유한차분법과 유한체적법을 이용한 1차원과 2차원 개수로 흐름해석

김 만 식*+ / 이 진 희** / 정 찬*** / 박 노 혁****

Analysis of 1D and 2D Flows in Open-Channel with FDM and FVM

Kim, Man Sik*+ / Lee, Jin Hee** / Jeong, Chan*** / Park, Roh Hyuk****

요약 : 하천에서의 부정류 해석을 위해서 1차원 유한차분법(FDM)인 Abbott-Ionescu scheme과 2차원 유한체적법(FVM)인 근사의 Riemann solver(Osher scheme)에 대하여 살펴보았다. 두 모형은 직선 하도, 약간 굽어진 사행하도 및 사행하도에서의 흐름 문제들에 적용되었으며 결과의 비교는 균일한 직사각형 수로에 대하여 이루어졌다. 하천의 복잡한 형상의 표현하기 위해서는 이를 고려할 수 있는 유한체적법을 이용하였다. 유한차분법과 유한체적법 결과는 수위 및 유량 수문곡선에 대하여 매우 만족스러운 것으로 나타났다. 균일한 직선하도에 대해서는 1차원분석으로도 충분하다는 사실을 파악할 수 있었으며, 사행하도의 경우 흐름을 정확하게 모형화하기 위해서는 2차원 또는 3차원 모형을 사용하여야 할 것이다.

핵심용어 : 유한차분법, 유한체적법, 개수로 흐름, 부정류 흐름

Abstract : The one-dimensional (1D) finite-difference method (FDM) with Abbott-Ionescu scheme and the two-dimensional (2D) finite-volume method (FVM) with an approximate Riemann solver (Osher scheme) for unsteady flow calculation in river are described. The two models have been applied to several problems including flow in a straight channel, flow in a slightly meandering channel and a flow in a meandering channel. The uniform rectangular channel was employed for the purpose of comparing results. A comparison is made between the results of computation on 1D and 2D flows including straight channel, slightly meandering channel and meandering channel application. The implementation of the finite-volume method allows complex boundary geometry represented. Agreement between FVM and FDM results regarding the discharge and stage is considered very satisfactory in straight channel application. It was concluded that a 1D analysis is sufficient if the channel is prismatic and remains straight. For curved (meandering) channels, a 2D or 3D model must be used in order to model the flow accurately.

Keywords : FDM, FVM, Open Channel Flow, Unsteady Flow

1. Introduction

Water flow in open channels is classified as steady or unsteady flow. If the flow velocity at a given point does not change with respect to time, the flow is called steady flow. The flow of water in river channels, canals, reservoirs, lakes, pools, and free surfaces flow in drains, conduits, pipes, galleries, tunnels and culverts, for which the velocities change with time, is defined as unsteady flow. Water flow in natural channels is almost always unsteady. There are number of

+ Corresponding author : kmspj@shinsung.ac.kr
* 정회원 · 신성대학교 도시건설과 교수
** 정회원 · 교신저자 · 인하대학교 사회기반시스템공학과 연구교수
*** 비회원 · 한국수자원공사 기술관리실 차장
**** 비회원 · 한국수자원공사 차장

open channel problems involving unsteady flows: that is, flows in which the velocity and depth of flow vary with time. The computation of unsteady flows is required for the analysis of tidal oscillations, flow produced by the operation of control structures such as sluice gates, pump or turbines, and flood wave generated by storms or failure of dams, dykes, or other structures. Few of these problems can be solved analytically, so they have to be solved numerically. For such analyses, numerical model may be used as a management tool if the water level and wave speed can be accurately predicted over the wide range of conditions.

The flow phenomena in river or a river network is complex because uncertainties are associated with related parameters and the differential equations cannot be integrated in closed forms except under very simplified conditions. The analysis of flow phenomena includes the computation of flood water level and discharge which determine the required height of structures such as bridges and levees and size of detention pond.

Computational models are less expensive than the equivalent physical scale model Many alternative design can be readily tested, quickly and cheaply although they can be only applied where the main underlying physics of the flow are known and can be included in the model. In addition, all desired quantities can be simulated with high resolution in space and time and the models do not suffer from any scaling effects. There are three most popular computational methods for the solution of governing equations including finite-difference method(FDM), finite-volume method(FVM), and finite-element method (FEM). The computational methods includes the processes of discretization(coupling governing equations) and solver(solving the discretized algebraic equations).

The purpose of this paper is to compare 1D and 2D flows in river modeling with FDM and FVM and describe the numerical methods according to theoretical development, applicability to different shape of rivers. A sensitivity analysis of the computational stability of the methods is also performed with regard to the time intervals. To compare the two methods, the Abbott-Ionescu scheme (1D) for finite-difference method and the Osher-scheme (2D) for finite-volume method are introduced.

2. Theological Background and Governing Equations

Open channel flows can be governed by the one-dimensional (1D) St. Venant equations or two-dimensional (2D) shallow water equations. The St. Venant equations include a continuity equation and an one-dimensional momentum equation and the relative assumptions can be found in Cunge et al.(1980), Abbott and Basco(1990), and Chaudhry(1993).

Continuity equation: $$\frac{\partial Q}{\partial x}+b_s\frac{\partial h}{\partial t}=0 \qquad (1)$$

Momentum equation:
$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\beta\frac{Q^2}{A}\right)+gA\left(\frac{\partial h}{\partial x}-I_b\right)+gA\frac{|Q|Q}{K^2} \qquad (2)$$

where t: time; x: longitudinal distance; Q: discharge(m^3/s); h: water depth(m); b_s: storage width(m); A: cross-sectional area(m^2); K: conveyance($K=CA\sqrt{R}$, m^3/s); C: Chezy resistance coefficient($m^{0.5}/s$); R: hydraulic radius (m); β: Boussinesq coefficient; I_b: Bottom slop. The assumpsions to develop the continuity and momentum equations mentioned above is found in Chaudhry (1993).

The most widely used approaches of fee-surface flows is that of shallow water, which can be obtained from the depth-averaged Navier-Stokes equations. The 2D shallow water equations can be expressed by vector form, which the terms describing turbulent diffusion are ignored (Zhao et al., 1994).

$$\frac{\partial q}{\partial t}+\frac{\partial f(q)}{\partial x}+\frac{\partial g(q)}{\partial y}=b(q) \tag{3}$$

where $q=[h,\ hu,\ hv]^T$:conserved physical vector; $f(q)=\left[hu,\ hu^2+\frac{gh^2}{2},\ huv\right]$:flux vector in the x-direction; $g(q)=\left[hu,\ huv,\ hv^2+\frac{gh^2}{2}\right]$: flux vector in the y-direction; u and v: depth-averaged velocity components along the x and y directions respectively. The source/sink term $b(q)$ is written as

$$b(q)=[0,\ gh(s_{0x}-s_{fx}),\ gh(s_{0y}-s_{fy})]^T \tag{4}$$

where s_{0x} and s_{fx}:bed slope and friction slope in the x-direction respectively; s_{0y} and s_{fy}:bed slope and friction slope in the y-direction respectively. The friction slope can be estimated using the Manning resistance law.

$$s_{fx}=\frac{un^2\sqrt{u^2+v^2}}{h^{\frac{4}{3}}},\ s_{fy}=\frac{vn^2\sqrt{u^2+v^2}}{h^{\frac{4}{3}}} \tag{5}$$

where n: Manning's resistance coefficient.

Computational models of river flows based on the St. Venant equations and shallow water equations are well-established tools in engineering practice. There have been a great number and variety of numerical techniques to solve unsteady flow problems. However, there is still not a single answer to which method is best. The answer to that question depends upon the particular application.

For many hydraulic engineering problems, the analysis of flow in open channel is a fundamental prerequisite. Numerical solutions of the one-dimensional (1D) open-channel flow equations are very common and have been in use for many years. In some situations, however, these flows are better described by two-dimensional models, e.g., interaction between the main channel flow and the floodplain flow(Zhao, 1994).

Most current hydrodynamic models of rivers are based on finite-difference solutions of the depth-averaged shallow water equations. These methods generally use space-staggered approximations on regular Cartesian grids overlaid on the region of interest. The lack of alignment of coordinate lines with boundaries can lead to inaccuracies in flow solution. Boundary-fitted grids provide one method of circumventing this problem by making the boundary a coordinate surface (Mingham and Causon 1998). Wijbenga (1985) developed one of the mathematical models capable of solving 2D on curvilinear grids (boundary-fitted coordinates), depth-averaged equations using the finite-difference scheme.

Another modeling approach called finite-volume method has been proposed by Zhao et al.(1994, 1996). This approach is to cast the flow equations in integral form with the solution variables stored at cell centres rather than at node points. The spatial discretization then involves simple flux balances across cell interfaces. Finite-volume methods based on the integral form of the conservation laws have several advantages over finite-difference methods. Finite-difference method is not valid at flow discontinuities such as bore waves since it is based on the differential form of the shallow water equations. In contrast, the integral form of the equations is valid both at flow discontinuities and in smooth region of the flow field. Finite-volume method combines the simplicity of finite-difference method with the geometric flexibility of finite-element method (Mingham and Causon 1998).

The solution of the St. Venant equations in a single channel with an implicit finite-difference method is described by Abbott (1979), Chaudhry (1993), Cunge et al. (1980) and Montes (1998). The second-order accurate finite-difference explicit schemes were used by Chaudhry and Fennema (1990) for the solution of two-dimensional shallow-water equations. They controlled some numerical oscillations occurred near the sharp-fronted waves using by selective addition of artificial viscosity. The historical overview of an implicit scheme and explicit scheme is shown by Montes (1998).

The solution of the 2 dimensional shallow water equation by Abbott-Ionescu scheme is described in detail by Abbott and Basco (1990) and Rillaer (1997). The general solution of shallow water equations with the finite-volume method is given by Toro (1997). A first-order finite-volume method using the Osher scheme is given by Zhao et al. (1994). A high-order finite-volume method based on MUSCL variable extrapolation and slope limiters for the resolution of 2D free-surface flow equations is shown by Alcrudo and Garcia-Navarro (1993).

The finite-volume methods using the three different schemes - the flux vector splitting (FVS), the flux difference splitting (FDS), and the Osher scheme - are compared by Zhao et al. (1996). The Osher scheme is the most accurate while FVS is least accurate. On the other hand, the FVS scheme required the least computer time while FDS required most computer time (Zhao et al. (1996)). The general problem in meandering channel is described in detail by Chang(1992), Chow (1959) and Petersen (1986). The modelling in meandering channel is described by Kalkwijk and Vriend (1980). They took into account the extra terms for transverse momentum exchange due to the secondary flow.

3. Solution of governing equations by FDM and FVM

3.1. Finite-difference solution with Abbott-Ionescu scheme

The Abbot-Ionescu scheme is depicted schematically in Figure 1.

This is a staggered-grid and implicit scheme, in which the two dependent variables are calculated at alternative grid points. Upon finite difference approximation the general form of the equations of (1) and (2) are as follows.

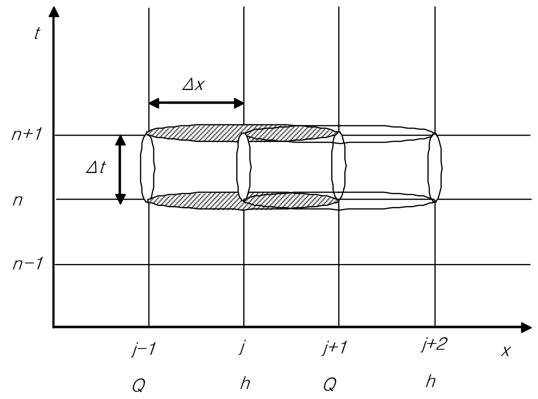

Figure 1. Schematic of the Abbott-Ionescu staggered scheme

$$A1_j Q_{j-1}^{n+1} + B1_j h_j^{n+1} + C1_j Q_{j+1}^{n+1} = D1_j \qquad (6)$$

where $A1_j = \left(\frac{-\theta}{2\Delta x}\right) = constant$; $B1_j = \left(\frac{b_{s_j}^{n+1/2}}{\Delta t}\right)$;

$C1_j = \left(\frac{\theta}{2\Delta x}\right) = constant$;

$D1_j = \frac{b_{s_j}^{n+1/2}}{\Delta t} h_j^n - \left(\frac{1-\theta}{2\Delta x}\right)\left(Q_{j=1}^n - Q_{j-1}^n\right).$

$$A2_j Q_{j-1}^{n+1} + B2_j h_j^{n+1} + C2_j Q_{j+1}^{n+1} = D2_j \qquad (7)$$

where $A2_j = -\frac{\beta}{\Delta t}\frac{b_{s_j}^{n+1/2} Q_j^{n+1/2}}{A_j^{n+1/2}} - \frac{\theta}{2\Delta x}\left(gA_j^{n+1/2} - \frac{b_{s_j}^{n+1/2}\left(Q_j^{n+1/2}\right)^2}{\left(A_j^{n+1/2}\right)^2}\right)$;

$B2_j = -\frac{1}{\Delta t} + \frac{gA_j^{n+1/2}}{K_j^{n+1/2}}|Q_j^n|$;

$C2_j = -\frac{\beta}{\Delta t}\frac{b_{s_j}^{n+1/2} Q_j^{n+1/2}}{A_j^{n+1/2}} + \frac{\theta}{2\Delta x}\left(gA_j^{n+1/2} - \frac{b_{s_j}^{n+1/2}\left(Q_j^{n+1/2}\right)^2}{\left(A_j^{n+1/2}\right)^2}\right)$;

$D2_j = \frac{Q_j^n}{\Delta t} - \frac{\beta}{\Delta t}\frac{b_{s_j}^{n+1/2} Q_j^{n+1/2}}{A_j^{n+1/2}}\left(h_{j-1}^n + h_{j+1}^n\right) - \frac{1-\theta}{2\Delta x}\left(gA_j^{n+1/2} - \beta\frac{b_{s_j}^{n+1/2}\left(Q_j^{n+1/2}\right)^2}{\left(A_j^{n+1/2}\right)^2}\right)\left(h_{j+1}^n + h_{j-1}^n\right) + gA_j^{n+1/2} I_b$

The use of algorithmic from and shifting of center points results in a tridiagonal matrix structure for the equation system equation (6) and (7). This permits a solution by double-sweep technique. The model uses implicit Abbott-Ionescu scheme and called New Improved Gem (Kutija and Murray, 1999). The model represent supercritical flow, lateral inflow and outflow, and irregular cross sections.

3.2. Finite-Volume Method by Osher Scheme

Upon integrating equation (3) over an arbitrary element Ω, the basic equation of the FVM can be obtained by the divergence theorem.

$$\iint_{\Omega} q_t \, d\omega = -\int_{\partial\omega} F(q) \cdot n\, dL + \iint_{\Omega} b(q)\, d\omega \quad (8)$$

where n = a unit outward vector normal to the boundary $\partial\Omega$; and $\partial\omega$ and ∂dL= area and arc elements. The integrand $F(q) \cdot n$ = normal flux vector in which $F(q)$ = $[f(q) \quad g(q)]^T$. The geometry of finite volume Ω is shown in Figure 2.

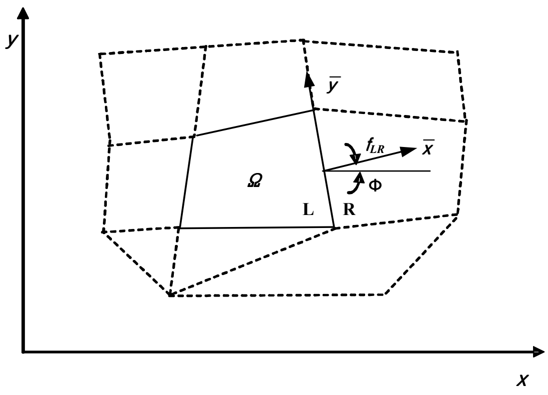

Figure 2. Geometry of Finite Volume Ω

The resolution of the flux vector in the normal direction allows the treatment of the 2D problem as a local 1D problem and the vector quantity q is assumed constant over and element. Thus, the discretized basic equation of the FVM is given by Zhao et al.(1994).

$$A\frac{dq}{dt} = -\sum_{j=1}^{m} T(\Phi)^{-1} f(\bar{q}) L^j + Ab(q) \quad (9)$$

where A = area of an element; m = total number of sides for an element; j = index for the side of the element; L^j = length of the side; $\bar{q}$ = transformed quantity of q. The estimation of the transformed flux $f(\bar{q})$ in equation (9) translates into the problem of solving a local 1D Riemann problem in the direction normal to the element interface.

The local 1D Riemann problem is an initial-value problem written as

$$\bar{q}_t + [f(\bar{q})]_{\bar{x}} = 0 \quad (10)$$

with

$$\bar{q}(\bar{x},\ 0) = \begin{cases} \bar{q}_L; & \bar{x} < 0 \\ \bar{q}_R; & \bar{x} > 0 \end{cases}$$

where $\bar{q}_L$ and $\bar{q}_R$ are the properties on the left and right of the element interface. The flux $f(\bar{q})$ in equation (10) can be split as forward and backward fluxes(Spekreijse, 1988) such that

$$(\bar{q}) = f^+(\bar{q}) + f^-(\bar{q}) \quad (11)$$

Then the approximate solution of the Riemann problem is given by

$$f_{LR}(\bar{q}_L, \bar{q}_R) = \begin{cases} f^+(\bar{q}_L) + f^-(\bar{q}_R) \\ f(\bar{q}_L) + \int_{\bar{q}_L}^{\bar{q}_R} J^-(\bar{q}) d\bar{q} \\ f(\bar{q}_R) - \int_{\bar{q}_L}^{\bar{q}_R} J^+(\bar{q}) d\bar{q} \end{cases} \quad (12)$$

where $J^+(\bar{q})$ and $J^-(\bar{q})$ = Jacobian matrices associated to the positive and negative eigenvalues of J. The Jacobian matrix J is given by

$$J = \frac{df}{dq} \begin{pmatrix} 0 & 1 & 0 \\ c^2 - u^2 & 2u & 0 \\ -uv & v & u \end{pmatrix} \qquad (13)$$

where $c = (gh)^{1/2}$.The estimate of the normal flux $f_{LR}(\bar{q}_L, \bar{q}_R)$ for given values of hydraulic properties (u, c) can be obtained by Osher approximate Riemann solver (Zhao et al., 1994).

4. Application and Results

A hypothetical example has been chosen to compare the results. The model is applied to the hypothetical area that is uniform rectangular channel 1800m long and 30m wide. The channel has a bed slope equal to 0.00001 and a Manning resistance coefficient n = 0.02.

The initial conditions at time t = 0 for all methods are shown in Figure 3. The flow depths and velocities at all grid points and elements at t = 0 are assumed to be 2 m and 0.231 m/s, respectively. The upstream boundary condition is an inflow hydrograph as shown in Figure 4. The downstream boundary condition is given by rating curve as given in Figure 5.

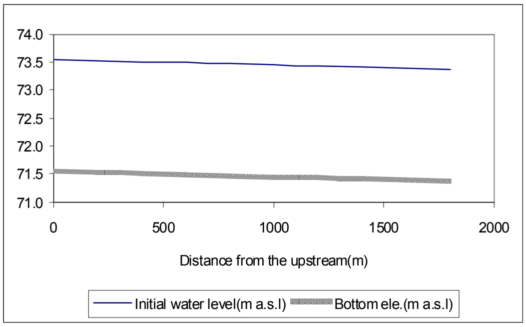

Figure 3. Initial conditions along the length of the channel

The method suggested by Kutija and Murray (1999) is used for finite-difference method. For the simulation of flows with the finite-volume method suggested by Erduran (1999) is used. The finite-volume method is applied to three different shapes of rivers, i.e., straight channel, slightly meandering channel and meandering channel (Figure 6 and Figure 7). The results are compared with computational results obtained by finite-difference method.

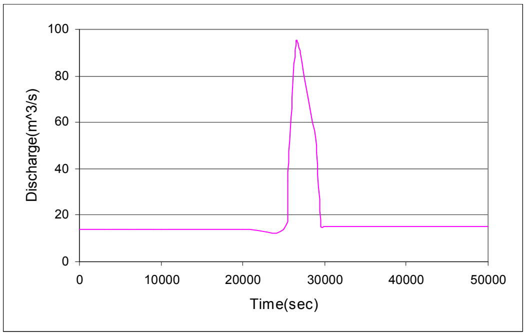

Figure 4. Upstream boundary condition

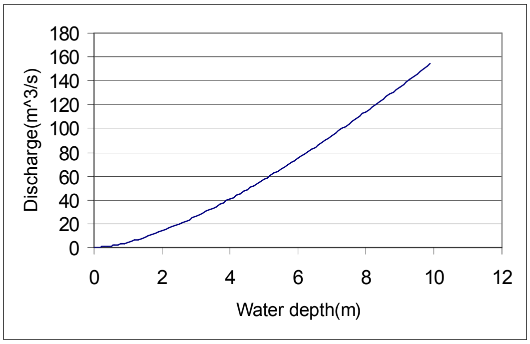

Figure 5. Downstream boundary condition (Rating curve)

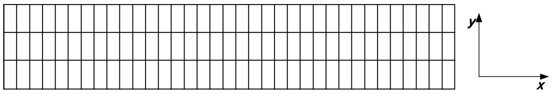

Figure 6. Generated grids for the straight channel

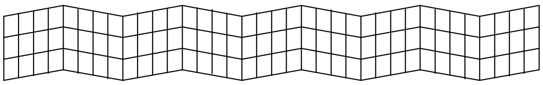

Figure 7. Generated grids for the slightly meandering channel

The preparation of data for finite-difference method is relatively easy compared with finite-volume method. The only one set of data file is required for the simulation of the flow regardless of the shape of the rivers because of the underlying assumption of the discretization in the finite-difference method.

As mentioned earlier, the scheme used for this study is the implicit scheme. This means that the scheme has no time step limit for stability and can be run successfully at Courant number greater than unity. However, for Cr > 1, phase error can be excessive so that a control has to be kept on Cr in practical computations.

To select the reasonable time step for the simulation, the simulated profile at the certain time may be used. The plot of water depth after 27000 sec with different time is shown in Figure 8. The computational results for the comparison with the finite-volume method will be obtained using a time step dt = 50 sec in the numerical scheme.

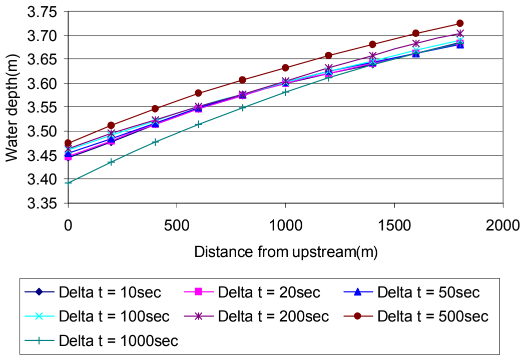

Figure 8. Water depth after 27000 sec with different time step

The computational domain consists of a 1800m x 30m region that has been subdivided into a 3 x 36 (108) quadrilateral grid with each element 50m x 10m for a straight channel application in finite-volume method.

At the centre of each grid, the Manning resistance coefficient, water depth, and velocity must be specified. The Manning resistance coefficient is assumed as 0.02 everywhere. The area of the grid, length of each side of the grid, the angle between vector n and the x-axis and slope in each direction for the different shapes of channels are obtained by using MicroSoft Excel. These quadrilateral grids can be generated by using an efficient commercial package. The unit discharge for the inflow hydrograph and rating table is used.

Since the scheme for the finite-volume method is explicit scheme, a time step dt is restricted by CFL condition. The results for the comparison with the finite-difference method will be obtained using a time step dt = 1 sec and 0.5 sec in the numerical scheme.

(1) Straight channel

Plots of water level for times t = 25000, 27000, 29000, and 32000 sec are shown in Figure. 9 through Figure. 12, respectively. These Figures show that the results from the two methods agree closely. Plots of discharge are shown in Figure. 13 through Figure. 16 for times t = 25000, 27000, 29000, and 32000 sec. It is clear from these figures that the agreement between the results computed by FVM and FDM is satisfactory. Figure. 17 shows the time variation of flow depth 1000m distance from upstream. Figure. 18 shows the time variation of discharge computed by the two methods at 1100m distance from upstream. It is clear from these figures that the agreement between the results computed by FVM and FDM is satisfactory. The finite-difference method produces oscillations at the early stage of calculation. These oscillations are produced because FDM takes time to reach the steady-state condition.

(2) Slightly meandering channel

Plots of water level for times t = 25000, 27000, 29000, and 32000 sec are shown in Figure. 19 through Figure. 22, respectively. It is clear from these figures that the agreement between the results computed by FVM and FDM is satisfactory. Figure. 23 shows the time variation of flow depth at 1100m distance from upstream. Agreement between FVM and FDM results regarding the stage hydrograph is considered satisfactory.

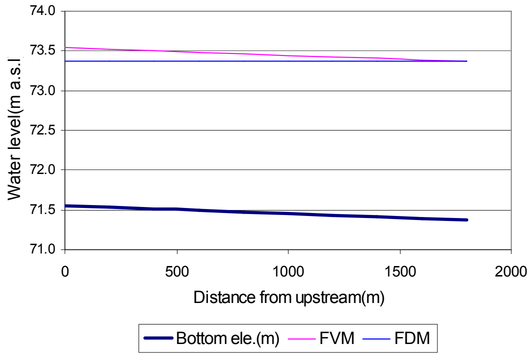

Figure 9. Comparison of water level after 25000sec

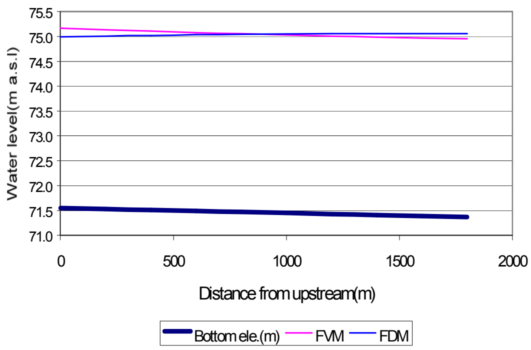

Figure 10. Comparison of water level after 27000sec

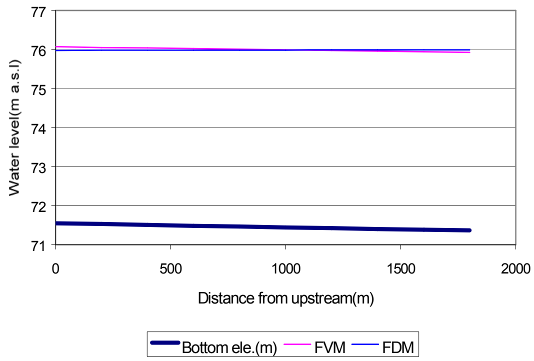

Figure 11. Comparison of water level after 29000sec

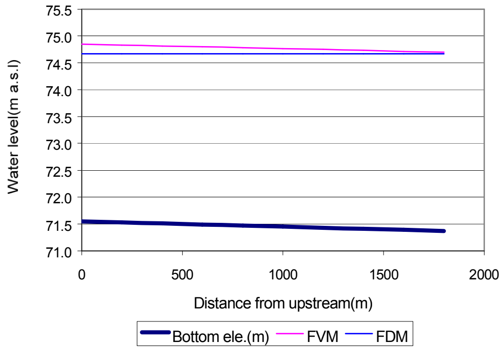

Figure 12. Comparison of water level after 32000sec

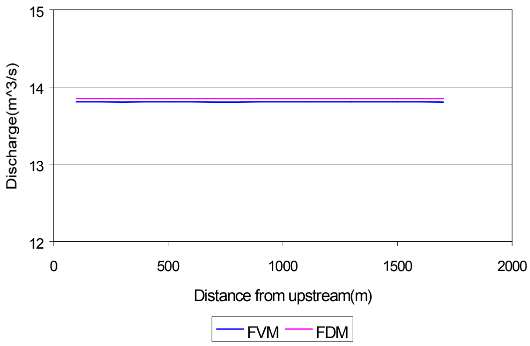

Figure 13. Comparison of discharge after 25000sec

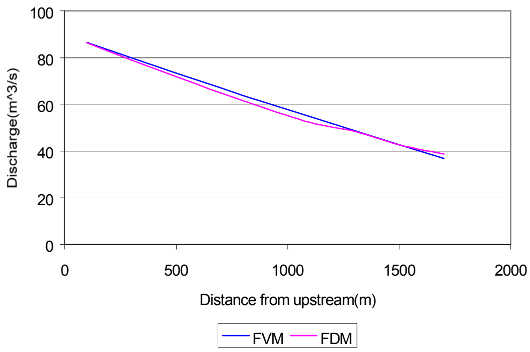

Figure 14. Comparison of discharge after 27000sec

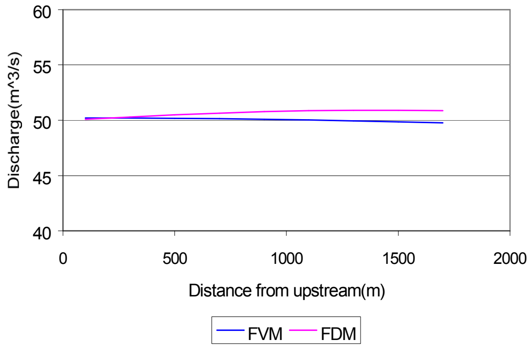

Figure 15. Comparison of discharge after 29000sec

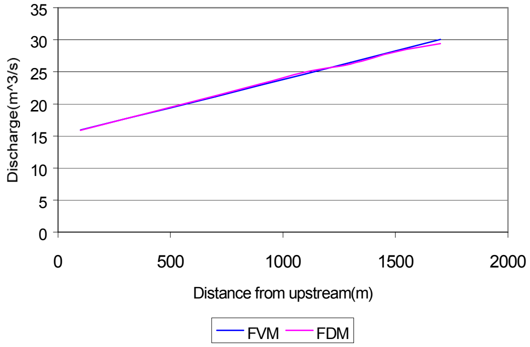

Figure 16. Comparison of discharge after 32000sec

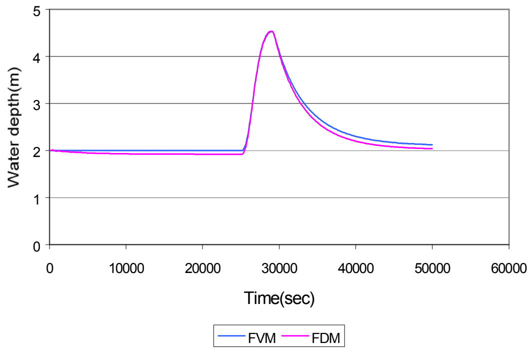

Figure 17. Comparison of variation of flow depth with time at 1000m

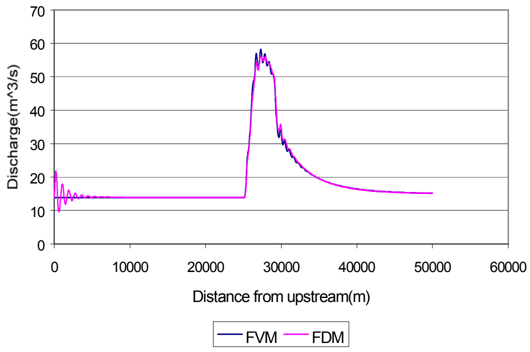

Figure 18. Comparison of variation of discharge with time

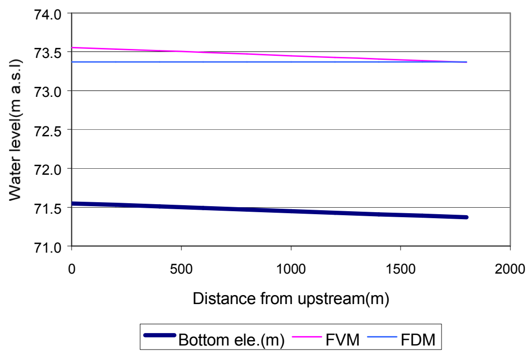

Figure 19. Comparison of water level after 25000 sec

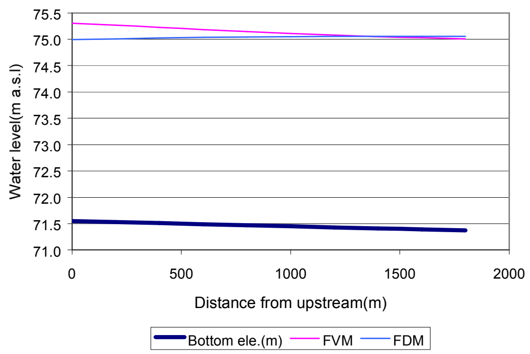

Figure 20. Comparison of water level after 27000 sec

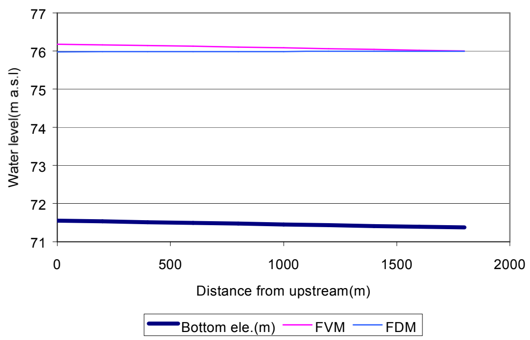

Figure 21. Comparison of water level after 29000sec

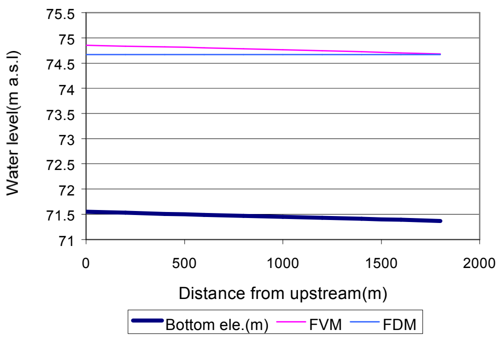

Figure 22. Comparison of water level after 32000sec

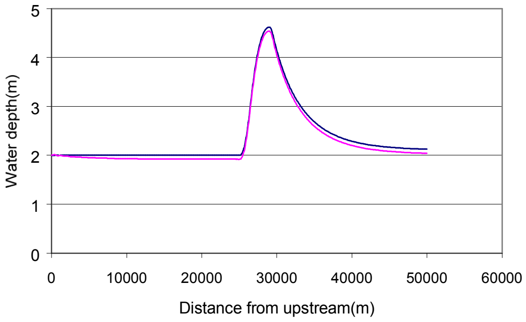

Figure 23. Comparison of variation of computed flow depth with time

(3) Meandering channel

Plots of water level for times t = 25000, 27000, 29000, and 32000 s are shown in Figure. 24 through Figure. 27, respectively. In the finite-volume method, water level is presented by using average value across the three cells. It is clear from these figures that the agreement between the results computed by FVM and FDM is satisfactory for practical purpose. Figure. 28 shows the time variation of water depth in the middle of channel. Agreement between FVM and FDM results regarding the stage hydrograph is considered satisfactory.

(4) Overall results

Figure. 29 shows the time variation of flow depth at 1000m from upstream computed by the two different methods. A systematic discrepancy in water surface profiles between the FVM and FDM is observed as expected. The result in

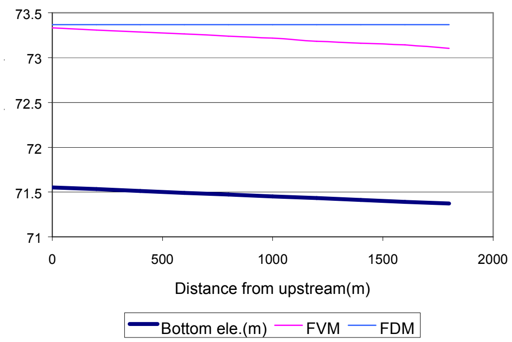

Figure 24. Comparison of water level after 25000sec

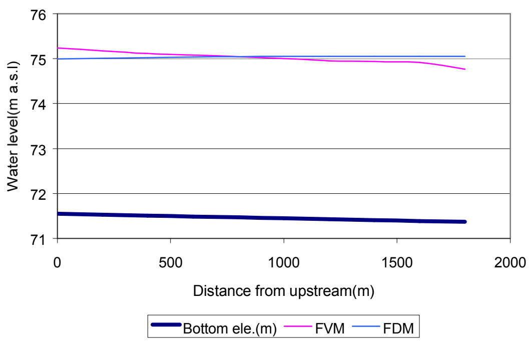

Figure 25. Comparison of water level after 27000sec

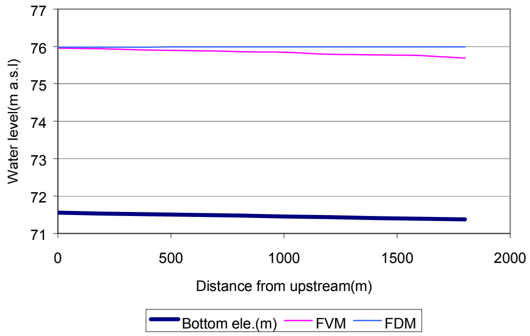

Figure 26. Comparison of water level after 29000sec

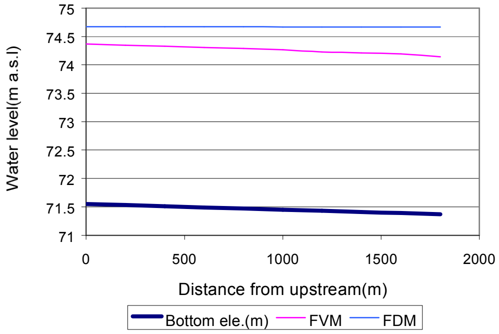

Figure 27. Comparison of water level after 32000sec

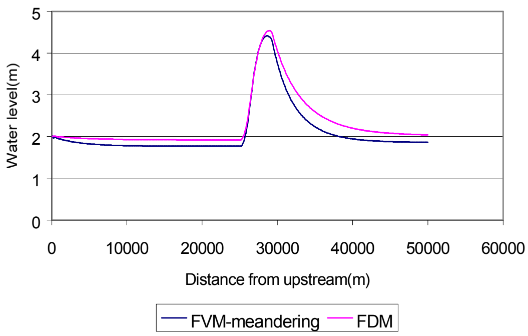

Figure 28. Comparison of variation of computed flow depth with time

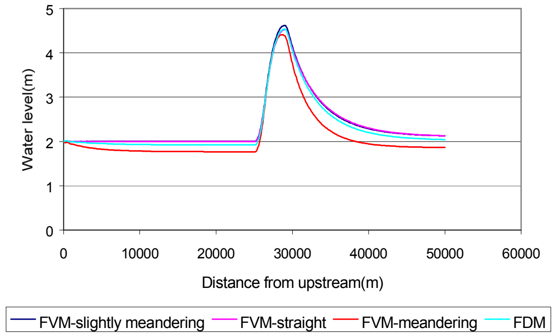

Figure 29. Comparison of variation of computed flow depth with time

meandering channel gives a lower water level than in straight channel. The reason for this that the influence of the secondary flow might be significant in meandering channel. This result implies that the FDM can be used limitedly in the case of meandering channel and the FVM takes the merits of FDM due to applicability of unstructured grid system for geometrical descrition.

5. Summary and conclusion

In the study, two different methods - finite difference and finite volume method - are introduced for the solution of unsteady free-surface flows. These methods are applied to analyze typical hydraulic flow problem-passage of a flood wave through a channel-including flow in a straight rectangular channel and flow in a meandering channel. A comparison of results for calculations using FVM and FDM has been carried out. Based on the results obtained the following conclusions may be drawn.

As a first comparison of numerical scheme, the straight channel was applied to compare the results. In straight channel application, one-dimensional depth-averaged flow calculation with finite-difference method yields results that have similar accuracy as calculations with finite-volume method. The similarity between the water surface profile/ discharge for FVM and FDM is almost perfect in straight channel. As a consequence, the discrepencies which can be observed between the two methods can be ignored. In other words, a one-dimensional model can be used successfully if the channel is straight and prismatic. One possible reason for the small discrepancy between FVM and FDM results could be the different algorithm in determining hydraulic variables. For example, FDM can take into account the friction of the side wall in a channel while FVM considers the friction of channel bed only.

In the application to meandering channel, it is difficult to compare the results directly since the flow is not 1D problem but 2D problem. If the two-dimensional finite-difference method would be used to analyze the flow, the comparison will be very interesting. However a systematic discrepancy in water surface profile between the FVM and FDM were observed. The reason for this deviation might be that the influence of the secondary flow on the main flow may be significant.

There were no systematic discrepancies in velocity distribution across the channel width at the certain location although the higher velocity in the outer bend is expected. The shapes of velocity profiles in x and y-direction along the channel are dependent on the shape of the channel. If the different shape of channel were used for calculation, different profile would be obtained. Difference in computational results between FVM and FDM is expected where curvature is large. It is not clear which of the velocity and water level is nearest to reality due to the lack of measurements. 1D analysis is sufficient if the channel is prismatic and remains straight. For curved channels, a 2D or 3D model must be used in order to accurately model the flow.

The major advantage of the finite-volume method is that it can be applied to any unstructured grid. However, grid generation and preparing the data for the finite-volume method were very time consuming and subject to error. In case of flow calculations where flow separation may have a significant influence on the expected flow field, the computational results have to be considered with much care. For the simulation of flow separation, advanced turbulence model may be used. When significant influence from secondary flow is expected, the results of depth-averaged flow models should be interpreted carefully. Accounting for the convective influence of the secondary flow on the main flow may be essential to a mathematical model of meandering channel.

Obviously the comparison in this thesis has not been through enough to fully investigate the different possible situations. Therefore, more work on the model is required as below.

To check the sensitivity of grid distribution, several different grid system may be tested, i.e., one is orthogonal and another is nonorthogonal. The numerical results obtained from a finite-volume method will increase in accuracy as the grid spacing is decreasing. In general, it is desirable to increase grid resolution in regions where the flow variables exhibit large gradients. A refined grid system can be employed to see the effect of refinement and numerical accuracy. Further on it is necessary to calibrate these models with measured results.

Both two methods may be extended to channels with arbitrary cross section, prismatic or nonprismatic, for solving any problem of unsteady flow in an open channel by incorporating the appropriate initial and boundary conditions. The comparison in different situation is recommended in the future.

References

Abbott, M. B. (1979) Computational Hydraulics; Elements of the Theory of Free Surface Flows. London: Pitman Publishing Limited.

Abbott, M. B., and Basco, D. R. (1990) Computational Fluid Dynamics. New York: John Wiley.

Alcrudo, F., and Garcia-Navarro, P. (1993) A high-resolution Godunov-type scheme in finite volumes for the 2D shallow-water equations.

Int. J. Numer. Methods in Fluids, 16, 489-505.

Chadwick, A., and Morfett, J. (1998) Hydraulics in Civil and Environmental Engineering (3 ed.). London: E & FN Spon.

Chang, H. H. (1992) Fluvial Processes in River Engineering. Malabar, Fla: Krieger publishing company.

Chaudhry, M. H. (1993) Open-Channel Flow. New Jersey: Prentice-Hall.

Chow, V. T. (1959) Open-Channel Hydraulics. McGraw-Hill. New York.

Cunge, J. A., Holly, F. M. and Verwey, A. (1980) Practical Aspects of Computational River Hydraulics. London: Pitman.

Erduran, K. S. (1999) Internal Report. Newcastle upon Tyne: University of Newcastle upon Tyne.

Fennema, R. J. and Chaudhry, M. H. (1990) Explicit Methods for 2-D Transient Free-Surface Flows. J. Hydr.Engrg, 116 (8), 1013-1034.

Jeong, Chan(1999) Comparison of Finite-Difference and Finite-Volume Method in Open-Channel Flows, MSc Thesis, University of Newcastle upon Tyne, Newcastle upon Tyne, UK.

Kutija, V. (1993) On the Numerical Modelling of Supercritical Flow. J. Hydraulic Research, 31 (6), 841-858.

Kutija, V., and Murray, M. G. (1999) New Improved Gem. Newcastle upon Tyne: University of Newcastle upon Tyne.

Mingham, C., G and Causon, D. M. (1998) High-resolution finite-volume method for shallow water flows. J. Hydr. Engrg., 124 (6), 605-614.

Montes, S. (1998) Hydraulics of Open Channel Flow.USA:ASCE

Petersen, M. S. (1986) River engineering. New Jersey: Prentice-Hall.

Rillaer, L. V. (1997) One-Dimensional Model of Unsteady Multy-Regime Flow in Combined Networks. MSc Thesis, University of Newcastle upon Tyne, Newcatle upon Tyne.

Spekreijse, S. P. (1988) Multigrid solution of steady euler equations. CWI Tract 46, Center for Mathematics and computer Science, Amsterdam, The Netherlands.

Silva, A. M. A. F.(1999) Friction Factor of Meandering Flows. J. Hydr.Engrg, 125 (7), 779-783.

Toro, E. F. (1997) Riemann solvers and numerical methods for fluid dynamics. Germany: Springer.

Wijbenga, J. H. A. (1985) Steady Depth-Averaged Floe Calculations on Curvilinear Grids. Paper presented at the Hydraulics of Floods and Flood Control, Cambridge, England.

Zhao, D. H., Shen, H. W., Tabios, G. Q., Lai, J. S., Tan, W. Y. (1994) Finite-Volume Two-Dimensional Unsteady-Flow Model for River Basins. J. Hydr. Engrg., 120 (7), 863-882.

Zhao, D. H., Shen, H. W., Tabios, G. Q., Lai, J. S., Tan, W. Y. (1996) Approximate Riemann Solvers in FVM for 2D Hydraulic Shock Wave Modelling. J. Hydr. Engrg., 122(12), 692-702