韓國經營科學會誌
第33권 第1號
2008年　3月

가격차에 의해 발생하는 수요대체효과를 고려한 정태적 최적가격결정 모형 수립

김 상 원*

A Deterministic Model for Optimal Pricing Decisions with Price-Driven Substitution

Sang-Won Kim*

■ Abstract ■

Market segmentation is a key strategic factor in increasing the expected profits, especially in the practice of revenue management. A manufacturing firm should manage both manufacturing quantities and pricing decisions over its segmented markets to maximize the expected profits, setting different price for each different segment. Also, market segments should be kept separate in order to prevent demand leakages between different market segments. In fact, even though the markets for different products are firmly segmented, it is not easy to keep separate segmentation because many products might be substitutable by customer buying behavior. That is, customers respond to price changes by purchasing other market's products instead of purchasing the originally requested products, which causes demand substitution effect ; This kind of substitution is referred to as price-driven substitution. Therefore, decisions on optimal prices should take into account the differences in customers' valuation of the different products. We consider a deterministic model for deciding optimal prices in the presence of price-driven substitution, and we compare both symmetrical-and asymmetrical-type demand substitutions between two segmented markets. The objective of this study is to develop analytical and numerical models to examine the impact of price-driven substitution on the optimal price levels and the total expected profits.

Keyword : Price-driven Substitution, Pricing, Market Segmentation, Customer Buying Behavior

논문접수일 : 2007년 03월 05일　논문게재확정일 : 2008년 01월 09일
* 울산대학교 경영학과

1. Introduction

Many companies produce a variety of products which are substitutable by customers. Production decisions depend heavily on the accuracy of demand estimates. The observed demand is composed of pure demand plus effect of demand and product substitution. When a product stocks out or its price changes many customers buy another product, which results in distorted demand. Perfect demand estimation is difficult because of these customer behaviors. Therefore, manufacturing management problems become more complicated by this phenomenon. Also, production decisions for these substitutable products are more difficult because demand for a particular product is not only affected by its inherent characteristics, e.g. color, design etc, but also by the price-differences. Demand substitution occurs when consumers respond to price changes by purchasing less expensive products in place of relatively more expensive products, which is called price-driven substitution. Consequently, the changing price levels of both products may affect customers' preferences for one product or the other product. For example, Coke and Pepsi Cola are substitutes. As the price of Coke increases, some people stop buying Coke and buy Pepsi instead. This type of substitution is called price-driven substitution, which is usually found in economics literature. Several researchers have studied the substitution effects of a price change (e.g., Paul, 1981 ; Taheri, 1994 ; Schmidt and Druehl, 2005). Paul (1981) developed a gasoline price control model. In this model, he argued that the U.S. Environmental Protection Agency opposed gasoline price decontrol because a wider posted price differential between leaded and unleaded gasoline would induce more consumers to switch illegally to leaded gasoline. Taheri (1994) investigated substitution among such fuels as coal, natural gas, and electricity during the actual oil price increases of the 1970s. Schmidt and Druehl (2005) studied a substitution model for developing new products, where customers who previously preferred the old products might later prefer the new products due to the changes in product reservation price over time.

A second type of substitution is referred to as inventory-driven substitution. A customer who cannot buy a product because it is out of stock might be willing to buy a similar product instead. This situation is frequently found in the retail industries. For example, some customers prefer to buy blue shirts while others prefer green shirts. When blue shirts are not available, customers who prefer to buy blue shirts may buy green ones. Stock-outs of frequently purchased products may be common when substitutable demand may make up a portion of the unmet demand of the stocked-out products. If this kind of consumer behavior occurs frequently, then the sales will depend significantly on demand substitution. The better a company can forecast demand substitution, the better it can balance supply and demand in an effort to maximize profits. Inventory-driven substitution has been studied most commonly in management science and operations research literature. The single-period inventory management problem has been frequently used for analyzing inventory levels for substitution problems. Analysis of single-period two product substitution problems appeared in Pasternack and Drenzer (1991), Gerchack, Tripathy, and Wang (1996) and Khouja et al. (1996). Bassok, Anupindi and Akella (1999), Hsu and

Bassok (1994), and Gerchak, Tripathy and Wang (1996) studied ordering policies with one-way substitution. Recent works of single-period stochastic inventory models for one-way inventory-driven substitution include Bassok et al. (1999), Hsu and Bassok (1999), Smith and Agrawal (2000), Rajaram and Tang (2001) and Rao et al. (2005). In these lines of research for inventory-driven substitution, the authors assumed that the manufacturer or manager was a price taker for all of the products. In management science literature, little research on price-driven substitution has been found.

Revenue management uses pricing and inventory control strategies to balance supply and demand. Although pricing and inventory control decisions are closely related, limited published research on demand substitution exists that investigates their interdependence. Traditionally, a marketing department takes care of pricing whereas a manufacturing or purchasing department determines the inventory level. Many studies have considered substitution problems without the pricing decision. However, price also changes the demand and should be an important part of the overall optimal decision because customers usually select a product based on price level. Extensive research on pricing and inventory level decisions (or capacity allocation) has been carried out under different assumptions. However, techniques developed in the two areas have rarely been integrated.

Research on joint pricing and inventory control and/or capacity management with demand substitution can be found in Bitran et al. (2006), Birge et al. (1998), Xu and Hopp (2004), and Kuyumcu and Popescu (2006). Birge et al. (1998) studied optimal price and production decisions with stochastic demand, where substitution was driven by price differences. The authors considered a single-period two-item production management model where there were two markets with a single product in each market. Xu and Hopp (2004) studied an inventory replenishment problem with dynamic pricing and stochastic demand substitution, where the customer arrival process is piecewise deterministic and follows a geometric Brownian motion. Kuyumcu and Popescu (2006) studied joint pricing and inventory control by considering static deterministic optimization models for substitutable products. They showed that the deterministic model with substitution could be reduced to a pure pricing problem.

In summary, there are a considerable number of substitution models in inventory or capacity management areas. However, most of these studies have considered substitution problems without the issue of price decisions. In addition, there are many pricing models in revenue management, but most of the models do not consider inventory/production decisions with or without substitution. The motivation of our research is the lack of this kind of investigation in a revenue management context. The objective of the models is to develop both analytical and numerical methods for solving these problems and to examine the impact of substitution on the optimal production levels and prices, and we investigate managerial insights and implications.

2. The Model

Our study attempts to connect traditional production management with the pricing issues of the marketing area. We consider how a firm that has two substitutable products should decide the optimal production levels and prices for these products. The purpose of this research is to examine the potential impact of substitution on the optimal price, production quantity and total expected revenue. We investigate the managerial implications from the deterministic substitution models, which can be used for subroutines for approximating stochastic models, and provide numerical examples, attempting to analyze how optimal pricing and production decisions are affected simultaneously for substitutable products. Product substitutability makes pricing and production decisions more difficult due to the fact that a change in the price of one product affects the demand level and optimal production level for another.

2.1 Notation

We consider the case where customers had preferences for one product or another and this preference was affected by the price levels of both products. Specifically, we consider two segmented markets (A & B) with a different product in each market : market A has a high priced product (A) and market B has a low priced product (B). We assume that the demand is deterministic and price dependent and that the demand for product is dependent on the price of both products. The mean demand of product A is decreasing in the price of product A and increasing in the price of product B. Also, the mean demand of product B is decreasing in the price of product B and increasing in the price of product A. As far as notation throughout the paper, let :

P_a : unit price of product A

P_b : unit price of product B$(P_a > P_b)$

C_a : unit production cost of product A$(P_a > C_a)$

C_b : unit production cost of product B$(P_b > C_b)$

D_a : demand in market A

D_b : demand in market B

Q_a : production level of product A

Q_b : production level of product B

2.2 A Substitution Model with Deterministic Demand

An objective of revenue management is to balance demand and supply to increase the efficiency and profitability. Many industries use dynamic pricing to match demand with inventory or capacity to maximize revenue or contribution. In many traditional business models price remains fixed or constant over the entire time horizon even if the demand is non-stationary over time. When price is not fixed, customers have preferences for one product or another as determined by the product's original price ; however, the price levels of both products can change this preference, and demand substitution motivated by price differences occurs between the two markets.

Now we present our price-driven substitution models. We assume that there are two markets with a different demand for each market, where demand substitution is realized from market with a high-priced product to market with a low-priced product. We investigate the impact of substitution on the decision variables and the total expected revenue by comparing the models with and without substitution. We use the notation in § 2.1. First, consider the case of two segmented markets with a different product in each market and independent demand. The demand model is

$$D_a = A_a - B_a P_a \quad (2.1)$$
$$D_b = A_b - B_b P_b$$

where A_a, A_b, B_a, $B_a > 0$(constant) are exogenous to the market. In order to estimate the demands, we need to decide the price elasticity of the demand which is a basis for price segmentation of the market. Therefore, each market has a different elasticity for its own market. The concept of elasticity of substitution measures the relative change in the ratio between the quantities of two goods consumed by a certain individual as a response to a relative change in the ratio of the prices of those goods. The elasticity of substitution between goods A and B can also be related to the cross price elasticity of demand for those goods. Cross price elasticity of demand is a related concept, measuring the percentage change in quantity (demand) from a percentage change in the price of different goods. Estimating demand in models (2.4) and (2.13), we need to estimate cross price elasticity, whereas demand model (2.1) does not consider cross elasticity because each market has a completely independent demand.

Methods for estimating price elasticity and cross price elasticity of demand include estimation by scorecards through interviews and secondary research from reliable sources e.g. academic journals, industry trade journals, professional pricing society, marketing science institute, textbooks, legal cases etc. Data to evaluate price sensitivity can be pulled from multiple sources—inside and outside the organization, e.g. customer (sales) transaction data, historical data, comparison data between competitive products, syndicated data, panel data.

Demand estimation uses these existing data to analyze the impact of price on sales demand, and regression analysis is a major demand estimation method using existing real world transaction data to analyze the impact of prices on sales demand. Also, a number of marketing research techniques are used for demand estimation by pricing researchers, e.g. "Monadic" price test, "Gabor Granger," "Van Westendorp," trade-off analysis etc. In Monadic pricing test, two or more different surveys are created to determine purchase intent at different prices for a particular product. Each survey uses a different price and provides a basis for price sensitivity measurement.

Then the contribution-maximization problem is

$$\underset{P_a,\, P_b}{Max}\, \pi = (P_a - C_a)Q_a + (P_b - C_b)Q_b \quad (2.2)$$

subject to

$$Q_a = D_a = A_a - B_a P_a$$
$$Q_b = D_b = A_b - B_b P_b$$
$$P_a > C_a > 0,\ P_b > C_b > 0,\ P_a > P_b$$
$$Q_a,\ Q_b,\ D_a,\ D_b \geq 0$$

The optimal prices and quantities are obtained from the first order conditions :

$$\overline{P}_a^* = \frac{A_a + C_a B_a}{2B_a} \quad (2.3)$$
$$\overline{P}_b^* = \frac{A_b + C_b B_b}{2B_b}$$
$$\overline{Q}_a^* = A_a - B_a \overline{P}_a^* = \frac{A_a - C_a B_a}{2}$$
$$\overline{Q}_b^* = A_b - B_b \overline{P}_b^* = \frac{A_b - C_b B_b}{2}$$

Next, we consider a substitution model as a result of price-differences. This form of substitution has been called demand leakage, where the magnitude of the leakage depended on the price differences in the two markets. We consider the impact of demand substitution between the two segmented markets, where the demand lost in market A becomes additional demand in market B. Therefore, the demand model is :

$$D_a = A_a - B_a P_a - L(P_a - P_b) \quad (2.4)$$
$$D_b = A_b - B_b P_b + L(P_a - P_b)$$
$$\text{where } P_a > P_b > 0,\ D_a \ge 0,\ D_b \ge 0,\ L > 0$$

We examine this contribution maximization problem where $L(P_a - P_b)$ is linear for the general case where $C_a \neq C_b$. The contribution maximization problem is :

$$\underset{P_a, P_b}{Max}\, \pi = (P_a - C_a)Q_a + (P_b - C_b)Q_b \quad (2.5)$$
subject to
$$Q_a = D_a = A_a - B_a P_a - L(P_a - P_b)$$
$$Q_b = D_b = A_b - B_b P_b + L(P_a - P_b)$$
$$P_a > P_b,\ P_a > C_a > 0,\ P_b > C_b > 0,\ L > 0$$
$$Q_a,\ Q_b,\ D_a,\ D_b \ge 0$$

If L is a constant, the first order conditions lead to optimal prices :

$$P_a^* = \frac{A_a + 2LP_b^* + C_a B_a + C_a L - C_b L}{2(B_a + L)} \quad (2.6)$$
$$P_b^* = \frac{A_b + 2LP_b^* + C_b B_b - LC_a + C_b L}{2(B_b + L)}$$

Eliminating decision variables leads to the closed-form optimal solutions :

$$P_a^* = \frac{C_a}{2} + \frac{A_a B_b + L(A_a + A_b)}{2[B_a B_b + L(B_a + B_b)]} \quad (2.7)$$
$$P_b^* = \frac{C_b}{2} + \frac{A_b B_a + L(A_a + A_b)}{2[B_a B_b + L(B_a + B_b)]}$$

The difference between the optimal prices is

$$P_a^* - P_b^* = \frac{C_a - C_b}{2} + \frac{A_a B_b - A_b B_a}{2(B_a B_b + L(B_a + B_b))} \quad (2.8)$$

If $C_a = C_b$, the existence condition for $P_a^* > P_b^*$ requires $A_a B_b > A_b B_a$. In a general case, the existence condition for $P_a^* > P_b^*$ requires $C_a - C_b > \frac{A_b B_a - A_a B_b}{(B_a B_b + L(B_a + B_b))}$. The demand model (2.4) can be rewritten :

$$D_a = A_a - (B_a + L)P_a + LP_b \quad (2.9)$$
$$D_b = A_b + LP_a - (B_b + L)P_b$$
$$\text{where } L > 0,\ P_a > P_b > 0,\ D_a \ge 0,\ D_b \ge 0$$

Comparing the model with independent markets (2.2) to the demand substitution model (2.5) leads us to obtain the price differences :

$$P_a^* - \overline{P}_a^* = \frac{L(A_b B_a - A_a B_b)}{2B_a(B_a B_b + L(B_a + B_b))} \quad (2.10)$$
$$P_b^* - \overline{P}_b^* = \frac{L(A_a B_b - A_b B_a)}{2B_b(B_a B_b + L(B_a + B_b))}$$

where $\overline{P}_a^*$ and $\overline{P}_b^*$ are the optimal prices when there is no substitution given by (2.3). These differences depend on the substitution factor, L, but do not depend on the unit production costs. If $C_a \le C_b$, then the existence condition for $P_a^* > P_b^* > 0$ requires $A_a B_b > A_b B_a$. In this case, $P_a^* < \overline{P}_a^*$ and $P_b^* > \overline{P}_b^*$ regardless of L. From (2.10), there are three potential cases for price decisions when $C_a > C_b$:

Case 1 : If $A_b B_a > A_a B_b$, then $P_a^* > \overline{P}_a^*$ and $P_b^* < \overline{P}_b^*$ (2.11)

Case 2 : If $A_bB_a < A_aB_b$, then $P_a^* < \overline{P}_b^*$ and $P_b^* > \overline{P}_b^*$

Case 3 : If $A_bB_a = A_aB_b$, then $P_a^* = \overline{P}_a^*$ and $P_b^* = \overline{P}_b^*$

In this model, the demand loss in market A equals the demand gain in market B, which is called symmetrical substitution. The differences between the optimal production levels are given by

$$Q_a^* - \overline{Q}_a^* = \frac{L(C_b - C_a)}{2} \qquad (2.12)$$

$$Q_b^* - \overline{Q}_b^* = \frac{L(C_a - C_b)}{2}$$

where $\overline{Q}_a^*$ and $\overline{Q}_b^*$ are the optimal production levels where there is no substitution (2.3). The difference between the optimal production levels in the model with substitution versus the one without substitution depends on the unit production cost, C_a, C_b, and substitution factor, L, but does not depend on the prices of the products.

Next, we consider an extended model where the demand loss in market A and the demand gain in market B are asymmetrical. Many researchers have proposed that only a fraction of the customers who could not buy a high priced product would substitute to a low priced product (e.g., Sen and Zhang, 1999 ; Birge et al., 1998). We consider the case where a fraction of the demand loss in market A becomes a demand addition to market B. We use the following demand models for the two markets :

$$D_a = A_a - B_aP_a - L(P_a - P_b) \qquad (2.13)$$

$$D_b = A_b - B_bP_b + \alpha L(P_a - P_b)$$

where $L \ge 0$, $P_a > P_b > 0$, $D_a \ge 0$, $D_a \ge 0$, $0 \le \alpha < 1$

$A_a, A_b, B_a, B_a > 0$

The contribution maximization problem is

$$\max_{P_a, P_b} \pi = (P_a - C_a)Q_a + (P_b - C_b)Q_b \qquad (2.14)$$

$$Q_a = D_a = A_a - B_aP_a - L(P_a - P_b)$$

$$Q_b = D_b = A_b - B_bP_b + \alpha L(P_a - P_b)$$

$$P_a > C_a > 0,\ P_b > C_b > 0,\ P_a > P_b$$

$$Q_a,\ Q_b,\ D_a,\ D_b \ge 0,\ L \ge 0,\ 0 \le \alpha < 1$$

We require that $A_a - B_aP_a - L(P_a - P_b) \ge 0$ and $A_b - B_bP_b \ge 0$. In order to show the existence of a unique optimal price for P_a and P_b, we derive sufficient conditions for the existence of a maximum. The first and second derivatives are

$$\frac{\partial \pi(P_a, P_b)}{\partial P_a} = A_a - B_aP_a - L(P_a - P_b) + (P_a - C_a)(-B_a - L) + (P_b - C_b)\alpha L \qquad (2.15)$$

$$\frac{\partial \pi(P_a, P_b)}{\partial P_b} = A_b - B_bP_b + \alpha L(P_a - P_b) + (P_b - C_b)(-B_b - \alpha L) + (P_a - C_a)L$$

$$H_{11} = \frac{\partial^2 \pi(P_a, P_b)}{\partial P_a^2} = -2(B_a + L) < 0$$

$$H_{22} = \frac{\partial^2 \pi(P_a, P_b)}{\partial P_b^2} = -2(B_b + \alpha L) < 0$$

$$H_{12} = L(1+\alpha)$$

The profit function is concave in P_a for any given P_b and is also concave in P_a for a given P_a. The sufficient condition for the existence of a unique maximum of (P_a^*, P_b^*) requires

$$H_{11}H_{22} - H_{12}^2 = 4(B_a + L)(B_b + \alpha L) - L^2(1+\alpha)^2 > 0 \qquad (2.16)$$

If $H_{11}H_{22} - H_{12}^2 > 0$ the profit function is jointly concave, and a unique optimal prices (P_a^*, P_b^*) exists. Next, we compare (2.14) to the model without substitution (2.2). The optimal solution is obtained from the first order conditions :

$$P_a^* = \frac{A_a + LP_b^* + C_a(B_a + L) + \alpha L(P_b^* - C_b)}{2(B_a + L)} (>0)$$
$$P_b^* = \frac{A_b + \alpha LP_a^* + C_b(B_b + \alpha L) + L(P_a^* - C_a)}{2(B_b + L)} (>0) \qquad (2.17)$$

leading to the closed-form optimal prices :

$$P_a^* = \frac{2(B_b + \alpha L)[A_a + (B_a + L)C_a - \alpha LC_b]}{4(B_a + L)(B_b + \alpha L) - L^2(1+\alpha)^2} + \frac{(1+\alpha)L[A_b + (B_b + \alpha L)C_b - LC_a]}{4(B_a + L)(B_b + \alpha L) - L^2(1+\alpha)^2} \qquad (2.18)$$

$$P_b^* = \frac{2(B_a + L)[A_b + (B_b + \alpha L)C_b - LC_a]}{4(B_a + L)(B_b + \alpha L) - L^2(1+\alpha)^2} + \frac{(1+\alpha)L[A_a + (B_a + L)C_a - \alpha LC_b]}{4(B_a + L)(B_b + \alpha L) - L^2(1+\alpha)^2}$$

The difference between P_a^* and P_b^* is

$$P_a^* - P_a^* = \frac{\begin{bmatrix} -L(1-\alpha)(A_a + B_aC_a + A_b + B_bC_b) + 2L(B_a + B_b)(C_a - \alpha C_b) \\ +2(A_aB_b - A_bB_a) + 2B_aB_b(C_a - C_b) \end{bmatrix}}{4(B_a + L)(B_b + \alpha L) - L^2(1+\alpha)^2} \qquad (2.19)$$

The necessary condition for the existence of $P_a^* > P_b^* > 0$ is

$$\begin{pmatrix} -L(1-\alpha)(A_a + B_aC_a + A_b + B_bC_b) \\ +2L(B_a + B_b)(C_a - \alpha C_b) \\ +2(A_aB_b - A_bB_a) + 2B_aB_b(C_a - C_b) \end{pmatrix} > 0 \qquad (2.20)$$

If $\alpha = 1.0$, we have the case of symmetrical substitution (2.5). If $A_aB_b > A_bB_a$ then $P_a^* - \overline{P}_a^* < 0$ from (2.10) and $P_b^* - \overline{P}_b^* > 0$. The differences between the optimal production quantities are

$$Q_a^* - \overline{Q}_a^* = \frac{1}{2}\{L(P_b^* - C_a) - \alpha L(P_b^* - C_b)\} \qquad (2.21)$$
$$Q_b^* - \overline{Q}_b^* = \frac{1}{2}\{\alpha L(P_a^* - C_b) - L(P_a^* - C_a)\}$$

where $\overline{Q}_a^*$ and $\overline{Q}_b^*$ are the optimal production levels for the model without substitution (2.3)

2.3 Managerial Implications

We now present managerial implications and insight of the models with symmetrical and asymmetrical substitution. Managers who make decisions of price and production in related markets must consider the impact of demand leakage. Even though two markets may be segmented, the demand in one market is affected by the prices of products in the other. When demand substitution is symmetrical, the demand loss of market A always equals the demand gain in market B. The price change of the product B affects the demand of the product A as much as the price change of product A affects the demand for product B. It is counterintuitive that the unit production cost of the product in one market has no effect on the optimal price of the other market's product. Intuitively, managers are willing to have incentives to vary the optimal prices of products in other market when the production cost changes. The model with asymmetrical demand substitution seems more realistic than symmetrical demand substitution in that the optimal prices should be determined by considering the relevant factors, i.e. production cost etc. As pointed out in Mulhern and Leone (1991)'s and Heath and Chatterjee (1995)'s empirical works, demand substitution patterns could be asymmetrical.

Therefore, when setting up prices and production quantities, managers should pay attention to the production costs of both markets.

Next, we consider the simultaneous decision of optimal prices and quantities. The pricing decision belongs to one of three possible cases, dependent on the demand parameters :

1 : If $B_aA_b > A_aB_b$, then $P_a^* > \overline{P}_a^*$ and $P_b^* < \overline{P}_b^*$

2 : If $B_aA_b < A_aB_b$, then $P_a^* < \overline{P}_a^*$ and $P_b^* > \overline{P}_b^*$

3 : If $B_aA_b = A_aB_b$, then $P_a^* = \overline{P}_a^*$ and $P_b^* = \overline{P}_b^*$

Given optimal production quantities and prices of the model without substitution, we can obtain the optimal solutions using the price differences $P_a^* - \overline{P}_a^*$ and $P_b^* - \overline{P}_b^*$ When α=1.0, $Q_a^* - \overline{Q}_a^* = -(Q_b^* - \overline{Q}_b^*)$ and $P_a^* - \overline{P}_a^* = -(P_b^* - \overline{P}_b^*)$. This means that if the optimal quantity of product A is increased from the optimal quantity of a model without substitution, then the optimal quantity of product B is decreased that much. The optimal production quantity of both products cannot be greater than those of the model without substitution simultaneously.

We now consider the impact of production cost on the optimal price and production quantity. When α=1.0 and $C_a > C_b$, we can make three possible optimal pricing decisions (2.11). There exists a unique optimum for each case, and the optimal values are dependent on the demand parameters. For example, if $B_aA_b < A_aB_b$, then the optimal solution has :

$$P_a^* < \overline{P}_a^*,\ P_b^* > \overline{P}_b^*,\ Q_a^* < \overline{Q}_a^*,\ Q_b^* > \overline{Q}_b^*$$

Both the optimal production quantity and the optimal price of product A are less than those of the model without substitution while the optimal production quantity and the optimal price of product B are greater than those of the model without substitution. It is a counterintuitive that the optimal quantity also increases while the optimal price increases. If $C_a \le C_b$, then the production quantity of product A is increased because the unit profit of product A is greater than that of product B : $Q_a^* > \overline{Q}_a^*$ and $Q_b^* < \overline{Q}_b^*$. Setting $P_a^* < \overline{P}_a^*$ and $P_b^* > \overline{P}_b^*$ limits demand leakage from market A to B. When $C_a = C_b$, then $Q_a^* = \overline{Q}_a^*$ and $Q_b^* = \overline{Q}_b^*$. Even though the optimal production quantities do not change from those of a model without substitution, the optimal price of product A is less than that of a model without substitution while the optimal price of product B is greater than that of a model without substitution. If the unit production costs are the same, changing the optimal prices does not affect the optimal production decisions.

If 0.0<α<1.0, then the differences, $Q_a^* - \overline{Q}_a^*$ and $Q_b^* - \overline{Q}_b^*$ depend not only on unit production costs but also on prices. If $\alpha C_b > C_a$, then the optimal production decision has $Q_a^* > \overline{Q}_a^*$ and $Q_b^* < \overline{Q}_b^*$; it is more profitable to produce more of product A. Hence, the optimal price decision has $P_a^* < \overline{P}_a^*$ and $P_b^* > \overline{P}_b^*$ to mitigate demand leakage from market A to B. If $C_a = C_b$, then the optimal production quantities have : $Q_a^* > \overline{Q}_a^*$ and $Q_b^* < \overline{Q}_b^*$. In this case, part of the substituted demand, $(1-\alpha)L(P_a - P_b)$ is lost, and $P_a - P_b$ should be decreased to protect market demand as α decreases. For a given α, as L increases, more demand will be lost ; therefore, $P_a - P_b$ should be decreased to protect the demand in market A. When α=0.0, there is no substituted demand in market B so that $P_a^* < \overline{P}_a^*$ and $P_b^* > \overline{P}_b^*$ for any demand parameters. For both cases where $\alpha=1.0$ and $0.0 \le \alpha < 1.0$, the most significant feature is that the decisions of the optimal prices and quantities have no definite patterns when $C_a > C_b$. Especially, if $C_a > C_b$ and $0.0 \le \alpha < 1.0$, we have the conditions for the optimal prices and production levels as

A. Optimal Prices

Condition 1 :

If $\alpha > \dfrac{-B_a(A_a + B_b C_b) + 2B_b A_a}{B_a(A_b - B_b C_a)}$ and $\alpha > \dfrac{B_b(A_a - B_a C_a)}{2B_a A_b - B_b(A_a + B_a C_a)}$,

then $\left(P_a^* \ge \overline{P}_a^* \text{ and } P_b^* \le \overline{P}_b^*\right)$ or $\left(P_a^* \le \overline{P}_a^* \text{ and } P_b^* \le \overline{P}_b^*\right)$

Condition 2 :

If $\alpha \le \dfrac{-B_a(A_a + B_b C_b) + 2B_b A_a}{B_a(A_b - B_b C_a)}$ and $\alpha \le \dfrac{B_b(A_a - B_a C_a)}{2B_a A_b - B_b(A_a + B_a C_a)}$,

then $\left(P_a^* \le \overline{P}_a^* \text{ and } P_b^* \ge \overline{P}_b^*\right)$ or $\left(P_a^* \ge \overline{P}_a^* \text{ and } P_b^* \ge \overline{P}_b^*\right)$

B. Optimal Production levels

Condition 1 :

If $\alpha < \dfrac{(P_b^* - C_a)}{(P_b^* - C_b)}$, then $Q_a^* > \overline{Q}_a^*$, or if

$\alpha = \dfrac{(P_b^* - C_a)}{(P_b^* - C_b)}$, then $Q_a^* = \overline{Q}_a^*$, or

if $\alpha > \dfrac{(P_b^* - C_a)}{(P_b^* - C_b)}$, then $Q_a^* < \overline{Q}_a^*$

Condition 2 :

If $\alpha < \dfrac{(P_a^* - C_a)}{(P_a^* - C_b)}$, then $Q_b^* > \overline{Q}_b^*$, or if

$\alpha = \dfrac{(P_a^* - C_a)}{(P_a^* - C_b)}$, then $Q_b^* = \overline{Q}_b^*$, or

if $\alpha > \dfrac{(P_a^* - C_a)}{(P_a^* - C_b)}$, then $Q_b^* < \overline{Q}_b^*$

From these conditions, the optimal prices and quantities of both products can be any values when $0.0 \le \alpha < 1.0$ and $C_a > C_b$, depending on the parameters and the substitution rate (α) while those with $\alpha=1.0$ have restricted values, i.e. if $P_a^* > \overline{P}_a^*$ then $P_b^* < \overline{P}_b^*$. When $0.0 \le \alpha < 1.0$ and $C_a > C_b$, it is also possible that the optimal prices have : $\left(P_a^* \ge \overline{P}_a^* \text{ and } P_b^* \ge \overline{P}_b^*\right)$, or $\left(P_a^* \le \overline{P}_a^* \text{ and } P_b^* \le \overline{P}_b^*\right)$. The optimal prices and quantities are heavily dependent on the unit production costs. <Table 2.1> summarizes the optimal solutions as functions of production costs. We see that the substitution rate (α) affects on the optimal solutions strongly. If $0.0 \le \alpha < 1.0$ and the unit production cost for the high-priced product is greater than that of the low-priced product, then we have different results from those with $\alpha=$ 1.0 When $C_a > C_b$ and $\alpha=1.0$, the optimal set of prices (P_a^*, P_b^*) can be one of the three possible cases (2.8). However, when $0.0 \le \alpha < 1.0$, it is more complex to understand the behavior of the optimal prices as a function of α.

We have carefully considered the three different cases of production costs in <Table 2.1>. In most manufacturing problems, we estimate that if $P_a > P_b$ then $C_a > C_b$. Our analysis demonstrates that if $C_a = C_b$ or $C_a < C_b$, the optimal prices and production levels have isolated solutions ; the optimal quantity of product A with substitution is always greater than that without substitution and the optimal quantity of product B with substitution is always less than that

〈Table 2.1〉 Optimal price and production decisions as a function of production cost

(α= 1.0)

Case 1 : $C_a < C_b$	Case 2 : $C_a = C_b$	Case 3 : $C_a > C_b$
Optimal production level : $Q_a^* > \overline{Q}_a^*$, $Q_b^* < \overline{Q}_b^*$ Optimal price : $P_a^* < \overline{P}_a^*$, $P_b^* > \overline{P}_b^*$	Optimal production level : $Q_a^* = \overline{Q}_a^*$, $Q_b^* = \overline{Q}_b^*$ Optimal price : $P_a^* < \overline{P}_a^*$, $P_b^* > \overline{P}_b^*$	Optimal production level : $(Q_a^* < \overline{Q}_a^*,\ Q_b^* > \overline{Q}_b^*)$ Optimal price : $(P_a^* < \overline{P}_a^*,\ P_b^* > \overline{P}_b^*)$ or $(P_a^* = \overline{P}_a^*,\ P_b^* = \overline{P}_b^*)$ or $(P_a^* > \overline{P}_a^*,\ P_b^* < \overline{P}_b^*)$

($0.0 \leq \alpha < 1.0$)

Case 1 : $C_a < C_b$	Case 2 : $C_a = C_b$	Case 3 : $C_a > C_b$
Optimal production level : $Q_a^* > \overline{Q}_a^*$, $Q_b^* < \overline{Q}_b^*$ Optimal price : $P_a^* < \overline{P}_a^*$, $P_b^* > \overline{P}_b^*$	Optimal production level : $Q_a^* > \overline{Q}_a^*$, $Q_b^* < \overline{Q}_b^*$ Optimal price : $P_a^* < \overline{P}_a^*$, $P_b^* > \overline{P}_b^*$	Optimal production level can be any value : $(Q_a^* < \overline{Q}_a^*$ or $Q_a^* = \overline{Q}_a^*$ or $Q_a^* > \overline{Q}_a^*)$ $(Q_b^* < \overline{Q}_b^*$ or $Q_b^* = \overline{Q}_b^*$ or $Q_b^* > \overline{Q}_b^*)$ Optimal price can be any value : $(P_a^* < \overline{P}_a^*$ or $P_a^* = \overline{P}_a^*$ or $P_a^* > \overline{P}_a^*)$ $(P_b^* < \overline{P}_b^*$ or $P_b^* = \overline{P}_b^*$ or $P_b^* > \overline{P}_b^*)$

without substitution. Also, from page 8~9, we need some restrictive conditions for the existence of the optimal prices with $P_a > P_b$. Example 2.1 presents numerical examples to illustrate the impact of α where $C_a > C_b$ and $C_a = C_b$. The optimal prices and production quantities are unique and are affected by α and L. However, if $C_a < C_b$, it violates the condition for $P_a^* > P_a^*$. We do not need to take much care of the two cases : $C_a = C_b$ or $C_a < C_b$.

Example 2.1

Let C_a =200, C_b =200, A_a =4,250, B_a =10, A_b = 1,440 and B_b = 5 for the contribution maximization problem (2.6). For the case of no-substitution, the optimal prices and production levels are : $\overline{P}_a^*$= 312.5, $\overline{P}_b^*$= 244.0, $\overline{Q}_a^*$= 1,125, $\overline{Q}_b^*$= 220, and $\overline{\pi}^*$ = 136,242.50. When we do not consider substitution, the profit from product A is 126,562.5, and the profit from product B is 9,680. <Table 2.2> summarizes the optimal solutions as a function of α(0 $\leq\alpha\leq$1) for L= 1 and 5 where $C_a = C_b$. As suggested in <Table 2.1>, when α= 1.0 $P_b^* > \overline{P}_b^*$, $P_a^* < \overline{P}_a^*$, $Q_a^* = \overline{Q}_a^*$, and $Q_b^* = \overline{Q}_b^*$. When 0.0 $\leq\alpha<$1.0, $P_a^* < \overline{P}_a^*$, $P_b^* > \overline{P}_b^*$, $Q_a^* > \overline{Q}_a^*$, and $Q_b^* < \overline{Q}_b^*$. Compared with the results which do not consider substitution, the benefit from product A becomes less whereas the benefit from product B becomes more as α increases. The optimal price of product B is greater than that without substitution while the optimal production quantity of product B with substitution is always less than that without substitution. When α is small the benefit from product B (= profit from product B with substitution-profit from product B without substitution) might be negative even though the optimal price of product B is greater than that without substitution. In case $C_a = C_b$ the optimal

〈Table 2.2〉 Numerical results for the optimal solutions as a function of L and α where $C_a = C_b$

($L=1$)

α	0.0	0.1	0.2	0.3	0.4	0.5
P_a^*	304.74	304.99	305.24	305.48	305.74	305.98
P_b^*	254.47	254.46	254.45	254.44	254.45	254.45
Q_a^*	1,152.33	1,149.54	1,146.79	1,144.15	1,141.30	1,138.67
Q_b^*	167.65	172.91	178.01	183.17	188.32	193.52
$P_a^*-P_b^*$	50.27	50.53	50.79	51.04	51.29	51.53
$\alpha L(P_a^*-P_b^*)$	0.00[1]	5.03	10.16	15.31	20.52	25.76
$(P_a^*-C_a)Q_a^*$	120,695.04	120,690.20	120,688.18	120,684.94	120,681.06	120,676.25
$(P_b^*-C_b)Q_b^*$	9,131.90	9,416.68	9,692.65	9,971.77	10,254.02	10,537.16
Benefit from product A[2]	-5,867.46	-5,872.30	-5,874.32	-5,877.56	-5,881.44	-5,886.25
Benefit from product B	-548.10	-263.32	12.65	291.77	574.02	857.16
π^*	129,826.94	130,106.88	130,380.83	130,656.71	130,935.08	131,213.41
$\pi^*-\bar{\pi}^*$	-6,415.56	-6,135.62	-5,861.67	-5,585.79	-5,307.42	-5,029.09

α	0.6	0.7	0.8	0.9	1.0
P_a^*	306.23	306.47	306.73	306.98	307.23
P_b^*	254.46	254.48	254.49	254.51	254.54
Q_a^*	1,135.93	1,133.30	1,130.46	1,127.73	1,125.00
Q_b^*	198.76	204.05	209.34	214.67	220.00
$P_a^*-P_b^*$	51.77	51.99	52.24	52.47	52.69
$\alpha L(P_a^*-P_b^*)$	31.06	36.39	41.79	47.47	52.69
$(P_a^*-C_a)Q_a^*$	120,669.84	120,662.45	120,654.00	120,644.56	120,633.75
$(P_b^*-C_b)Q_b^*$	10,824.47	11,116.64	11,406.94	11,701.66	11,998.80
Benefit from product A	-5,892.66	-5,900.05	-5,908.50	-5,917.94	-5,928.75
Benefit from product B	1,144.47	1,436.64	1,726.94	2,021.66	2,318.80
π^*	131,494.31	131,778.09	132,060.94	132,346.22	132,632.55
$\pi^*-\bar{\pi}^*$	-4,748.19	-4,464.41	-4,181.56	-3,896.28	-3,609.95

주) 1 : when $\alpha=0.0$, there is no demand leakage.
2 : benefit = profit from product A with substitution-from product A profit without substitution

quantity of product A with substitution should always be greater than that without substitution. The assumption of $C_a = C_b$ makes our model somewhat restrictive.

<Table 2.2> continued

(L=5)

α	0.0	0.1	0.2	0.3	0.4	0.5
P_a^*	289.82	290.64	291.47	292.34	293.21	294.10
P_b^*	288.90	285.32	282.41	280.01	278.03	276.38
Q_a^*	1,347.2	1,317.0	1,289.95	1,264.95	1,242.00	1,220.40
Q_b^*	0.00	16.06	37.07	58.45	80.21	102.40
$P_a^*-P_b^*$	0.92	5.32	9.06	12.33	15.18	17.72
$\alpha L(P_a^*-P_b^*)$	0.00[1]	2.66	9.06	18.50	30.36	44.30
$(P_a^*-C_a)Q_a^*$	120,328.33	119,372.88	117,991.73	116,805.48	115,766.82	114,839.64
$(P_b^*-C_b)Q_b^*$	0.00	1,370.24	3,054.94	4,676.59	6,258.79	7821.31
Benefit from product A	-6,234.17	-7,189.62	-8,570.77	-9,757.02	-10,795.68	-11,722.86
Benefit from product B	-9,680.00	-8,309.76	-6,625.06	-5,003.41	-3,421.21	-1,858.69
π^*	120,328.33	120,743.12	121,046.67	121,482.07	122,025.61	122,660.95
$\pi^*-\bar{\pi}^*$	-15,914.17	-15,499.38	-15,195.83	-14,760.43	-14,216.89	-13,581.55

α	0.6	0.7	0.8	0.9	1.0
P_a^*	295.0	295.92	296.86	297.82	298.80
P_b^*	275.0	273.85	272.87	272.07	271.30
Q_a^*	1,200.00	1,180.45	1,161.45	1,143.05	1,125.00
Q_b^*	125.00	147.99	171.61	195.53	220.00
$P_a^*-P_b^*$	20.00	22.07	23.99	25.75	27.50
$\alpha L(P_a^*-P_b^*)$	60.00	77.24	95.96	115.88	137.50
$(P_a^*-C_a)Q_a^*$	114,000.00	113,228.76	112,498.05	111,813.15	111,150.00
$(P_b^*-C_b)Q_b^*$	9,375.00	1,0784.03	12,367.93	13,941.29	15,686.00
Benefit from product A	-12,562.50	-13,333.74	-14,064.45	-14,749.35	-15,412.50
Benefit from product B	-305.00	1,104.03	2,687.93	4,261.29	6,006.00
π^*	123,375.00	124,012.79	124,866.98	125,754.34	126,836.00
$\pi^*-\bar{\pi}^*$	-12,867.50	-12,229.71	-11,375.52	-10,488.16	-9,406.50

주) 1 : There is no substituted demand.

<Table 2.3> summarizes the optimal solutions for the same problem except that $C_a(=200)>C_b$ $(=180)$. The assumption of $C_a > C_b$ makes our model more realistic in that the optimal prices and the optimal quantities to product can be greater or less than that without substitution,

<Table 2.3> Numerical results for the optimal solutions as a function of L and α where $C_a > C_b$

($L=1$)

α	0.0	0.1	0.2	0.3	0.4	0.5
P_a^*	304.29	304.58	304.87	305.17	305.46	305.75
P_b^*	244.43	244.42	244.39	244.39	244.40	244.41
Q_a^*	1,147.20	1,144.45	1,140.82	1,137.63	1,134.34	1,131.16
Q_b^*	218.05	224.07	230.15	236.28	242.42	248.61
$P_a^*-P_b^*$	59.86	60.16	60.48	60.78	61.06	61.34
$\alpha L(P_a^*-P_b^*)$	0.00	6.02	12.10	18.23	24.42	30.67
$(P_a^*-C_a)Q_a^*$	119,641.49	119,686.58	119,637.79	119,644.55	119,627.50	119,620.17
$(P_b^*-C_b)Q_b^*$	14,048.96	14,434.59	14,819.36	15,214.07	15,611.85	16,012.97
Benefit from product A	-6,921.01	-6,875.92	-6,924.71	-6,917.95	-6,935.00	-6,942.33
Benefit from product B	-531.04	-145.41	239.36	634.07	1,031.85	1,432.97
π^*	133,690.45	134,121.17	134,457.15	134,858.62	135,239.35	135,633.14
$\pi^*-\bar{\pi}^*$	-7,452.05	-7,021.33	-6,685.35	-6,283.88	-5,903.15	-5,509.36

α	0.6	0.7	0.8	0.9	1.0
P_a^*	306.04	306.34	306.63	306.93	307.23
P_b^*	244.43	244.45	244.47	244.51	244.53
Q_a^*	1,127.99	1,124.71	1,121.54	1,118.28	1,115.0
Q_b^*	254.81	261.07	266.37	273.63	280.05
$P_a^*-P_b^*$	61.61	61.89	62.16	62.42	62.70
$\alpha L(P_a^*-P_b^*)$	36.97	43.32	49.73	56.18	62.70
$(P_a^*-C_a)Q_a^*$	119,612.06	119,601.66	119,589.81	119,577.68	119,561.45
$(P_b^*-C_b)Q_b^*$	16,417.41	16,825.96	17,172.87	17,845.40	18,071.63
Benefit from product A	-6,950.44	-6,960.84	-6,972.69	-6,984.82	-7,001.05
Benefit from product B	1,837.41	2,245.96	2,592.87	3,265.40	3,491.63
π^*	136,029.47	136,427.62	136,762.68	137,423.08	137,633.08
$\pi^*-\bar{\pi}^*$	-5,113.03	-4,714.88	-4,379.82	-3,719.42	-3,509.42

and there is no restrictive condition for the existence of the optimal prices. The optimal prices and production levels without substitution are $\bar{P}_a^*=312.5$, $\bar{P}_b^*=234.0$, $\bar{Q}_a^*=1,125$, $\bar{Q}_b^*=270$ and $\bar{\pi}^*=1.411\times10^5$. When we do not consider substitution effect, the profit from product A is 126,562.5, and the profit from product B is 14,580. As α increases, the optimal production levels

<Table 2.3> continued

($L=5$)

α	0.0	0.1	0.2	0.3	0.4	0.5
P_a^*	288.00	288.99	290.00	291.03	292.07	293.14
P_b^*	278.00	274.50	271.70	269.36	267.47	265.90
Q_a^*	1,320.00	1,287.65	1,258.50	1,231.35	1,206.3	1,182.40
Q_b^*	50.00	74.75	99.80	125.71	151.85	178.60
$P_a^*-P_b^*$	10.00	14.49	18.30	21.67	24.60	27.24
$\alpha L(P_a^*-P_b^*)$	0.00	7.245	18.30	32.51	49.20	68.10
$(P_a^*-C_a)Q_a^*$	116,160.00	114,587.97	113,265.00	112,089.79	111,064.04	110,128.74
$(P_b^*-C_b)Q_b^*$	4,900.00	7,063.88	9,151.66	11,233.46	13,282.32	15,341.74
Benefit from product A	-10,402.50	-11,974.53	-13,297.50	-14,472.71	-15,498.46	-16,433.76
Benefit from product B	-9,680.00	-7,516.12	-5,428.34	-3,346.54	-1,297.68	761.74
π^*	121,060.00	121,651.85	122,416.66	123,323.25	124,346.36	125,470.48
$\pi^*-\bar{\pi}^*$	-20,082.50	-19,490.65	-18,725.84	-17,819.25	-16,796.14	-15,672.02

α	0.6	0.7	0.8	0.9	1.0
P_a^*	294.23	295.34	296.47	297.63	298.80
P_b^*	264.61	263.55	262.70	261.97	261.40
Q_a^*	1,159.60	1,137.65	1,116.45	1,095.40	1,075.00
Q_b^*	205.81	233.52	261.58	290.62	320.00
$P_a^*-P_b^*$	29.62	31.79	33.77	35.66	37.40
$\alpha L(P_a^*-P_b^*)$	88.86	111.265	134.80	160.47	187.00
$(P_a^*-C_a)Q_a^*$	109,269.11	108,463.55	107,703.93	106,943.90	106,210.00
$(P_b^*-C_b)Q_b^*$	17,413.58	19,510.60	21,632.67	23,822.12	26,048.00
Benefit from product A	-17,293.39	-18,098.95	-18,858.57	-19,618.60	-20,352.50
Benefit from product B	2,833.58	4,930.60	7,052.67	9,242.12	11,468.00
π^*	126,682.69	127,974.15	129,336.60	130,766.02	132,258.00
$\pi^*-\bar{\pi}^*$	-14,459.81	-13,168.35	-11,805.90	-10,376.48	-8,884.50

change significantly compared with the optimal prices. Also, the profit from product A decreases while the profit from product B increases. From the examples, if we consider substitution effect the total profit is less than that without substitution. Therefore, managers should carefully decide the optimal prices.

The optimal production quantities are significantly affected slightly by α. As L increases firms should reduce the price of product A and increase the price of product B because the demand for product A decreases and the demand for product B increases. From <Tables 2.2> and <Tables 2.3> the expected optimal profit decreases as L increases, as anticipated. The optimal production quantities are sensitive to changes in C_a and C_b. The results show that the optimal prices are related to α with $\frac{\partial(P_a^* - P_b^*)}{\partial\alpha} > 0$. As α decreases, the expected profit decreases because a part of demand is lost. To mitigate this loss, $P_a^* - P_b^*$ should be decreased as α decreases.

3. Conclusion

We investigated the impact of demand substitution on the optimal pricing and production quantity levels for the two scenarios : symmetrical and asymmetrical demand substitution between the two segmented markets. We found some structural properties of the deterministic models with substitution as a result of price-differences, and managerial implications. Simultaneous decisions on pricing and production quantities of the both products would be more complicated. Our major contribution lies in investigating of the potential impact of asymmetrical demand substitution on prices and production levels. The noticeable difference is that the difference between the two optimal prices of the symmetrical substitution model are affected by the substitution factor, which is the cross elasticity L, not by the production cost of other market's product whereas those of the asymmetrical substitution model are influenced by both cross elasticity and the production costs of both markets' products. For the case of symmetrical demand substitution, the cost of producing one more product in market A does not affect the optimal price of product B and vice versa. However, decisions on the optimal solutions for the asymmetrical substitution are quite complex, and the cost of producing one more product in market A does affect the optimal price of product B. Our model helps decision makers understand the analytic procedures for the optimal solutions and decide the optimal solutions.

By clarifying the limitations of this paper, we suggest directions for future research. First, we examined the optimal decisions for a deterministic without considering a capacity constraint. A substitution model with a capacity constraint is more desirable, which is thought to provide additional and meaningful implications. Second, we articulated models based on single-period cases, which reflects a fact that manufacturers hardly take a long view in production planning. Based on the single-period result, this study awaits for an extended study including a multi-period scenario. Third, our analytical and numerical results are based on the assumption of demands being deterministic. Future research should lead to additional insights on models with stochastic demand and with general demand distributions.

References

[1] Bassok, Y., R. Anupindi, and R Akella., Infinite horizon multi-product inventory models with substitution. Operations Research Vol.40, No.5(1993), pp.999-1017.

[2] Bassok, Y., R. Anupindi, and R Akella., Single

-period multiproduct inventory models with substitution. Operations Research Vol.47, No.4 (1999), pp.632-642.

[3] Birge, J., Drogosz, J., and Duenyas, I., Setting single-period optimal capacity levels and prices for substitutable products. International Journal of Flexible Manufacturing Systems Vol.10, 1998, pp.407-430.

[4] Bitran, G., Caldentry, C., and Vial R., Pricing policies for perishable products with demand substitution. Working paper, MIT, Cambridge, USA, 2005.

[5] Gerchak, Y., A. Tripathy, and K. Wang, Co-production models with random functionality yields. IIE Transactions Vol.328, No.5(1996), pp.2391-403.

[6] Heath, T.B, and S.C. Chatterjee, Asymmetrical decoy effects on lower-quality versus higher-quality brands. Journal of Consumer Research Vol.22, No.3(1995), p.268.

[7] Hsu, A. and Y. Bassok, Single period substitution problems with single raw material input. Operations Research Vol.3, 1996, pp.267-275.

[8] Khouza, M., A. Mehrez, and G. Raibinowitz, A two-item newsboy problem with substitutability," International Journal of Production Economics Vol.44, 1996, pp.267-275.

[9] Kuyumcu, H.A. and I. Popescu, Deterministic price-inventory management for substitutable products. Journal of Revenue and Pricing Management Vol.4, No.4(2006), pp.354-367.

[10] Mulhern, F.J. and R.P. Leone, Implicit price bundling of retail products : A multi-product approach to maximizing store profitability. Journal of Marketing Vol.55, No.4(1991), p.63.

[11] Pasternack, B. and Z. Drezner, Optimal inventory policies for substitutable commodities with stochastic demand. Naval Research Logistics Vol.38, 1991, pp.221-240.

[12] Paul, K., Fuel, Switching, gasoline price controls, and the leaded-unleaded gasoline price differential. Journal of Environmental Economics and Management Vol.8, No.3 (1981), pp.287-305.

[13] Rao, U.S., Swaminathan, J.M., and Zhang, J., Multi-product inventory planning with downward substitution, stochastic demand and setup costs. IIE Transactions Vol.36, No.1(2005), p.59.

[14] Rajaram, K. and Tang, C.S., The impact of product substitution on retail merchandising. European Journal of Operational Research Vol.135, 2001, pp.582-601.

[15] Schmidt, G.M. and Druehl, C.T., Changes in product attributes and costs as drivers of new product diffusion and substitution. Production and Operations Management Vol.14 No.3(2005), pp.272-286.

[16] Sen, A. and Zhang, A., The Newsboy problem with multiple demand classes. IIE Transactions Vol.31, 1999, pp.431-444.

[17] Smith, S. and Agrawal, N., Management of multi-item retail inventory systems with demand substitution Vol.48 No.1(2000), p.50.

[18] Taheri, A. A., Oil shocks and the dynamics of substitution adjustments of industrial fuels in the US. Applied Economics Vol.26 No.8(1994), pp.751-757.

[19] Xu, X. and Hopp, W.J., Dynamic pricing and inventory control with demand substitution : The value of pricing flexibility. Working paper, Department of Industrial Engineering, Northwestern University, Evanston, IL USA, 2004.