Journal of the Korean Institute of Industrial Engineers
Vol. 34, No. 1, pp. 1-12, March 2008.

Effectiveness of an Exponentially Smoothed Ordering Policy as Compared with Kanban System

Takayoshi Tamura[1†] · Tej S. Dhakar[2] · Katsuhisa Ohno[3]

[1]Nagoya Institute of Technology, Gokiso-cho, Syowa-ku, Nagoya 466-8555, Japan
[2]Southern New Hampshire University, 2500 North River Road, Manchester, NH 03106, USA
[3]Aichi Institute of Technology, 1247 Yachigusa, Yakusa-cho, Toyota 470-0392, Japan

The Kanban system in Just-In-Time (JIT) production is very effective in reducing the inventories when consumption rate of the final product is relatively stable. When large fluctuations exist in the consumption rate, a new production ordering policy in which the production order quantity is determined by smoothing the demands exponentially is more suitable. This new ordering policy has not been investigated sufficiently. In this research, a multi-stage production and inventory system with stock points for materials and finished items located at each stage is considered. Approximations of average inventories at each stage in the system are derived theoretically. Numerical simulations are carried out to assess the accuracy of approximations and to evaluate the effectiveness of the new ordering policy as compared with the Kanban system. As a result, it is shown that the new ordering policy can achieve significantly lower inventory costs than the original Kanban system. The new ordering policy thus emerges as a key concept for an effective supply chain management.

***Keywords*:** Multi-stage Production and Inventory System, Production Ordering Policy, Exponential Smoothing, Kanban, Jit, Supply Chain Management

1. Introduction

A typical tool for controlling production and procurement in the JIT system is the "Kanban" (Monden 1998). Many articles exist dealing with the performance of the Kanban system, e.g., Krajewski *et al*., 1987, Miltenburg 1997 and Spearman *et al*., 1990. In order to make the production and procurement more efficient using the Kanban system, the consumption rate of a part used for production is leveled at the final assembly line in the automobile industry (Monden 1998, Korkmazel *et al*., 2001 and Kubiak 1993). In practice, however, the number of kanbans removed according to part consumption in a time period fluctuates.

Kotani (1990) who worked as a middle manager at Toyota Motor Co. proposed a new production ordering policy in which the production order quantity in each period is determined by smoothing demands. However, he does not discuss how his policy is effective in reducing the total inventory. We call Kotani's concept "exponentially leveled ordering policy" (referred to as EXPLEVEL hereafter). So far, this ordering policy has been investigated in a very few papers (see Tamura *et al*., 2005, 2006). In Tamura *et al.* (2005) the effectiveness of the EXPLEVEL system is discussed for a single stage model as compared with Kanban and CONWIP by Spearman *et al*. (1990).

In this research, we derive approximations of average inventory at each stage in a multi-stage pro-

Authors would like to acknowledge Professor Ilkyeong Moon, Pusan National University, and Korean Institute of Industrial Engineers for giving us the opportunity to contribute this article to the Journal.
† Corresponding author : Takayoshi Tamura, Nagoya Institute of Technology, Gokiso-cho, Syowa-ku, Nagoya 466-8555, Japan
Tel : +81-52-735-5390, E-mail : tamura@nitech.ac.jp
Received December 2007; revision received january 2008; accepted january 2008.

duction and inventory system when an exponentially smoothed ordering policy is used to determine production order quantities. Using derived approximations together with simulation experiments, the effectiveness of the exponentially smoothed ordering policy is then compared with the original Kanban system.

The paper is organized as follows. Section 2 lists the assumptions and notation used in formulating the mathematical models. In section 3, we formulate the mathematical models for the original Kanban system and the EXPLEVEL system. In section 4, we derive approximations for variance and expectation of inventories at each stage when the production quantity is smoothed at the final stage. In section 5, we evaluate quantitatively the performance of the EXPLEVEL system as compared with the Kanban system using simulation experiments together with derived approximations for a three-stage production and inventory system. We also briefly discuss the application of the EXPLEVEL system to supply chain management. Section 6 concludes the paper.

2. Model Assumptions and Notation

The following assumptions are made for modeling the Kanban and EXPLEVEL systems:

(1) A single item is produced at each stage.
(2) Two stock points, one for materials and the other for the finished item, exist at each stage.
(3) The production capacity at each stage is unlimited.
(4) Product demand at the final stage follows an identical independent distribution during each period.
(5) Pro duct demand during each period is shipped to the customer at the end of the period, if inventory is available.
(6) Finished item at a stage is shipped to the successor stage at the end of each period.
(7) Production order at each stage is placed at the beginning of each period, while the material replenishment order at each stage is placed at the end of each period.
(8) Material needed for production during a period is required at the beginning of the period for each stage.
(9) Replenishment order quantity for material at the end of each period is equal to quantity of the material consumed for production during the period for each stage.
(10) Three types of lead-times are considered, which are production lead-time, lead-time to send replenishment order to the predecessor stage and shipment lead-time for conveying the finished item to the successor stage. They are known and constant.
(11) Shipment backlog of finished item from each stage to the successor stage is allowed.
(12) To produce one unit of item at each stage requires one unit of material.
(13) Kanban container size is set to one.
(14) No defectives are produced.

Notation is defined as follows :

M : The number of stages,
m : Stage index,
D_t : Demand for the final product in period t,
a_m : Production lead-time at stage m, where $a_m \geq 1$,
d_m : Lead-time to send replenishment order from stage m to stage $m-1$, where d_m is a given non-negative integer,
e_m : Shipment lead-time to send finished item from stage m to stage $m-1$, where e_m is a given non-negative integer,
$b_m = d_m + e_m - 1$: Replenishment lead-time for materials used at stage m, where b_1 is replenishment lead-time for the raw material required at stage 1,
$O_{m,t}$: Production order quantity at stage m in period t,
$X_{m,t}$: Production quantity realized under consideration of material constraint at stage m in period t,
$Y_{m,t}$: Material replenishment order quantity at stage m at the end of period t,
$S_{m,t}$: Shipment quantity of finished item sent from stage m to stage $m+1$ at the end of period t,
$P_{m,t}$: Inventory of finished item at stage m at the beginning of period t, where $P_{m,t}$ is final product inventory at the final stage,
$I_{m,t}$: Inventory of finished item at stage m at the end of period t,
$A_{m,t}$: Shipment backlog of finished item to be sent from stage m to stage $m+1$ in period t, which is caused by a shortage of

finished item at stage m, where $A_{m,t}$ is product shipment backlog for customers,

$B_{m,t}$: Production backlog at stage m in period t, which is caused by a shortage of material used for production at stage m,

$H_{m,t}$: Material inventory at stage m at the beginning of period t,

$J_{m,t}$: Material inventory at stage m at the end of period t,

$K_{m(\theta_m)}$: Safety factor for allowable stock-out probability θ_m with respect to finished item at stage m,

$L_m(\epsilon_m)$: Safety factor for allowable stock-out probability ϵ_m with respect to material used at stage m,

β_m : Exponential smoothing factor used for smoothing production quantity at stage m, $0 < \beta_m \leq 1$

$PB_{m,t}$: Production backlog caused by production smoothing at stage m,

$Var[X]$: Variance of random number, X

$E[X]$: Expectation of random number X, and

$[X]$: Smallest integer which is larger than or equal to x.

3. Model Description

3.1 Production and Inventory Model

Based on the assumptions stated in section 2, the production and inventory model can be formulated as follows:

(1) Production quantity realized under consideration of both available material and production order quantity:

$$X_{m,t} = \min\{O_{m,t} + B_{m,t-1},\ H_{m,t}\} \quad \text{for } m = 1, \cdots, M \tag{1}$$

(2) Production backlog at stage m:

$$B_{m,t} = B_{m,t} - 1 + O_{m,t} - X_{m,t} \quad \text{for } m = 1, \cdots, M \tag{2}$$

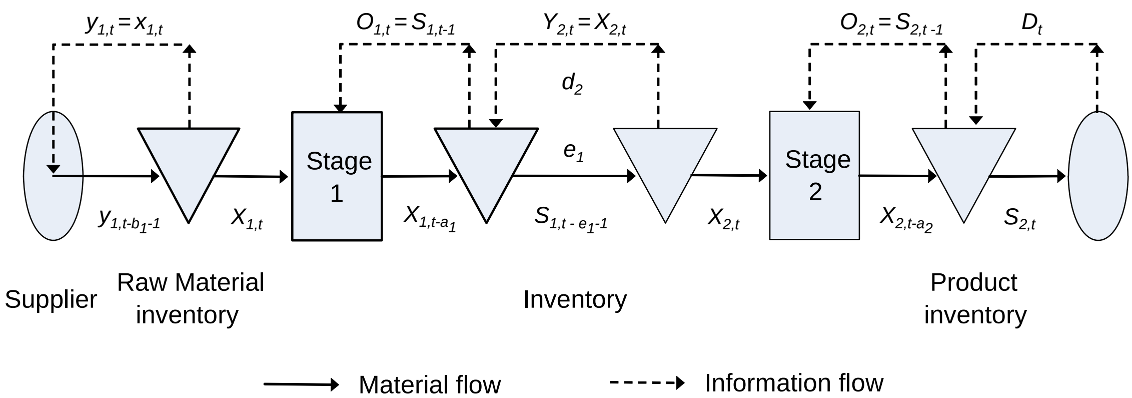

Figure 1. Two-stage production system managed by Kanban

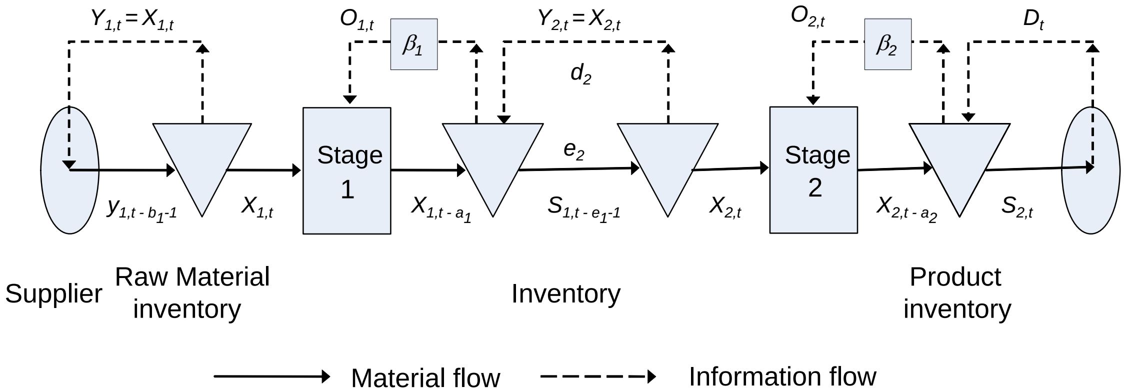

Figure 2. Two-stage production system managed by EXPLEVEL system

(3) Finished item inventory at stage m at the beginning of period t:

$$P_{m,t} = I_{m,t-1} + X_{m,t-am} \quad \text{for } m = 1, \cdots, M \tag{3}$$

(4) Material inventory at stage m at the beginning of period t:

$$H_{1,t} = J_{1,t-1} + Y_{1,t-1-b1}$$
$$H_{m,t} = J_{m,t-1} + S_{m-1,t-1-e_m-1} \quad \text{for } m = 2, \cdots, M \tag{4}$$

Note that the difference between subscripts $t-a_m$ in (3) and $t-1-e_m$ in (4) is because the production order is released at the beginning of each period while shipment is made at the end of each period.

(5) Material replenishment order quantity which is ordered at the end of each period (see Assumption 9) is given by the following equation:

$$Y_{m,t} = X_{m,t} \quad \text{for } m = 1, \cdots, M \tag{5}$$

(6) Shipment quantity of item sent from stage m:

$$S_{m,t} = \begin{cases} \min\{P_{m,t},\ D_t + A_{m,t-1}\} & \text{for } m = M \\ \min\{P_{m,t},\ Y_{m+1,t-d_{m+1}} + A_{m,t-1}\} & \text{for } m = 1, \cdots, M-1 \end{cases}$$

Note that equation (6) allows us to receive a replenishment order from the successor with zero lead-time when $d_{m+1} = 0$.

(7) Backlog of shipment quantity to be sent from stage m to the next stage:

$$A_{m,t} = \begin{cases} D_t + A_{m,t-1} - S_{m,t} & \text{for } m = M \\ Y_{m+1t-dm+1} + A_{m,t-1} - S_{m,t} & \\ & \text{for } m = 1, \cdots, M-1 \end{cases}$$

(8) Finished item and material inventories at stage m at the end of period t:

$$I_{m,t} = P_{m,t} - S_{m,t} \quad \text{for} \quad m = 1, \cdots, M \tag{8}$$

$$J_{m,t} = H_{mt} - X_{mt} \quad \text{for} \quad m = 1, \cdots, M \tag{9}$$

3.2 Production Order Quantity

3.2.1 Kanban system

<Figure 1> depicts the operation of the Kanban system. In the Kanban system, production order quantity at a period is simply set to the quantity of finished item consumed during the last period, i.e.

$$O_{mt} = S_{mt-1} \quad \text{for} \quad m = 1, \cdots, M \tag{10}$$

3.2.2 EXPLEVEL system

<Figure 2> depicts the operation of the EXPLEVEL system consisting of two stages. In this system, the production order quantity is determined by smoothing demands from the next stage exponentially.

For the material at each stage, it is assumed that the replenishment order quantity is set equal to the quantity consumed for production at the stage in each period. Based on the assumption, the EXPLEVEL system can be modeled as follows：

(1) Production order quantity at stage m:

$$O_{mt} = \lfloor \beta_m (S_{m,t-1} + PB_{m,t-1}) \rfloor \quad \text{for } m = 1, \cdots, M \tag{11}$$

As shown in section 4.1, the production order quantity given by (11) is identical to the quantity obtained by smoothing demands from the next stage exponentially if the production order quantity is not restricted to integer value. Note that the concept of production smoothing in (11) is different from the production smoothing in a mixed-model line achieved by the traditional goal chasing method.

(2) Backlogs caused by production smoothing：

$$PB_{m,t} = S_{m,t-1} + PB_{m,t-1} - O_{m,t} \quad \text{for } m = 1, \cdots, M \tag{12}$$

4. Approximations for EXPECTATION and Variance of Inventories

4.1 Variance of Inventory in EXPLEVEL System

In the EXPLEVEL system, the demands are exponentially smoothed to determine the production order quantity at each stage as given by (11). By ignoring any shortage of product and material as well as ignoring the integer condition for the order quantity, the production quantity (11) at the final stage is approximated by substituting (12) as follows:

$$\begin{aligned} O_{m,t} &\cong \beta_M (D_{t-1} + PB_{M,t-1}) \\ &= \beta_m D_{t-1} + \beta_M (D_{t-2} + PB_{M,t-2}) - \beta_M O_{M,t-1} \\ &\cong \beta_M D_{t-1} + (1-\beta_M) O_{m,t-1} \end{aligned} \tag{13}$$

The last result was given by Kotani (1990) and Tamura *et al*. (2005). This is the reason why the ordering policy is called "exponentially leveled ordering policy". Equation (13) can be expressed by

$$O_{M,t} \cong X_{M,t} \cong \sum_{j \geq 0} \beta_M (1-\beta M)^j D_{t-j-1} \tag{14}$$

In the following discussion, we assume that the production order smoothing is performed only at the final stage and the other stages utilize the Kanban rule to determine their order quantities.

Consider expectation and variance of inventories at the final stage. For the final stage, if we assume that the shortages for product and material are small enough to be ignored, for the final stage the shipment quantity given in (6) is approximated by demand, i.e.

$$S_{M,t} \cong D_t$$

By substituting this approximation and equations (3) and (14) into (8),

$$\begin{aligned} I_{M,t} &= I_{M,t-1} + X_{M,t-a_M} - S_{M,t} \\ &\cong I_{M,t-1} + X_{M,t-a_M} - D_t \\ &\cong I_{M,0} + \sum_{j \geq 0} X_{t-a_M-j} - \sum_{j \geq 0} D_{t-j} \\ &\cong I_{M,0} + \sum_{j \geq 0} \sum_{k \geq 0} \beta_M (1-\beta_M)^k D_{t-aM-j-k-1} \\ &\qquad - \sum_{j \geq 0} D_{t-j} \\ &\cong I_{M,0} + \sum_{j \geq 1} \sum_{k \geq 0}^{j-1} \beta_M (1-\beta_M)^k D_{t-a_M-j} \\ &\qquad - \sum_{j \geq 0} D_{t-j} \end{aligned}$$

is obtained. If we apply the following relation for any α, where $0 < \alpha < 1$, to the last equation.

$$\sum_{k=0}^{j-1} \alpha (1-\alpha)^k = 1 - (1-\alpha)^j \tag{15}$$

then we obtain the following approximation:

$$I_{M,t} \cong I_{M,0} - \sum_{j=0}^{a_M-1} D_{t-j} - \sum_{j \geq 0} (1-\beta_M)^j D_{t-a_M-j} \tag{16}$$

Since the demand is assumed to be i.i.d., the variance of the right hand side in (16) (denoted by $V_I(M:\beta_M)$) is given by

$$V_I(M:\beta_M) = a_M V_D + \frac{V_D}{1-(1-\beta_M)^2} \tag{17}$$

where V_D=Var[D_t]. The $V_I(M:\beta_M)$ can be used as an approximation for the variance of $I_{M,t}$ for the EXPLEVEL (denoted by Var[$I_{M,t} : \beta_M$]), i.e.

$$\text{Var}\left[I_{M,t} : \beta_M\right] \cong V_I(M : \beta_M)$$

For the material at the final stage, when the material replenishment is performed by the JIT rule, the following approximation holds.

$$\begin{aligned} J_{M,t} &\cong J_{M,t-1} + X_{M,t-b_M-1} - X_{M,t} \\ &\cong J_{M,0} - \sum_{j=0}^{b_M} X_{M,t-j} \end{aligned} \tag{18}$$

Substituting (14) into (18), we obtain

$$\begin{aligned} J_{M,t} &\cong J_{M,0} - \sum_{j=0}^{b_M} X_{M,t-j} \\ &\cong J_{M,0} - \sum_{J=0}^{b_M} \sum_{k \geq 0} \beta_M (1-\beta_M)^k D_{t-j-k-1} \end{aligned}$$

Since demand is assumed as being i.i.d., applying (15) the variance of the last equation (denoted by $V_J(M:\beta_M)$) is given by

$$\begin{aligned} V_J(M:\beta_M) &= \sum_{j=1}^{b_M} \left\{1-(1-\beta_M)^j\right\}^2 V_D \\ &+ \left\{1-(1-\beta_M)^{b_M+1}\right\}^2 \frac{1}{1-(1-\beta_M)^2} V_D \end{aligned} \tag{19}$$

And then the variance of $J_{M,t}$ denoted by Var [$J_{M,t} : \beta_M$] is approximated by

$$\text{Var}[J_{M,t} : \beta_M] \cong V_J(M:\beta_M)$$

Using the Kanban rule to determine production quantity and material replenishment quantity at each stage $m=1,\cdots,M-1$, variances of inventories at stage m can be derived in a manner similar to the derivation of (19) assuming that the shortages of finished item and material are small enough to be ignored. The results are given below：

$$V_I(m:\beta_M)=\sum_{j=1}^{a_m}\left\{1-(1-\beta_M)^j\right\}^2 V_D + \left\{1-(1-\beta_M)^{a_m+1}\right\}^2\frac{1}{1-(1-\beta_M)^2}V_D \quad \text{for } m=1,\cdots,M-1 \tag{20}$$

$$V_J(m:\beta_M)=\sum_{j=1}^{b_m}\left\{1-(1-\beta_M)^j\right\}^2 V_D + \left\{1-(1-\beta_M)^{b_m+1}\right\}^2\frac{1}{1-(1-\beta_M)^2}V_D \quad \text{for } m=1,\cdots,M-1 \tag{21}$$

where $V_I(m:\beta_M)$ and $V_J(m:\beta_M)$ are the approximated variances for product and material inventories, respectively, at stage m at the end of period when the smoothing factor at the final stage β_M is given.

Let E[$X:\beta_M$] denote the expectation of random variable X for each β_M. Using (17), (20) and (21), the following approximations for expectation and variance of product and material inventories in EXPLEVEL can be obtained, where notation $K_m(\theta_m)$ and $L_m(\epsilon_m)$ are safety factors for allowable stock-out rates θ_m of product and ϵ_m of material, respectively, at stage m:

$$\text{Var }[I_{m,t}:\beta_M]\cong V_1(m;\beta_M) \quad \text{for } m=1,\cdots,M \tag{22}$$

$$\overline{I_m(\beta_M)}=K_m(\theta_m)\sqrt{V_I(m;\beta_M)} \quad \text{for } m=1,\cdots,M \tag{23}$$

$$\text{E}[I_{m,t}:\beta_M]\cong\overline{I_M(\beta_M)} \quad \text{for } m=1,\cdots,M \tag{24}$$

$$\text{E}[P_{m,t}:\beta_M]\cong E(D_t)+\overline{I_m(\beta_M)}, \quad \text{for } m=1,\cdots,M \tag{25}$$

$$\text{Var }[J_{m,t}:\beta_M]\cong V_J(m:\beta_M) \quad \text{for } m=1,\cdots,M \tag{26}$$

$$\overline{J_m(\beta_M)}=L_m(\varepsilon_m)\sqrt{V_J(m:\beta_M)} \quad \text{for } m=1,\cdots,M \tag{27}$$

$$\text{E}[J_{m,t}:\beta_M]\cong\overline{J_m(\beta_M)} \quad \text{for } m=1,\cdots,M \tag{28}$$

$$\text{E}[H_{m,t}:\beta_M]\cong E(D_t)+\overline{J_m(\beta_M)} \quad \text{for } m=1,\cdots,M \tag{29}$$

In order to derive (17), (20) and (22), the mutual interaction between material shortage and production delay are not taken into consideration. For instance, (17) does not consider any shortage of material and hence an actual stock-out rate of final product will become larger than θ_M due to shortage of material. Conversely (21) will overestimate the safety stock level of the material. In order to obtain tighter approximations, both $K_m(\theta_m)$ and $L_m(\varepsilon_m)$ in (23) and (27) will be revised by simulation experiments (Tamura *et al*., 2005).

4.2 Expectation of Inventory in Kanban System

When the smoothing factor β_m is equal to one, the EXPLEVEL system becomes equivalent to the Kanban system. Substituting $\beta_m=1$, (17) and (20) are simplified to

$$V_I(m:1)\cong(a_m+1)V_D \quad \text{for } m=1,\cdots,M \tag{30}$$

and (21) is also simplified to

$$V_J(m:1)\cong(b_m+1)V_D \quad \text{for } m=1,\cdots,M \tag{31}$$

Substituting (30) and (31) into equations from (22) to (29), we obtain formulas for variance and expectation of product inventory as well as material inventory for the Kanban system.

4.3 Delay due to Production Smoothing

Ignoring any shortage of product and material as well as ignoring the integer condition for the order quantity, since $X_{M,t}$ and $S_{M,t}$ are approximately equal to $O_{M,t}$ and D_t, respectively, (11) and (12) at the final stage can be approximated by

$$X_{M,t}\cong\beta_M(D_{t-1}+PB_{M,t-1})$$

$$PB_{M,t} \cong D_{t-1} + PB_{M,t-1} - X_{M,t}$$

Substituting the first of these two equations given above into the second, we obtain

$$PB_{M,t} \cong (1-\beta_M)(D_{t-1} + PB_{M,t-1})$$

Then taking the expectation of both sides in the above equation leads to the following equation :

$$\mathrm{E}[PB_{M,t}] \cong \frac{1-\beta_M}{\beta_M}\mathrm{E}[D_t] \tag{32}$$

5. Numerical Experiments

This section discusses the accuracy of approximations derived in the previous section and the effectiveness of smoothing the production quantity at the final stage in the EXPLEVEL system as compared with the Kanban system.

5.1 Parameters and Conditions for the Simulation

The following parameters were used in the simulation experiments:

(1) Number of stages : The production system consists of three stages, i.e., $M = 3$.

(2) Demand : Binomial distribution is used, which is defined by two parameters (say, n and p) and the expectation and variance are given by np and $np(1\text{-}p)$ respectively. For the experiments, we assume that $p = 0.5$ and $n = 30$ or $n = 100$. Then $\mathrm{E}[D_t] = 15$, $V_D = 7.5$ for $n = 30$ and $\mathrm{E}[D_t] = 50$, $V_D = 25$ for $n = 100$.

(3) Lead-times : Production lead-time, replenishment order sending lead-time and shipment lead-time are set as follows:

$$a_1 = 3,\ a_2 = a_3 = 2,$$
$$b_1 = 5,$$
$$e_1 = d_2 = 2,\ b_2 = e_1 + d_2 = 4,$$
$$e_2 = 2,\ d_3 = 1,\ b_3 = e_2 = + d_3 = 3$$

(4) Stock-out rate : We assume $\theta_m = \varepsilon_m = 5\%$ for all $m = 1, 2, 3$ and then safety factor for each stage $m = 1, 2, 3$ is set to

$$K_m(\theta_m) = L_m(\varepsilon_m) = 1.645$$

which is obtained from the standard normal distribution.

(5) Exponential smoothing factors: Smoothing factors β_1 and β_2 for $m = 1$ and 2, respectively, are set to one and only β_3 at stage 3 is changed to consider the performance of the EXPLEVEL system. Note that at $\beta_3 = 1$ the EXPLEVEL system is equivalent to the Kanban system.

(6) Initial inventories for simulation runs : These initial values are set to the following values :

$$I_{m,0} = \overline{I_m(\beta_3)},\ \ J_{m,0} = \overline{J_m(\beta_3)}$$
$$\text{for } m = 1, 2, 3$$

(7) Initial values of production quantity, replenishment order quantity and shipment quantity : These initial values are set at the start of simulation run in order to take account of lead-times. These quantities are set as follows :

$$X_{m,t} = E[D_t]$$
$$\text{for } t = 1 - a_m, \cdots, 0 \text{ and } m = 1, 2, 3$$
$$Y_{m,t} = E[D_t]$$
$$\text{for } t = 1 - d_m, \cdots, 0 \text{ and } m = 2, 3$$
$$Y_{1,t} = E[D_t] \quad \text{for } t = -b_1, \cdots, 0$$
$$S_{m,t} = E[D_t]$$
$$\text{for } t = 1 - e_m, \cdots, 0 \text{ and } m = 1, 2$$

(8) Initial backlogs : These values are set as follows :

$$A_{m,0} = B_{m,0} = 0 \text{ for } m = 1, 2, 3$$
$$PB_{3,0} = \frac{1-\beta_3}{\beta_3}E[D_t]$$

(9) Iterations : Each simulation consists of 10,000 periods and the simulation is repeated ten times with different series of random numbers. Since the total iterations denoted by *Iter* becomes 100000, the confidence interval of stock-out rate θ_m with confidence level of 95% is given by

$$\theta_m \pm 2 \times \sqrt{\frac{1}{Iter}\theta_m(1-\theta_m)}$$

When the sample average of stock-out rate θ_m is 5%, the 95% confidence interval becomes 5% ±

0.14% and the relative interval is $\pm 0.0014/0.05 = \pm$ 2.8%.

5.2 Accuracy of Approximations

Stock-out rates for both systems at each stage obtained from ten simulation runs are shown in <Table 1>, where the smoothing factor β_3 for the EXPLEVEL system is set to 0.2. Note that the realized stock-out rates of product at the final stage are relatively close to 5% corresponding to the safety factor of 1.645, even though the demand takes on integral values. Stock-out rates at the other stock points however are fairly less than 5% for both ordering systems. The reason is that the distributions of requirements for both material and finished items at every stage except the final product, are all truncated by the production order quantity at the final stage since the production order quantity at the final stage is limited by the available material at that stage, and that $K_m(\theta_m)$ and $L_m(\varepsilon_m)$ in (23) and (27) for all m are set to a fixed value 1.645 for all m in the simulation. Due to truncation of the distribution, the stock-out rate becomes sensitive at each stage such that when we decrease the initial inventory by one unit the stock-out rate becomes fairly large. If we reduce the material initial inventory at $m = 1$ by one unit, then $J_{1,0}\overline{J_1(\beta_3)}-1$. If other values are not changed, then the stock-out rate changes from the values in <Table 1> to the values shown in <Table 2>, where β_3 equals 0.2 and n is set to 30. This discussion implies that the stock-out rate is sensitive to the initial stock and for the demand and production quantity taking on integral values, it will be almost impossible to realize the exact targeted stock-out rate, for an example 5%, at every stage by adjusting the initial inventory.

Table 2. Stock-out rates for the EXPLEVEL system when initial inventory is adjusted

Stage	$m = 1$	$m = 2$	$m = 3$
Finished item	0.039	0.011	0.056
Material	0.076	0.025	0.002

<Table 3> shows average end-of-period inventories for both of Kanban and EXPLEVEL systems. In the table "Simulation" gives results obtained by simulation using the same data as in <Table 1> and "Theoretical" are obtained by theoretical computation using (23), (24), (27) and (28). Also, no adjustment is made to the initial inventories. Although stock-out rates given in <Table 1> are different from the theoretical values for reasons given earlier, the average inventories in <Table 3> are relatively close to the theoretical values. As a conclusion, we can say that while stock-out rates are sensitive to the initial stock level, the estimation of the average inventory levels by (23), (24), (27) and (28) are fairly accurate.

Results for each of ten simulation runs are given in <Table 4>. From the table, we can estimate the confidence interval for the average total inventory as follows. Since the degree of freedom for ten simulation runs is nine, the t-value is equal to 2.262.for the confidence level of 95%. Since the standard deviations of data given in <Table 4> are 0.736 and 0.845 for the Kanban and EXPLEVEL systems respectively, the 95% confidence interval of the average total inventory at the end of period is computed as [54.04, 55.10] for the Kanban system and [35.98, 37.19] for the EXPLEVEL system.

5.3 Performance of EXPLEVEL System

Although the stock-out rates are a little different

Table 1. Stock-out rates for Kanban and EXPLEVEL systems

Demand	Ordering System	Smoothing Factor	Finished item/Product			Material		
			$m = 1$	$m = 2$	$m = 3$	$m = 1$	$m = 2$	$m = 3$
$p = 0.5$, $n = 30$	Kanban	$\beta_3 = 1.0$	0.001	0.001	0.043	0.026	0.031	0.032
	EXPLEVEL	$\beta_3 = 0.2$	0.002	0.004	0.055	0.014	0.016	0.001
$p = 0.5$, $n = 100$	Kanban	$\beta_3 = 1.0$	0.001	0.001	0.055	0.028	0.034	0.037
	EXPLEVEL	$\beta_3 = 0.2$	0.000	0.042	0.056	0.000	0.002	0.002

Table 3. Comparison of the EXP + LEVEL system with the Kanban system

Demand	Ordering System	Simulation/ Theoretical	Finished item/Product			Material			Total
			$m=1$	$m=2$	$m=3$	$m=1$	$m=2$	$m=3$	
$p=0.5$ $n=30$	Kanban	Simulation	8.91	7.91	7.91	10.94	9.958	8.96	54.57
		Theoretical	9.01	7.80	7.80	11.03	10.07	9.01	54.73
	EXPLEVEL	Simulation	4.95	3.95	9.84	6.94	5.95	4.96	36.59
		Theoretical	5.28	4.11	9.85	7.43	6.39	5.28	38.34
$p=0.5$ $n=100$	Kanban	Simulation	15.87	13.86	13.85	19.93	17.94	15.95	97.41
		Theoretical	16.45	14.25	14.25	20.15	18.39	16.449	99.92
	EXPLEVEL	Simulation	9.96	6.97	17.87	13.93	11.94	9.91	70.59
		Theoretical	9.65	7.50	17.98	13.57	11.67	9.65	70.00

Table 4. Total inventory for each of ten simulation runs when $n=30$

Run No.	1	2	3	4	5	6	7	8	9	10
Kanban	54.44	55.36	53.63	55.52	54.89	54.18	54.94	53.99	53.47	55.28
EXPLEVEL	36.42	37.50	35.48	37.65	36.96	36.16	37.02	35.91	35.34	37.43
Reduction	33.1%	32.3%	33.8%	32.2%	32.7%	33.3%	32.6%	33.5%	33.9%	32.3%

between the two systems due to their sensitivity to the initial stock, we will ignore that difference and compare the performance of the two systems based on the average total inventory. With $n=30$ for the binomial distribution for demand in <Table 3>, the average total inventory is reduced from 54.57 in the Kanban system to 36.59 in the EXPLEVEL with $\beta_3=0.2$. This means that by leveling the production order quantity under the EXPLEVEL system, the average total inventory is reduced by 33.0% with a 95% confidence interval of [32.50%, 33.43%] based on the last row in <Table 4>.

With $n=100$ for the binomial distribution, the average total inventory is reduced from 97.41 for the Kanban system to 70.59 for the EXPLEVEL system, a 27.5% reduction, in the simulation as shown in <Table 3>.

5.4 Optimal Value of β_3

We will now discuss the optimal value of β_3 which minimizes the expectation of total inventory over all stages of the system. Since it is difficult to realize a predetermined stock-out rate precisely in the simulation as mentioned earlier, our discussion will be based on the theoretical values computed by (23), (24), (27) and (28). This will also allows us to depict the smoothing curves in the following figures.

<Table 5> gives the expectation of inventory level theoretically computed at the end of period at each stock point for each value of β_3, when the average and variance of the demand are 15 and 7.5, respectively. <Figure 3> and <Figure 4> are depicted based on the values given in the table. <Figure 3> shows the changes in product inventory, material inventory and the total inventory at stage 3 as the smoothing factor β_3 is changed. <Figure 4> shows the changes in total inventory at each stage as well as the total system inventory as β_3 is changed. Note that the EXPLEVEL system is equivalent to the Kanban system when $\beta_3=1.0$. From these tables and figures we can draw the following conclusions：

a) Total inventory at the final stage obtained theoretically seems to be convex as shown in <Figure 3> and it has a minimum value at around $\beta_3=0.2$ in this example. The total inventories obtained by simulation are 16.36 (4.9%), 14.80 (5.5%) and 14.83 (5.2%), when β_3 equals 0.1, 0.2 and 0.3, respectively. Values in the parentheses are stock-out rates of the product. The corresponding theoretical values for the total inventories are 16.02, 15.13 and 15.20. When β_3 equals 0.2, the theoretical total inventory at the final stage is 16.81 for

Kanban system as compared to 15.13 for the EXPLEVEL system, which is a reduction of 10.0%.

b) From (23) and (27) we can see that optimal value of β_3 changes according to the production and material replenishment lead-times, but is not influenced by the variability of the demand distribution.

c) The average total inventory over the entire system is reduced when β_3 is reduced. The minimum value for the average total system inventory is achieved when $\beta_3 = 0.04$ as shown in the last column of <Table 5> and in <Figure 4>. From the values given in the last column of <Table 5>, we can see that the total system inventory is reduced from 54.73 at $\beta_3 = 1.0$ (Kanban system) to 31.05 at $\beta_3 = 0.04$, i.e. 43.3% reduction is obtained. For $n = 100$, although any data is not shown, the total system inventory is reduced from 99.92 at $\beta_3 = 1.0$ (Kanban system) to the minimum value of 56.59 at $\beta_3 = 0.04$ for the EXPLEVEL system, i.e. the same reduction rate as 43.3% is achieved.

Table 5. Average inventory by stage for different β_3 values

β_3	Finished item/Product			Material			Total
	$m=1$	$m=2$	$m=3$	$m=1$	$m=2$	$m=3$	
0.03	2.18	1.65	19.59	3.24	2.71	2.18	31.56
0.04	2.51	1.90	17.30	3.71	3.12	2.51	31.05
0.05	2.80	2.12	15.77	4.12	3.46	2.80	31.07
0.06	3.05	2.31	14.66	4.48	3.77	3.05	31.33
0.07	3.28	2.49	13.81	4.81	4.06	3.28	31.74
0.08	3.50	2.66	13.14	5.11	4.31	3.50	32.21
0.09	3.70	2.82	12.59	5.38	4.55	3.70	32.73
0.10	3.88	3.296	12.14	5.63	4.77	3.88	33.26
0.20	5.28	4.11	9.85	7.43	6.39	5.28	38.34
0.30	6.23	4.93	8.96	8.52	7.42	6.23	42.30
0.40	6.93	5.57	8.50	9.25	8.15	6.93	45.33
0.50	7.47	6.10	8.22	9.75	8.67	7.47	47.69
0.60	7.90	6.54	8.05	10.12	9.07	7.90	49.58
0.70	8.24	6.92	7.93	10.41	9.39	8.24	51.12
0.80	8.53	7.24	7.86	10.64	9.64	8.53	52.44
0.90	8.78	7.54	7.82	10.85	9.87	8.78	53.62
1.00	9.01	7.80	7.80	11.03	10.07	9.01	54.73

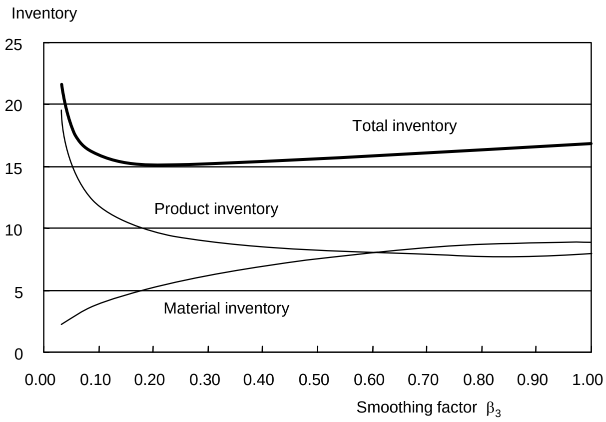

Figure 3. Inventory at stage 3 vs. smoothing factor β_3

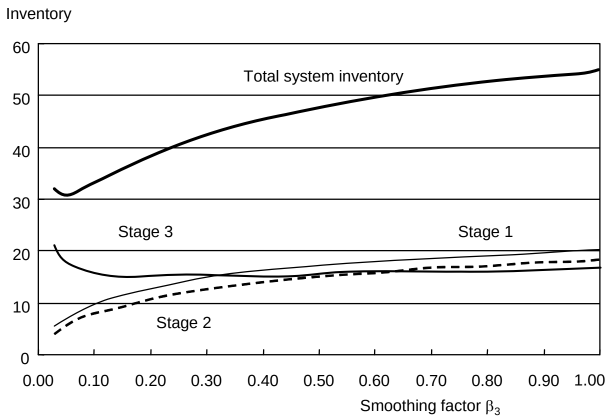

Figure 4. Total stage inventories and total system inventory vs. smoothing factor β_3

5.5 Total Inventory Holding Cost

Now consider the total inventory cost reduction by production smoothing. As in most literature, we will use the average inventory at the end of period to evaluate the inventory holding cost. We will thus compute the total inventory cost by multiplying the average inventory and its unit holding cost together at each stage. We will use four types of unit holding costs as in <Table 6>. For example, in cost type 4 the unit holding cost of raw material is 0.1 while the unit holding cost of product at the final stage is 1.0. It is assumed in the table that the same items located at two stock points between two successive stages m and $m+1$ have the same unit holding cost.

Using unit costs as shown in <Table 6>, the total inventory holding cost is computed given in <Table 7>. Since cost type 1 has been used in the earlier tables, the total inventory cost for cost type 1 is identical to the last column of <Table 5>. The smallest value of the total inventory cost for each cost type is shown with "*" in <Table 7>. The maximum cost reduction for each cost type for the EXPLEVEL system as compared to the Kanban system ($\beta_3 = 1$) is shown in the last row of the table. From <Table 7> we conclude as follows:

a) As the unit material holding cost decreases from type 1 to type 4, overall system inventory cost reduction achieved by the EXPLEVEL system in comparison to the Kanban system goes down from 43.3% to 26.0%.

b) Optimal value of β_3 is 0.04 for cost type 1 while 0.10 for cost type 4. In other words, smaller the material unit holding cost, larger the optimal value of β_3.

c) Optimal value of β_3 which minimizes the total inventory holding cost at the final stage is 0.2 for cost type 1 while it is 0.3 for cost type 4 at which 5.54% cost reduction is achieved at the final stage.

Table 6. Types of unit inventory holding cost values used in experiments

Cost type	Finished item			Material		
	$m = 1$	$m = 2$	$m = 3$	$m = 1$	$m = 2$	$m = 3$
Type 1	1	1	1	1	1	1
Type 2	0.8	0.9	1	0.7	0.8	0.9
Type 3	0.6	0.8	1	0.4	0.6	0.8
Type 4	0.4	0.7	1	0.1	0.4	0.7

Table 7. Total inventory cost for different types of unit holding costs

β_3	Cost type 1	Cost type 2	Cost type 3	Cost type 4
0.03	31.56	29.22	26.89	24.56
0.04	31.05*	28.37	25.69	23.01
0.05	31.07	28.09*	25.11	22.13
0.06	31.33	28.09	24.84	21.59
0.07	31.74	28.25	24.76*	21.27
0.08	32.21	28.50	24.79	21.09
0.09	32.73	28.81	24.90	20.99
0.1	33.26	29.16	25.05	20.95*
0.2	38.34	32.84	27.33	21.83
0.3	42.30	35.90	29.49	23.09
0.4	45.33	38.29	31.25	24.21
0.5	47.69	40.18	32.67	25.16
0.6	49.58	41.70	33.83	25.95
0.7	51.12	42.96	34.80	26.63
0.8	52.44	44.04	35.64	27.23
0.9	53.62	45.01	36.39	27.78
1.0	54.73	45.92	37.11	28.31
Reduction	43.27%	38.84%	33.28%	25.98%

5.6 Application to a Supply Chain Management

The Kanban system is an effective tool to control the production and inventory in a supply chain. The smoothing of production at the final stage as in the EXPLEVEL system can improve the performance of the Kanban system significantly as we have shown in this paper. Consider a supply chain consisting of three different companies, which correspond to the three stages in the paper. When a parent company corresponding to the final stage (final assembly station) produces a finished product using production smoothing at $\beta_3 = 0.2$, the inventory level in the parent company is minimized. At the same time, the other companies composing the supply chain, which would correspond to stages 1 and 2 in the paper, will also reduce their inventory levels and inventory cost owing largely to the production smoothing at the parent company.

Thus, a win-win relation between companies in a supply chain will be attainable by production smoothing at the final assembly company. Although there exists much research concerning quantitative approaches to supply chain management, e.g. Klose *et al*. (2002) Graves and Willems (2003) and Dudek (2004), insofar as we know, there may be no way to reduce the total inventory in the supply chain as much as can be achieved by smoothing production at the final stage as shown by the EXPLEVEL system.

6. Conclusion

In this paper, a new ordering policy EXPLEVEL, in which the production order quantity is leveled using an exponential smoothing factor, was discussed for a multi-stage production system. In this research, we smoothed the production order quantity only at the final stage while for the other stages, the production order quantities and material replenishment quantities were set according to the Kanban rule.

The contributions of the paper can be summarized as follows : (1) Approximations of average inventories both for the finished item and the materials were derived theoretically. (2) By carrying out simulations, the accuracy of approximations was examined at each stage. Approximations were relatively accurate although stock-out rates realized in the simulation are smaller than the pre-set value especially for the materials. (3) The EXPLEVEL system resulted in much lower total inventory cost in a three-stage production and inventory system as compared to the Kanban system. For the production and inventory systems with more than three stages, smoothing production should be even more effective for inventory reduction. (4) For a wide rage of unit inventory holding costs a smoothing factor smaller than 0.1 leads to the best results in reducing the total inventory cost for the entire production system. Reducing the smoothing factor impacts the total inventory cost reduction at stages 1 and 2 much more than at the final stage. (5) If a parent company using JIT uses production smoothing at the final assembly stage and the suppliers manage their production and inventory according to the Kanban rule, then the parent company as well as the suppliers will incur lower inventory holding costs. Thus production smoothing by the parent company, as demonstrated by this paper through the EXPLEVEL system, would result in lower total inventory cost for the entire supply chain, a win-win situation for all the players in the supply chain.

References

Dudek, G. (2004), *Collaborative planning in supply chain: A negotiation-based approach*, Springer.

Graves, S. C. and Willems, S. P. (2003), Supply chain Design : Safety stock placement and supply chain configuration, in de Kok, A. G. and Graves, S. C (Eds.), *Handbook in Operations Research and Management Science Vol.11*, Elsevier, 95-132.

Klose, A, Speranza, M. G. and Van Wassenhove, L. N. Eds. (2002), *Quantitative approaches to Distribution logistics and supply chain management*, Springer.

Korkmazel, T. and Meral, S. (2001), Bicriteria sequencing methods for the mixed-model assembly line in just-in-time production systems, *European J. of Operational Research* **131**(1), 188-207.

Kotani, S. (1990), Analysis of Kanban System by the Exponential Smoothing *Method, Proceedings of 1st APORS within IFORS*, 305-318.

Krajewski, L. J., King, B. E., Ritzman, L. P. and Wong, D. S. (1987). Kanban, MRP, and shaping the manufacturing environment, *Management Science* **33**(1), 39-57.

Kubiak, W. (1993), Minimizing variation of production rates in just-in-time systems : a survey, *European J. of Operational Research* **66**, 259-271.

Miltenburg, J. (1997), Comparing JIT, MRP and TOC, and embedding TOC into MRP, *International Journal of Production Research* **35**(4), 1147-1169.

Monden, Y. (1998), *Toyota Production System : An Integrated Approach to Just-In-Time*, 3rd Ed., Engineering & Management Press.

Spearman, M. L., Woodruff, D. L. and Hopp, W. J. (1990), CONWIP : A pull alternative of kanban, *International Journal of Production Research* **28**(5), 879-894.

Tamura, T., Kojima, M., Fujita, S. and Kumagai, C. (2005), An effectiveness of an exponentially smoothed ordering policy, *Proceedings of Int. Conference on Operations and Supply Chain Management*, Bali-Indonesia, B.1-B.8.

Tamura, T., Dhakar, T. S., Ohno, K. and Kojima. M. (2006), Effectiveness of an Exponentially Smoothed Ordering Policy in a Multi-Stage Production System, *Proceedings of the 7th Asia Pacific IE and Management Systems Conference* 2006, Bangkok-Thailand, 1959-1970.