taken at the base as shown in Fig. 1. Each load increment remained on the J1 specimen for 3 days.

4. Results and Discussion

4.1 Constitutive Relationships

Fig. 2 shows measured relationships for kaolinite hydraulic conductivity obtained from test J0 and compressibility obtained from test J1. Fig. 2 (a) indicates a linear relationship between hydraulic conductivity \( k \) and void ratio \( e \) on the semi-log plot and is characterized by a constant hydraulic conductivity index \( C_k = \frac{d e}{d \log k} = 0.877 \). The measured compressibility data in Fig. 2 (b) shows an essentially constant value of compression index \( C_c = 0.674 \). The corrected compressibility curve using Eq. (5) is also presented. Interestingly, the corrected compression index \( C'_c = 0.653 \) is only slightly smaller. This may tend to suggest that the effect of side friction is relatively constant. However, Fig. 2 (c) shows the same compressibility curves plotted on an arithmetic scale and indicates that side friction clearly increases with increasing vertical effective stress and decreasing void ratio. The compressibility curve is shifted upward as side friction acts to increase the measured void ratio for any given applied vertical effective stresses.

4.2 Settlement and Pore Pressure

Fig. 3 shows measured specimen height \( H \) as a function of time \( t \) for J1. Primary consolidation was generally finished in approximately 2 days for each load increment and was followed by a negligible amount of secondary compression. Initial and final heights for the J1 specimen were 71.7 and 45.2 mm, which corresponds to an average vertical strain of 37%. Thus, large strain was achieved in the J1 consolidation test.

Numerical simulations were performed using the CS2 piecewise-linear model (Fox and Berles, 1997), which can simulate large strain consolidation. CS2 accounts for vertical stain, self-weight, the relative velocity of fluid and solid phase, and variable hydraulic conductivity and soil compressibility.
during the consolidation process. The constitutive relationships for the soil are specified using discrete data points without requiring mathematical approximations or the calculation of derivative functions. The comparatively simple formulation of CS2 makes it particularly advantageous for the incorporation of additional modifications, including layered systems, time-dependent applied loads and piezometric groundwater levels, depth-dependent and pre-existing initial excess pore pressures, log-linear constitutive relationships including a preconsolidation pressure and unloading/reloading effects. Solid mass conservation can be strictly enforced and, as such, the method is actually a Lagrangian approach that follows the motion of the solid phase throughout the consolidation process. Subsequent upgrades to CS2 have included effects of compressible pore fluid (Fox and Qiu, 2004) and high gravity conditions in a geotechnical centrifuge (Fox et al., 2005; Lee and Fox, 2005). Simulations for the current paper assume normal gravity and incompressible pore water, and were conducted using the J0 hydraulic conductivity relationship and the J1 compressibility relationships with and without side friction. Eq. (1) was used in CS2 to simulate the effect of side friction during consolidation with a single top drained boundary, uniformly distributed initial void ratio (\(e_0 = 2.48\)), and 100 elements. The internal friction angle of kaolinite was estimated as \(\phi' = 31^\circ\) based on the plastic index \((I_p = 25.8)\) and using correlations provided by Mitchell (1993). Side friction is activated when relative displacement (i.e., settlement) occurs between a clay element in CS2 and the sidewall of the consolidometer.

Results of the CS2 simulations for settlement are also presented in Fig. 3. Although the results are similar, some differences are observed. The simulation without side friction generally shows a faster consolidation rate than the measured values. When the corrected compressibility relationship and side friction are included, however, the simulated consolidation rate decreases and the numerical results are in excellent agreement with measured values for both final height and consolidation rate. Simulated specimen heights \((H)\) and excess pore pressures \((u_e)\) at the base of the J1 specimen are presented for the first and last load increments in Figs. 4 and 5, respectively. Although it was originally expected based on results of Tan et al. (1988) that the effect of side friction would be minor at low vertical effective stress, Fig. 4 (a) indicates the rate of consolidation settlement is

![Fig. 3. Experimental and numerical settlement curves for test J1](image)

![Fig. 4. Experimental measurements and numerical simulations for the first load increment of test J1: (a) settlement and (b) pore pressure](image)
Fig. 5. Experimental measurements and numerical simulations for the last load increment of test J1: (a) settlement and (b) pore pressure

significantly affected by side friction during the first load increment. In Fig. 4 (b), simulated excess pore water pressures that include the effect of side friction also show better agreement with measured values. Close agreement of maximum measured and predicted pore pressures in Fig. 4 (b) also suggests that air bubbles were not trapped in the tubing and valves of the pore pressure measurement system.

Corresponding curves for measured and simulated settlement and excess pore pressure for the last load increment of J1 are shown in Fig. 5. Fig. 5 (a) suggests that side friction has a less important effect on the rate of settlement at high effective stress. In Fig. 5 (b), CS2 simulations show identical maximum pore pressure of 38.9 kPa for both with and without side friction, but simulation with side friction indicates that pore pressure drops sharply after consolidation starts (i.e., once side friction is activated). The measured maximum pore pressure of 29.4 kPa in Fig. 5 (b), however, is significantly less than simulated value of 38.9 kPa. This discrepancy occurs as a result from the assumption that side friction activates when settlement occurs within the CS2 model. Settlement from previous load increments results in fully developed side friction such that the new load increment is carried by both side friction and pore water pressure. As a result, the maximum pore pressure is less than the applied load increment, which corroborates a similar to the conclusion reached by Leonards and Girault (1961).

It is interesting that, with increasing vertical stress, side friction forces increase but the effect of side friction on consolidation rate decreases. This is caused by the relationship between hydraulic conductivity and compressibility on consolidation as shown in Fig. 6. For the first loading step, void ratio decreases from initial ($e_i$) to final ($e_f$) values as vertical effective stress increases from $\sigma'_{inf}$ to $\sigma'_{uf}$. In this range of void ratio, hydraulic conductivity is very sensitive to the change of void ratio as depicted on Fig. 6. Therefore, small shifting of compressibility due to side friction causes significantly different consolidation rate. On the other hand, hydraulic conductivity is not sensitive to changes in the compressibility curve when the last load is applied. Therefore, the effect of side friction has an important effect on the rate of consolidation for early increments even though vertical effective stress is low.

5. Conclusions

An experimental and numerical investigation was conducted into the effect of side friction on the results of a consolidation test. Consolidation tests were performed to measure
settlement, pore pressure, compressibility, and hydraulic conductivity for kaolinite slurry. Measured compressibility was corrected to include the effect of side friction using a modified form of Taylor’s analytical solution that incorporates self-weight of a specimen. The compressibility curve is shifted upward as side friction acts to increase the measured void ratio for any given applied vertical effective stresses.

Numerical simulations without side friction predicted a faster consolidation settlement rate than the experimental measurements. However, numerical results obtained using a modified version of CS2 with side friction and corrected compressibility show good agreement with settlement and pore pressure measurements. Experimental and numerical results revealed that maximum pore pressure upon loading may be significantly less than the magnitude of the load increment when side friction acts to support the additional load. Simulations also revealed that pore pressure decreases more rapidly in the presence of side friction. Side friction effects were more pronounced in the early stages of consolidation due to the sensitivity of small void ratio changes on soil hydraulic conductivity.

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References


Notation

The following symbols are used in this paper:

- $C_c$ compression index
- $C_h$ hydraulic conductivity index
- $e$ void ratio
- $e_v$ initial void ratio
- $e_{v0}$ final void ratio of first load increment
- $f$ coefficient of side friction
- $G_s$ specific gravity of solids
- $H$ height of specimen
$H_o$ initial height of specimen  
$H_f$ final height of specimen  
$K_o$ coefficient of earth pressure at rest  
k hydraulic conductivity  
$LL$ liquid limit  
$PL$ plastic limit  
$q$ vertical applied stress  
r radius of specimen  
$t$ time  
$u_c$ excess pore pressure  
z depth coordinate  

$\delta$ angle of interface friction  
$\phi'$ effective stress friction angle of specimen  
$\gamma_w$ unit weight of water  
$\mu$ coefficient of interface friction  
$\sigma'_v$ vertical effective stress  
$\sigma'_{v,avg}$ average vertical effective stress  
$\sigma'_{v,o}$ initial vertical effective stress of the first load increment  
$\sigma'_{v,f}$ final vertical effective stress of the last load increment  
$\sigma'_{v,Self}$ effective stress by self weight of specimen