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An improved Ordering and Recovery Policy for Reusable Items

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재활용품 제고시스템에 대한 주문 및 재생정책의 개선방안

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This paper studies a joint EOQ and EPQ model in which a stationary demand can be satisfied by recycled products as well as newly purchased products. The model assumes that a fixed quantity of the used products are collected from customers and later recovered for reuse. The recovered products are regarded as perfectly new ones. We also assume that the number of orders for newly purchasing items and the number of recovery setups in a cycle can be mutually independent integers. Under these assumptions, we develop an optimization model obtaining the economic order quantity for newly procured products, the optimal lot size for the recovery process, and the sequence of the orders and the setups, simultaneously. And then a simple solution procedure to find a local optimal control parameter set is proposed. To validate the model and the solution procedure, finally, some computational experiments are presented.

Keywords: Inventory, Recycling, Recovery, Reuse, EOQ, EPQ

1. Introduction

Product recovery is an area that is receiving increasing attention. Metal scrap brokers, waste paper recycling and deposit systems for soft drink bottles are the examples that have been around for a long time. In these cases recovery of the used products is economically more attractive than disposal. In the recent past, furthermore, the growth of environmental concerns has given increasing attention to reuse (Fleischmann *et al*., 1997). The environmental costs during the whole lifecycle of industrial products already play an increasingly important role in the calculation of total production costs (Spengler *et al*., 1997).

There are different types of recovery : repair, refurbishing, remanufacturing, cannibalization and recycling (Thierry *et al*., 1995). In this paper we focus on the third type, in which an item is brought up to an "as-new" quality. <Figure 1> shows the general framework of the situation studied in this paper. The supplier meets a stationary demand for an item, and he/she receives used products returned from the customers. For fulfilling the demand, he/she has two alternatives : Either he/she orders new products externally or he/she recovers old products and brings them back to "as-new" condition. In this system, the supply of new products is from the outside of the system and the replenishment rate is infinite (i.e. instant replenishment), while the recovery process is performed inside the system and the replenishment rate is finite (i.e. gradual replenishment). Washed-and-sterilized and newly purchased

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bottles in a soft drink company are good examples. The objective of this inventory management system is to control external component orders as well as the internal component recovery process to guarantee a required service level and to minimize fixed and variable costs.

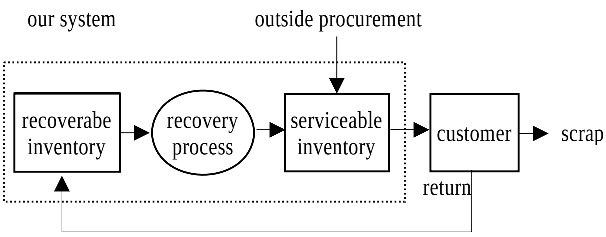

Figure 1. Framework of the inventory system

A number of authors have proposed inventory models and policies for these systems. Review papers are provided by Guide *et al*. (1997), Fleischmann *et al*. (1997), van der Lann *et al*. (1999b) and Guide (2000). According to these review papers, the models can be classified into two groups; deterministic and stochastic models.

The first deterministic model is studied by Schrady (1967). He assumes constant demand and return rates and fixed lead times for external orders and recovery. The costs considered are fixed setup costs for orders and recovery process and linear holding costs for serviceable and recoverable inventories. For this model he proposes a control policy with fixed lot sizes serving demand as far as possible from recovered products. Expressions for the optimal lot sizes for order and recovery are derived similar to the classical EOQ formula.

Nahmias and Rivera (1979) extend Schrady's model to the case that the recovery rate is finite. Another extension to the Schrady's model is proposed by Mabini *et al*. (1992). They consider stockout service level constraints and a multi-item system where items share the same repair facility. For these extended models numerical solution methods are proposed.

Richter (1996a, b) considers a slightly different model. He assumes that there is no continuous flow of used items to the recovery shop. Used items are collected in a location and brought back to the recovery shop at the end of each collection interval. This collection interval coincides with a recovery cycle in the recovery shop. The recovery cycle consists of a number of recovery batches followed by a number of orders. This policy leads to extra holding costs, since recovery of returned items has to be postponed until the end of the interval.

Teunter (2001) also generalizes the Schrady's (1967) result in two ways. First, he considers more general policies. These policies alternate a number of manufacturing batches with a number of recovery batches. Second, he distinguishes between the holding cost rates for manufactured and recovered items, whereas Schrady used the same rate for both.

Koh *et al*. (2002) generalize the work of Nahmias and Rivera (1979). They allow the recovery rate to be both smaller and larger than the demand rate, and then they consider the system that has one setup for recovery (or one order for new products) and many orders for new products (or many setups for recovery). For all four combinations, they derive a closed-form expression for the average total cost which can be used to determine the optimal lot sizes numerically. But their policy has a drawback in recovery schedule. Teunter (2004) points out the problem and proposes a partly heuristic approach to solve it. Later, Konstantaras and Papachristos (2007) propose an exact solution method to solve the same problem.

Stochastic models that treat demands and returns as stochastic processes have been also widely discussed too. Cho and Parlar (1991) survey the literature related to optimal maintenance and replacement models for multi-unit systems. Cohen et al. (1980) study a system where recoverable and serviceable inventory coincide because returned products can be reused directly. They assume that a fixed share of the products issued in a given period is returned after a fixed lead time. Later, their system are modified to the system with random returns by Kelly and Silver (1989).

In contrast to Cohen's model, Heyman (1977) deals with a system where recoverable and serviceable inventories are distinct. Simpson (1978) proves that the optimum solution structure for an n-period repairable inventory problem is completely defined by three period dependent values and proposed a solution methodology. Muckstadt and Isaac (1981) consider a continuous review model with explicit modelingof a remanufacturing facility with non-zero lead times. In their research the demands and returns occur as a Poisson process and there are no assumptions about repair time distribution or the number of repair servers. Kim and Shin (1993) develop an algorithm to find the optimal spare inventory level for multiechelon repairable inventory system, and their result are extended by Kim et al. (1998) to the system in which the inventories are at the central depot as well as the several bases.

Van der Laan *et al*. (1996a) present more general model that has four control parameters and they (1996b) develop two approximations for the average costs of an (s, Q) remanufacturable inventory model. Recently, van der Laan *et al*. (1997, 1999a, 1999b) study various manufacturing/remanufacturing systems with PUSH and PULL disposal strategies. Wong *et al*. (2005) develop a multi-dimensional Markovian model to estimate several performance measures in a single-item, multi-company, repairable inventory system where complete pooling of stock is permitted among the companies.

This paper deals with the deterministic model, which can also be classified into several groups by two criteria. The first one is the replenishment rate of new and recovered products. As stated earlier, this study deals with infinite replenishment for new products and finite recovery rate. Nahmias and Rivera (1979) deals with the same situation. On the other hand, the studies assuming both the infinite replenishment rates include Schrady (1967), Mabini *et al*. (1992), Richter (1996a, b), and Teunter (2001). And the studies with both the finite replenishment rates are Teunter (2004) and Konstantaras and Papachristos (2007).

The second criterion is the order and recovery policies used. Using the notation of Teunter (2004), the policies can be represented by (P, R), in which P means the number of orders for newly purchasing (or producing) items and R means the number of recovery setups in a cycle. Most of existing studies have used $(1, R)$ and/or $(P, 1)$ policies except Richter (1996a, b) who considered the (P, R) policy that alternates P orders for new items and R recovery setups. Since Richter (1996a, b) assumed that all of the recoverable items collected in a cycle are brought to the recovery shop at the end of the cycle, the consecutive recovery setups can be reasonable. But, in general case, the sequence of P orders and R setups should be arbitrary. This study deals with this general case. Under this situation, we obtain i) the economic order quantity for newly procured products, ii) the optimal lot size for the recovery process, iii) the sequence of the orders and the setups, simultaneously.

The paper is organized as follows. In the following section an optimal cost model is derived and a search algorithm to find an optimal ordering and recovery policy is proposed. Section 3 reports the results of computational experiments to validate the models and a solution algorithm proposed in this paper. Finally, we conclude the paper with a summary and some directions for future research in section 4.

2. Optimal Ordering and Recovery Problem

In this section, we deal with the problem of finding an optimal order policy for newly purchased items and an optimal recovery schedule for recoverable items, simultaneously. Before generation of the model, we state the assumptions and notations.

2.1 Assumptions

(1) Demand for serviceable items in a unit time is a known constant (d).
(2) Quantity of used items collected in a unit time from customers is a known constant (r).
(3) The repair quantity in a unit time is a known constant (p).
(4) All of the cost parameters are known constants.
(5) Lead time for purchasing new items and setup time for recovery process are known constants. And we can ignore them.
(6) Shortages for serviceable items are not allowed.
(7) It is more economical to repair items than to purchase new ones.
(8) Demand rate is greater than collection rate ($d > r$).
(9) The repair rate is greater than demand rate ($p > d$).
(10) Lot sizes of the recovery process are same through time (Q_1).
(11) Order sizes of newly procured items are same through time (Q_2).

2.2 Notations

Known parameters :

r : collection rate, [units]/[time]

p : repair rate, [units]/[time]

d : demand rate of the serviceable items, [units]/[time]

C_S : setup cost for recovery process, [$]/[setup]

C_O : ordering cost for new items, [$]/[order]

C_{H1} : inventory holding cost for the recoverable items, [$]/[unit]/[time]

C_{H2} : inventory holding cost for the serviceable items, [$]/[unit]/[time]

Decision variables :

T : cycle time of the model, i.e. time interval between time points when the inventory level of recoverable items is zero (See <Figure 2>).

m : number of orders for new items during a cycle, $m \geq 1$

n : number of setups in the recovery shop during a cycle, $n \geq 1$

Dependent variables:

Q_1 : lot size for the recovery process

Q_2 : order quantity for newly procured items

I_0 : Inventory level of the serviceable item when a recovery processing batch is finished

R_i : inventory level of recoverable items when i^{th} recovery process begins, $i = 1, 2, \cdots, n$

T_i : time point when i^{th} recovery process begins in a cycle, $i = 1, 2, \cdots, n$

t_1 : time to use I_0 units of serviceable items, $T_1 = I_0 / d$

t_2 : time to use one lot of newly purchased items, $t_2 = Q_2 / d$

t_3 : time length of a recovery processing run, $t_3 = I_0 / (p - d) = Q_1 / p$

2.3 Model

The problem in this paper is an extension to Koh *et al*. (2002). They used the (1, *R*) and (*P*, 1) policies, but in this paper, there can be one or more orders for newly purchasing items (i.e. $P \geq 1$, or $m \geq 1$ by the notation in this paper) as well as one or more setups for recovery process in a cycle (i.e. $R \geq 1$, or $n \geq 1$ by the notation in this paper). <Figure 2> shows an example of the system in which two orders and three setups occur in a cycle.

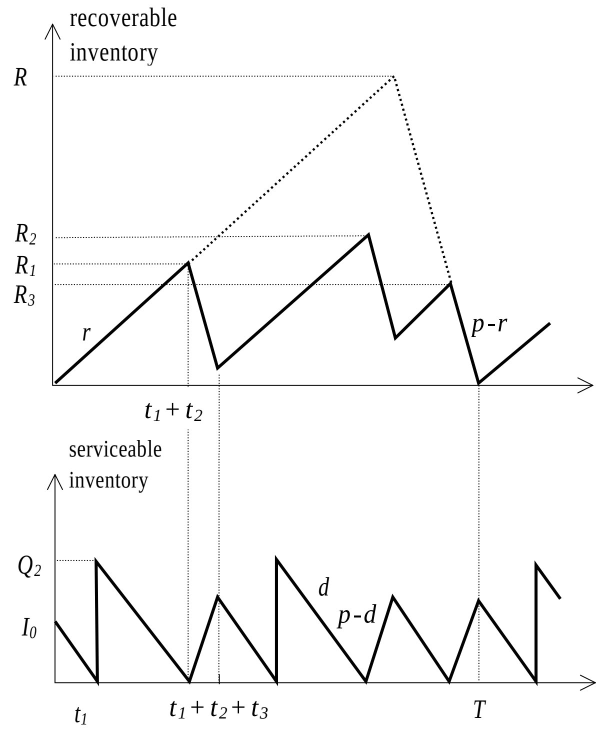

Figure 2. An inventory flow of the system when $m = 2$ and $n = 3$

In this figure, one can see the total demand of serviceable items and the total quantity of recovered items in a cycle are dT and rT, respectively. Since we assumed shortages for serviceable items are not allowed, the procurement of new items is needed when $rT < dT$. From the assumption (8), we can make the following observation.

Observation 1 (Ordering rule for newly purchasing items). There should be one or more orders in a cycle (i.e. $m \geq 1$). And the order size is as follows :

$$Q_2 = \frac{T(d-r)}{m} \tag{1}$$

From the lower graph in <Figure 2>, furthermore, one can see the inventory holding cost of serviceable items is the same regardless of the sequence of the setups and orders. But the upper graph in the figure shows that the inventory holding cost of recoverable items depends on the sequence. In this figure, for example, the inventory holding cost of recoverable items will be increased if the second order for newly purchasing items is prior to the first setup for recovery process. This leads us to the following observation.

Observation 2 (Recovery schedule). If the inventory level of recoverable items, when the inventory level of serviceable items is zero, is less than R_n, an order for newly purchasing items should be released. On the other hand, if the inventory level is greater than or equal to R_n, it is more economical to start a recovery process.

Since we do not allow shortages for serviceable items, the annual revenue is a constant, and thus it is enough to consider the system costs only. To develop the cost model, we have to find the relationships between system variables in <Figure 2>. First of all, from the assumption (10),

$$Q_1 = \frac{rT}{n} \tag{2}$$

From the definition of t_3, the recovery quantity during t_3 period is Q_1, i.e. $Q_1 = pt_3$ and therefore,

$$t_3 = \frac{Q_1}{p} = \frac{rT}{np} \tag{3}$$

Also, the inventory level of recoverable items decreases from R_n (for instance, R_3 in <Figure 2>) to zero during t_3 period, i.e. $R_n = (p-r)t_3$, and so,

$$R_n = \frac{rT(p-r)}{np} \tag{4}$$

Next, the inventory level of serviceable items increases from zero to I_0 during t_3 period, i.e. $I_0 = (p-d)t_3$, and so,

$$I_0 = \frac{rT(p-d)}{np} \tag{5}$$

Similarly, from the definition of t_1 and t_2,

$$t_1 = \frac{I_0}{d} = \frac{rT(p-d)}{ndp} \tag{6}$$

and

$$t_2 = \frac{Q_2}{d} = \frac{T(d-r)}{md} \tag{7}$$

The system costs can be classified into four categories and each cost term during a cycle can be obtained as follows :

(1) Setup cost of recovery process

$$nC_s \tag{8}$$

(2) Ordering cost of new items

$$mC_0 \tag{9}$$

(3) Inventory holding cost for serviceable items

$$\left[m\frac{Q_2 t_2}{2} + n\frac{I_0(t_1+t_3)}{2}\right]C_{H2} = \left[\frac{(T^2(d-r)^2}{2md} + n\frac{r^2\,T^2(p-d)}{2ndp}\right]C_{H2} \tag{10}$$

(4) Inventory holding cost for recoverable items (detailed procedure to get the equation is underlying)

$$\frac{rT^2(p-r)}{2p}C_{H1}, \quad \text{if } n=1,$$
$$\left[\frac{(p-r)T}{2} - \frac{1}{n}\sum_{i=1}^{n-1}\left|(p-r)(T_i - T) + R_i\right|\right]\frac{rT}{p}C_{H1}, \text{ if } n \geq 2 \tag{11}$$

To find the inventory holding cost for recoverable items, one should know the area under the thick line in the upper graph of <Figure 2>, which can be obtained by subtracting the area of $n-1$ parallelograms from the area of big triangle whose area is $\frac{RT}{2}$. Using equations of two upper lines in this big triangle, i.e. $y = rx$ and$y = -(p-r)x + (p-r)T$, one can easily find the value of *R* as follows:

$$R = \frac{r(p-r)T}{p} \tag{12}$$

Also, the $n-1$ parallelograms are corresponded to i^{th} setup (i =1, 2, $\cdots$, $n-1$) in the recovery shop as in <Figure 3>.

Once the values of R_i and T_i are given, the area of dashed parallelogram in <Figure 3> can be obtained by multiplying the length of line AB by the distance from point *A* to line *CD*. First, because

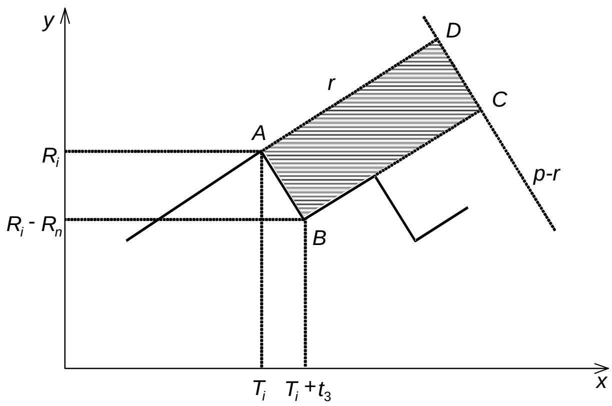

Figure 3. Area of the i^{th} parallelogram

the coordinates of points *A* and *B* are (T_i, R_i) and (T_i+t_3, R_i-R_n), respectively, the length of the line AB is

$$len(AB) = \sqrt{t_3^2 + R_n^2} = \frac{rT}{np}\sqrt{1+(p-r)^2} \tag{13}$$

On the other hand, it is well known that the distance from a point (x_1, y_1) to a line of equation $A_x + B_y + C = 0$ is $\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$. Therefore, since the coordinate of the point A and the equation of the line *CD* are (T_i, R_i) and $(p-r)x+y-(p-r)T=0$, respectively, the distance from point A to line *CD* is

$$dist(A,\ CD) = \frac{|(p-r)(T_i-T)+R_i|}{\sqrt{1+(p-r)^2}} \tag{14}$$

Therefore, the area of the i^{th} parallelogram is the product of equations (13) and (14) as follows :

$$area(ABCD) = \frac{rT}{np}|(p-r)(T_i-T)+R_i| \tag{15}$$

Therefore, the inventory holding cost for recoverable items in a cycle can be expressed as equation (11).

The values of R_i and T_i, $i = 1,\ 2,\ \cdots,\ n-1$, vary by system parameter values and cannot be expressed in a closed form. According to Observation 2, however, it is enough to compare the inventory level of recoverable items with R_n when the inventory level of serviceable items is zero. At the first time point of zero inventory of serviceable items, t_1, however, the inventory level of recoverable items is $rt_1 = \frac{r^2T(p-d)}{ndp}$, which is less than $R_n = \frac{rT(p-r)}{np}$. Therefore, the first time point we have to check the inventory level of recoverable items is t_1+t_2. We propose an algorithm to find the recovery schedule, when $n>1$, as follows :

Algorithm Get_Recover_Schedule

Step 1 : Determine R_1 and T_1.
Step 1.0 : Set $k = 0$.
Step 1.1 : Set $k = k + 1$.
If $r(t_1+kt_2) < R_n$, go to Step 1.1.
Else, $T_1 = t_1 + kt_2$, $R_1 = rT_1$, and go to Step 2.
Step 2 : Determine R_i and T_i, i = 2, 3, $\cdots$, $n-1$.
Step 2.0 : Set $i = 1$.
Step 2.1 : If $i = n-1$, stop.
Else, $i = i+1$, $T_i = T_{i-1}+t_3+t_1$ and $R_i = R_{i-1} - R_n + rt_1$.
Step 2.2 : If $R_1 < R_n$, set $k = 0$ and go to Step 2.3.
Else, go to Step 2.1.
Step 2.3 : Set $k = k+1$, $T_i = T_{i-1}+t_3+t_1+kt_2$, and $R_i = R_{i-1} - R_n + r(t_1+kt_2)$.
If $R_i < R_n$, go to Step 2.3.
Else, go to Step 2.1.

End_Of_Algorithm

Once the R_i's and T_i's are found by the algorithm *Get_Recover_Schedule*, we can get the total cost in a unit time by dividing the sum of these four cost terms by the cycle time length as follows :

$$TC(n,\ m,\ T) = \frac{nC_S + mC_O}{T} + \left[\frac{T(d-r)^2}{2md} + \frac{r^2T(p-d)}{2ndp}\right]C_{H2} + \left[\frac{(p-r)T}{2} - \frac{1}{n}\sum_{i=1}^{n-1}|(p-r)(T_i-T)+R_i|\right]\frac{r}{p}C_{H1} \tag{16}$$

2.4 Solution Procedure

Although the equation (16) is too complicated to be shown its convexity for the three decision variables, i.e. m, n and T, after calculating many example problems, we believe that the equation is a unimodal convex function of each variable when the other variables are fixed. But it seems to be very difficult that the optimal values of the three decision variables are given in closed forms. Therefore, we propose a phased search procedure to find them. First, it is necessary to find a local optimal length of cycle time when the number of orders for newly purchasing items (m) and the number of recovery setups (n) are fixed. To do that, we propose a very simple search procedure as follows :

Algorithm Get_Cycle_Time

Step 0 : Set $T^* = T = 0$ and TC_2 (current minimum total cost) = ∞.
Set the optimal cycle time when $m = n = 1$ to the initial search step, T_0, as follows :

$$T_0 = \sqrt{\frac{2dp(C_S + C_O)}{rd(p-r)C_{H1} + [p(d-r)^2 + r^2(p-d)]C_{H2}}} \quad (17)$$

Step 1 : Set $T = T + T_0$.
Using equation (16), calculate TC for given m, n and T.
If $TC \le TC_2$, go to Step 2.
Else, go to Step 3.

Step 2 : Set $T^* = T$ and $TC_2 = TC$.
Go to Step 1.

Step 3 : If $T_0 < \varepsilon$ (a sufficiently small positive number), stop.
Else, go to Step 4.

Step 4 : Set $T = \max\{0,\ T - 2T_0\}$.
Set $T_0 = T_0/10$ and $TC_2 = \infty$
Go to Step 1.

End_Of_Algorithm

In this algorithm, the equation (17) is the optimal cycle time when $m = n = 1$. Because the equation (16) is a convex function of T when $m = n = 1$, one can easily get this equation.

Next, using the cycle time calculated by the algorithm *Get_Cycle_Time* and its cost, we can find local optimal values of m and n by another simple search procedure as follows :

Algorithm Get_Best_Policy

Step 0 : Determine the values of the cost parameters C_S, C_O, C_{H1} and C_{H2} as well as the system
parameters r, p and d.
Set $m = 1$ and $TC^* = \infty$.

Step 1 : *Determine optimal value of n for given m.*

Step 1.0 : Set $n = 1$ and TC_1 (current minimum total cost for given *m*) = ∞.

Step 1.1 : For given m and n, find TC_2 using the algorithm *Get_Cycle_Time.*

Step 1.2 : If $TC_2 < TC_1$, set $n_1 = n$, $TC_1 = TC_2$ and $n = n + 1$. And go to Step 1.1.
Else, go to Step 2.

Step 2 : If $TC_1 < TC^*$, set $m^* = n$, $n^* = n_1$, $TC^* = TC_1$ and $m = m + 1$. And go to Step 1.
Else, Stop.

End_Of_Algorithm

In this algorithm, TC_2 is the current best total cost calculated by the algorithm *Get_Cycle_Time* when the cycle time *T* is a variable and the values of *m* and *n* are fixed, while TC_1 is the current minimum total cost when *n* is a variable and *m* is given and TC^* is the current minimum total cost value for the variable *m*.

2.5 Numerical example

To validate the proposed model and solution procedure, a number of sample problems were solved. One of these problems is as follows :

$r = 15$, $d = 30$, $p = 150$,
C_S = \$1,000 / recovery setup,
C_O = \$500 / order for new items,
C_{H1} = \$1 / recoverable item/time and
C_{H2} = \$10 / serviceable item/time.

Applying these data to the solution procedure, the best decision variables can be obtained as follows :

$m^* = 3$, $n^* = 2$, $T^* \cong 10.54$ and $TC^* \cong \$664.08$.

To compare this result with Koh *et al*. (2002), we tried to find the best recovery and ordering schedule under the condition that restricts to only one setup or order. As a matter of course, we did not use the original model of Koh *et al*. (2002), but used a modified model that reflected the comment of Teunter (2004). The result with the same parameter values is as follows :

$m^* = 2$, $n^* = 1$, $T^* \cong 6.00$ and $TC^* \cong \$666.33$.

The difference of the two systems in total cost is \$2.25, which is about 0.34% of the best value of the proposed system in this paper.

3. Computational Experiments

This section shows how the system characteristics respond to changes of the parameters. To do this we perform a sensitivity analysis, in which one parameter varies over a range, while all other parameters are held constants. Using the data in Section 2.5, the following are set as standard values of the system parameters: $d = 30$, $r = 0.5d$, $p = 5d$, $C_S =$ \$1,000 / recovery setup, $C_O =$ \$500 / order for new items, $C_{H1} =$ \$1 / recoverable item / unit time, and $C_{H2} = 10C_{H1}$ / serviceable item / unit time. <Tables 1> through 5 show the effects of the changes in r, p, C_S, C_o, and C_{H2}, respectively. For example, the results of <Table 1> are calculated for $r \in \{0.1d, 0.2d, \cdots, 0.9d\} = \{3, 6, 9, \cdots, 27\}$ when the other parameters are fixed to the standard values.

In each table one can see the optimal number of orders for new items in a cycle (i.e. m^*), optimal number of recovery setups in a cycle (i.e. n^*), and total cost in a unit time under the optimal policy (i.e. TC^*). These tables also show the comparison results of the (P, R) policy with $(P, 1)$ and $(1, R)$ policies. To do this, we calculated the cost improvement of the (P, R) policy, and the results are given at the rightmost column in each table. For example, the fifth row in <Table 1> shows that (P, R) policy can reduce the total cost in unit time by 0.34%. As one can see in the example case of Section 2.5, optimal (m^*, n^*) values for $(1, R)$ and $(P, 1)$ policies under the same condition are (2, 1), which are expressed in the parentheses at the m^* and n^* columns of the table. Except for the fifth and the seventh rows, <Table 1> has no value in the rightmost columns, which mean that the cost reduction quantity is zero (in other words, the results for (P, R) policy are same to the results for $(1, R)$ and $(P, 1)$ policies).

Table 1. Effects of r on the system

r	m^*	n^*	TC^*	improvement (%)
3	10	1	596.4	-
6	5	1	613.4	-
9	3	1	628.7	-
12	2	1	643.2	-
15	3(2)	2(1)	664.1	0.34
18	1	1	671.4	-
21	2(1)	3(1)	697.5	0.15
24	1	2	711.6	-
27	1	4	729.7	-

Table 2. Effects of p on the system

p	m^*	n^*	TC^*	improvement (%)
60	2	1	587.4	-
90	2	1	632.5	-
120	3(2)	2(1)	653.1	0.11
150	3(2)	2(1)	664.1	0.34
180	1	1	673.6	-
210	1	1	678.0	-
240	1	1	681.2	-
270	1	1	683.7	-
300	1	1	685.7	-

Now, the effects of varying parameters are investigated. According to <Table 1>, as the quantity of items collected in a unit time (i.e. r) increases, 1) the ratio of the optimal number of orders for newly purchasing items to the number of setups in the recovery shop (i.e. m^*/n^*) is decreasing and 2) the optimal total cost (i.e. TC^*) is increasing. Because the quantity of newly purchasing items decreases and the number of recovery setups needed increase when the collection quantity becomes large, the decrease of the ratio seems reasonable.

In <Table 2>, we can see the number of orders for new items in a cycle (i.e. m^*) tends to increase when the repair quantity in a unit time (i.e. p) is small. If p is small, the serviceable inventory level when the recovery shop is just finished its process is small, and therefore, one more order for new item can be needed. From the viewpoint of total cost, since the serviceable inventory level becomes relatively higher than recoverable inventory when p becomes large, TC^* becomes large as p increase.

<Table 3>～<Table 5> show the system responses to changes of cost parameters. In <Table 3> and <Table 4>, we can see that the m^*/n^* value becomes large as the setup cost in the recovery shop, C_S, increases or the ordering cost for newly purchasing items, C_O, decreases. <Table 5> shows that both of the integer variables (i.e. m^* and n^*)

become large when C_{H2} is increasing. All of these observations are predictable results.

Table 3. Effects of C_S on the system

C_S	m^*	n^*	TC^*	improvement (%)
200	2(1)	3(2)	454.3	0.36
400	1	1	517.0	-
600	1	1	571.6	-
800	1	1	621.4	-
1,000	3(2)	2(1)	664.1	0.34
1,200	2	1	698.9	-
1,400	2	1	729.9	-
1,600	2	1	759.7	-
1,800	2	1	788.4	-

Table 4. Effects of C_O on the system

C_O	m^*	n^*	TC^*	improvement (%)
100	3	1	506.1	-
200	2	1	557.5	-
300	2	1	596.0	-
400	2	1	632.1	-
500	3(2)	2(1)	664.1	0.34
600	1	1	689.3	-
700	1	1	710.6	-
800	1	1	731.2	-
900	1	1	751.2	-

Table 5. Effects of C_{H2} on the system

C_{H2}	m^*	n^*	TC^*	improvement (%)
2	1	1	348.6	-
4	1	1	450.0	-
6	1	1	532.4	-
8	1	1	603.7	-
10	3(2)	2(1)	664.1	0.34
12	3(2)	2(1)	719.7	0.39
14	3(2)	2(1)	771.4	0.42
16	3(2)	2(1)	819.8	0.45
18	3(2)	2(1)	865.4	0.47

4. Conclusions

We proposed a model to analyze an inventory system where the stationary demand can be satisfied by recovered products and newly purchased products. This model extended previous studies to the case of variable number of orders for newly purchasing items as well as variable number of setups for recovery process within a cycle. We developed an optimization model obtaining the economic order quantity for newly procured products, the optimal lot size for the recovery process, and the sequence of the orders and the setups, simultaneously. And then a simple solution procedure to find the optimal control parameters was proposed. From the computational experiments, we found that the results of this study is better than the earlier study results in some cases.

The results in this paper may be extended to the following cases. One is the case in which the recoverable items are deteriorating. In second case, one may assume that the collection rate and/or demand rate are random variables. Final suggestion is the problem of dynamic version, in other words, the return rate of recoverable items and the demand rate of serviceable items are known but not constant through time.

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