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# Numerical Analysis of Seepage Induced Earthern Slope Failures

# 침투가 고려된 토사사면파괴의 수치해석

Seo, Young-Kyo[1] 서 영 교


## 요 지

침투에 의한 토사사면의 붕괴는 기상학적인 현상과 더불어 많은 양의 지하수의 유입에 의하여 발생한다. 토사사면 속에 존재하는 지하수의 흐름은 심각한 재산 및 인명손실의 잠재적인 요인으로 작용한다. 이러한 침투에 의한 토사사면의 안정성 문제는 지반공학에서 중요한 문제로 인식되어져 오고 있다. 본 연구는 기존의 유체 및 고체의 상호 작용에 대한 수치모델링 기법을 이용하여 침투에 의한 토사사면붕괴의 이해 및 이를 예측하기 위하여 수행되었다. 본 연구는 지반공학에서 중요히 다루는 사면안정화기법 연구에 효과적인 기술적 기여에 중점이 있다.

## Abstract

Seepage induced earthern slope failures occurs in concert with meteorological events when large quantities of groundwater are channeled into slopes through infiltration. The presence of flowing groundwater in earthern slopes can induce ground failures that result in significant property damage and potential loss of life. Seepage induced earthen slope failures represent a serious problem in geotechnical engineering. This research applies existing fluid-solid numerical modeling capabilities to the study and prediction of seepage induced earthen slope failures. Study of the targeted application holds potential for much needed advances in geotechnical engineering analysis technology which could be used to design more effective engineering slope stabilization interventions.


**Keywords** : Finite element method, Numerical method, Seepage, Slope stability

## 1. Introduction and Motivation

Pore water in soils can strongly influence the physical interactions between soil grains. Changes of pore pressures can directly impact the effective stresses which in turn impact both the shear strength and consolidation behaviors of soils. Moreover, the water in the void spaces of soils is not static, particularly in slopes. Therefore, the analysis of pore fluid seepage plays an important role in the solution of many geotechnical problems, especially those concerning the stability analysis of slopes and retaining structures. Stability analysis of slopes in which seepage is occurring involves solving boundary value problems for coupled field equations on spatial domains part of whose boundaries (the so called free surface or phreatic line) are unknown and remain to be determined as a part of the solution. The major difficulty in solving free-boundary problems numerically is associated with the nonlinearity introduced by the unknown free surfaces. Solving stability analysis problems for slopes in which unconfined seepage occurs involves mainly two difficulties. The first involves the fact that the soil can undergo inelastic deformation under gravity and seepage forces, while the second involves locating the equilibrium

1 Assistant Prof., Div. of Ocean Development Engrg., Korea Maritime Univ., yseo@hhu.ac.kr

free boundary of fluid in the soil. In the slope stability analyses considered in this paper only steady state seepage effects are considered, but transient effects could also be considered if one knew the changing hydrologic conditions.

The objective of this paper is first to develop a general methodology for solving the coupled slope stability analysis problem, which involves simultaneously solving the fluid pressure and velocity fields as well as the phreatic surface, and also the stress and deformation fields in the slope including the limit state failure mechanism. As an approximation to the fully coupled problem, a simplified two-step decoupling is then introduced, implemented and solved. The first step involves solving the unconfined seepage problem in the soil while assuming that the soil skeleton is rigid and does not deform. The fluid pore pressure field is then imposed on the slope and fixed while the slope stability problem is solved using the 'Strength Reduction Method' introduced in Swan and Seo (1999). The de-coupled procedure is then applied to assess the stability of slope systems in which steady state, unconfined seepage is occurring.

## 2. Literature Survey

Fluid flow through porous media occurs and plays an important role in many geotechnical problems. Due to the intrinsically irregular geometries associated with most of real problems, analytical solutions can be obtained only for relatively simple situations (Harr, 1962). The analysis of more complex cases can be carried out through numerical procedures based on various discretization techniques which are becoming increasingly popular and are replacing traditional procedures like hand-drawn graphical flow nets (Cedergren, 1967). Among numerical techniques, the finite element method and the boundary integral equation method are those most widely used. Confining attention here to the finite element approaches, Zienkiewicz et al. (1966) first presented the solution of confined seepage flow problems. Thereafter, adaptive mesh methods (Desai, 1972; Chen et al., 1973) and fixed domain methods (Desai, 1976; Lancy et al., 1987; Cividini et al., 1989) have been widely used to find free surfaces.

The adaptive mesh methods solve the seepage problem with a trial free surface, iteratively modifying the geometry of the saturated soil mesh so that the free surface coincides with element boundaries until a sufficient approximation of the correct shape of the flow domain is reached. In the first step, the mesh is usually defined between given physical boundaries and an assumed location of free surface, then the Laplace equation is solved for the domain below the trial free surface, then the flow domain is modified based on computed velocities at the free surface. With the modified flow domain and free-surface, the problem is then re-meshed and solved again. The iterative procedure continues until the flow domain converges. While this method is general and can define the free surface very accurately (Isaacs, 1980), it requires significant amounts of computational effort and potential human intervention in the re-meshing at each iteration. Moreover, this method, as pointed out by Oden and Kikuchi (1980), often presents stability problems during the iterative solution process, which in some cases leads to apparently non-uniqueness of solutions. Difficulties have also been encountered in problems involving inhomogeneous permeabilities. In order for these methods to work reliably, one must typically start with a mesh that very closely approximates the actual flow domain.

In order to overcome these difficulties, progress has been made in formulating and solving the problems on the entire domain. These so-called fixed domain methods do not change the geometry of the finite element mesh during the iterative solution process. Instead, the conditions on the free boundary are observed in the field quantities, which are then enforced within the spatial problem domain. Once a trial pressure field is computed, the free surface is then computed a posteriori as some suitable level set within this fixed domain. For the spatial region above the trial phreatic surface the permeability is then decreased (penalized) to model the lack of flow in this region.

## 3. Problem Statement for Unconfined Seepage in a Coupled Porous Medium

The description of the problem is shown in Figure 1.

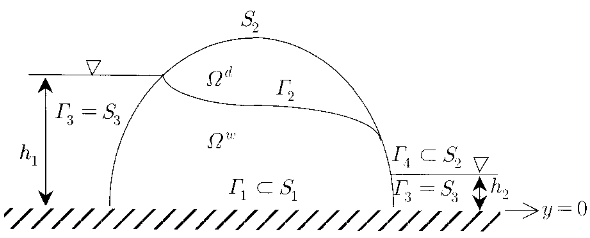


Fig. 1. The problem geometry

Let $\Omega \subset R^3$ represent the porous medium domain in 3-dimensional space. Let $\Omega^w \subset \Omega$ represent the saturated part of $\Omega$ which is the flow region, and let the complementary part, $\Omega^d$ represent a dry region. With capillarity, partial saturation and evaporation effects neglected, the soil domain $\Omega$ is decomposed strictly into fully saturated ($\Omega^w$) and fully dry ($\Omega^d$) regions. The external boundary of the porous medium domain consists of three parts: $S_1$ is the impermeable part, $S_2$ is the part in contact with the air, and $S_3$ is the part in contact with water reservoirs. The boundary of the saturated region $\Omega^w$ is assumed to have four parts; $\Gamma_1 \subset S_1$ is the impermeable part; $\Gamma_2 \subset \Omega$ is the internal free surface boundary; $\Gamma_3 = S_3$ is the boundary with the water reservoir; and $\Gamma_4 \subset S_2$ is the seepage face [Refer from (Borja, 1991)].

## 3.1 Strong Form

With steady state seepage of an incompressible fluid assumed, the coupled boundary value problem is stated as follows: Find the skeletal displacement field $u^s(\Omega \times T$ $R^3)$ and the pressure field $p(\Omega \times T \rightarrow R^3)$ such that the following equations are satisfied:

Balance of linear momentum of the fluid and solid media moving together;

$$\nabla \cdot (\sigma' - p1) + \rho b = 0 \text{ in } \Omega, \text{ and} \tag{3.1}$$

Conservation of mass for the fluid phase;

$$-\nabla \cdot \left[ k \cdot \nabla \left( \frac{p}{\gamma_w} + y \right) \right] + \nabla \cdot v^s = S \tag{3.2}$$

where $\sigma'$ is the effective stress tensor, $p$ is the pore pressure, $k$ is the permeability tensor, $b$ is the body force vector, $\rho$ is the total density of the soil mass, $1$ is the second order unit tensor and $S$ is a fluid source term.

The displacement and force boundary conditions for theis problem are stated as following:

$$u^s + \overline{u^s} \quad \text{on} \quad S_1 \tag{3.3}$$

$$n \cdot (\sigma' - p1) = \overline{h^s} \quad \text{on} \quad S_2 \cup S_3 \tag{3.4}$$

where $\overline{u^s}$ and $\overline{h^s}$ are prescribed displacements and surface tractions, respectively, and $n$ is the outward unit normal to $\Omega$.

The pressure and fluid flow boundary conditions in and on $\Omega$ are as follows;

$$p > 0 \text{ in } \Omega^w \;;\; p = 0 \quad \text{elsewhere} \tag{3.5}$$

$$n \cdot v^w = 0 \quad \text{on} \quad \Gamma_1 \tag{3.6}$$

$$p = 0 \quad \text{and} \quad n \cdot v^w = 0 \quad \text{on} \quad \Gamma_2 \tag{3.7}$$

$$p = \overline{p} \quad \text{on} \quad \Gamma_3 \tag{3.8}$$

$$p = 0 \quad \text{and} \quad -n \cdot v^w \leq 0 \quad \text{on} \quad \Gamma_4 \tag{3.9}$$

where, as an example associated with Figure 1,

$$\overline{p} = \begin{cases} \gamma_w (h_2 - y) & \text{on the right side of the dam} \\ \gamma_w (h_1 - y) & \text{on the left side of the dam} \end{cases} \tag{3.10}$$

The fluid velocity field $v^w$ is determined from Darcy's law as

$$v^w = -k \cdot grad\left(\frac{p}{\gamma_w} + y\right) \tag{3.11}$$

where $k$ is the permeability tensor, $\gamma_w$ is unit weight of water and $y$ is elevation head.

## 3.2 Penalized Problem and Matrix Equations

To define the weak form, a collection of trial solid displacement and fluid pressure solutions satisfying the two respective differential equations and boundary conditions are required, in addition to trial weighting functions which vanish on the regions where essential boundary conditions are imposed. Trial solutions for the skeletal displacement field, $u_i^s$ and the fluid pressure field, $p$ satisfy the following requirements

$$U_i = \left\{ u_i^s \;\; u_i \in H^1 ; \quad u_i = \overline{u_i^s} \quad on \quad S_1 \right\} \tag{3.12}$$

$$P=\{p \ p\in H^1; \quad p=\bar{p} \quad on \quad \Gamma_3\} \tag{3.13}$$

The virtual skeletal displacement functions $w_i$ and the virtual pressure function $q$ satisfy the following requirements

$$W_i=\{w_i \ w_i\in H^1; \quad u_i=0 \quad on \quad S_1\} \tag{3.14}$$

$$Q=\{q \ q\in H^1; \quad p=0 \quad on \quad \Gamma_3\} \tag{3.15}$$

Using these quantities, the variational equation of linear momentum balance (Eq. 3.1) can be written as follows

$$\int_\Omega w_i[\sigma'_{ij,j}-p_{,i}+\rho b_i]d\Omega$$
$$=-\int_\Omega w_{(i,j)}\sigma'_{ij}d\Omega+\int_\Omega w_{i,i}pd\Omega+\int_{S_2\cup S_3} w_i\bar{h}_i^s d\Gamma$$
$$+\int_\Omega w_i\rho b_i d\Omega=0 \tag{3.16}$$

where

$$\int_\Omega w_i p_{,i}\Omega=\int_{S_2\cup S_3} w_i pn_i d\Gamma-\int_\Omega w_{i,i}pd\Omega$$

In addition, the variational equation for mass balance of the fluid (Eq. 3.2) takes the form

$$\int_\Omega q[v_{i,i}^s+v_{i,i}^w-S]d\Omega$$
$$=-\int_\Omega q_{,i}v_i^s d\Omega+\int_\Gamma qn_i v_i^s d\Gamma-\int_{\Omega_w} q_{,i}v_i^w d\Omega-\int_\Omega qSd\Omega$$
$$=-\int_{\Gamma_4} qv_i^w n_i d\Gamma\le 0 \tag{3.17}$$

where the inequality implies that the pore fluid may be seeping outward across the unknown seepage face $\Gamma_4$. The domain of integration for the third term in Eq. (3.17) can be extended to the entire region $\Omega$ using Heavyside function, $H(p)$.

$$\int_\Omega q_{,i}k_{ij}\left[\frac{1}{\gamma_w}p_{,j}+H(p)y_{,j}\right]d\Omega\le 0 \tag{3.18}$$

where

$$H(p)=\lim_{\epsilon\to 0}\begin{cases} 0 & \text{if} \quad p\le\epsilon \\ \frac{p+\epsilon}{2\epsilon} & \text{if} \quad -\epsilon<p<\epsilon \\ 1 & \text{if} \quad p\ge\epsilon \end{cases} \tag{3.19}$$

the above inequality can be converted into equality using penalty function, $H(p_\epsilon)$.

$$\int_\Omega q_{,i}k[\frac{1}{\gamma_w}p_{\epsilon,i}+H(p_\epsilon)y_{,i}]d\Omega=0 \tag{3.20}$$

where $\epsilon>0$ and $p_\epsilon$ is defined by

$$p_\epsilon=p\ast H(p,\epsilon) \tag{3.21}$$

In the above equation, $\epsilon$ represents a small penalty parameter which smooths the step function. It is generally chosen as a function of mesh discretization size.

These coupled equations can be re-written in matrix form as follows

$$\begin{bmatrix}0 & 0\\ G & 0\end{bmatrix}\begin{bmatrix}v^s\\ p\end{bmatrix}+\begin{bmatrix}N^s(d^s)+M^w(p)\\ \Phi(p)\end{bmatrix}=\begin{bmatrix}F^{ext}\\ H^{ext}\end{bmatrix} \tag{3.22}$$

where

$$\begin{bmatrix}0 & 0\\ G & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\ \int_\Omega \overline{N_{A,i}}v_i^s d\Omega & 0\end{bmatrix} \tag{3.23}$$

$$\begin{bmatrix}N^s(d^s)+M^w(p)\\ \Phi(p)\end{bmatrix}=\begin{bmatrix}\int_\Omega B_{ji}\sigma'_j d\Omega-\int_\Omega N_{A,i}pd\Omega\\ \int_\Omega \overline{N_{A,i}}k_{ij}[\frac{1}{\gamma_w}p_{\epsilon,j}H_\epsilon(p_\epsilon)y_{,j}]d\Omega\end{bmatrix} \tag{3.24}$$

While the coupled field equations of equations (3.22)-(3.24) can be in principle be solved, such transient coupled problems are characterized by singular and non-symmetric stiffness matrices. To overcome such difficulties, direct simultaneous time integration of the coupled semi-discrete equation has been used (Prevost, 1983; Zienkiewicz, 1985) and the resulting algorithms have been shown to be unconditionally stable. However, such implementations have several limitations (Charbal et al., 1991). First, they require the development of special software modules to solve the coupled field equations. Second, result in very large matrix problems, especially for three-dimensional cases. For these reasons, staggered solution algorithms (Park et al., 1980; Zienkiewicz, 1988) have been suggested in which the skeletal displacement and pore pressure field equations are solved separately assuming that the field variables of the other subsystems are known (via a predictor) and temporarily frozen. There are many advantages to such staggered procedures: (1) modularity features which allow the coupled equations to be processed by separate program modules taking full advantage of specialized features and disciplinary expertise built into independently developed single-field analyzers; (2) resulting algorithmic structure which allows the set of analyzers to be synchronized to operate in sequential or parallel fashion. However, simple, straight forward implementations of staggered procedures are known to be at best only conditionally stable. Implicit integrations are used for the individual modules. To deal with this, various stabilization procedures have been proposed (Park et al., 1983; Park, 1983).

## 4. Problem Statement for De-coupled Seepage Analysis

In the previous section, the fully coupled seepage problem with free boundaries and the skeletal equilibrium problem with pore fluid pressure effects were developed, culminating in Eq. (3.22). While such a fully coupled system of equation can indeed be solved in principle, it would require the development of special purpose solution algorithms such as those noted. To avoid such complexities, a simplification is proposed here. First, the pore fluid pressure field equations are solved, including location of the free-surface, while assuming that the soil skeleton is rigid and does not deform ($i.e.\ v^s=0$). Once the pore pressure field is found in this manner, it is applied to the slope domain and used in finding the equilibrium deformation state of the slope. During the limit state structural analysis of the slope, it is assumed that deformations of the slope do no result in changes of the pore pressure field.

With the assumption that $v^s=0$, the strong form of the uncoupled equation governing conservation of the pore fluid is;

$$-\nabla\left[k\cdot\nabla\left(\frac{p_w}{\gamma_w}+y\right)\right]=0 \qquad (4.1)$$

which results in the following matrix systems of equations.

$$\int_\Omega \overline{N_{A,i}}k_{ij}\left[\frac{1}{\gamma_w}p_{\epsilon,j}H(p_\epsilon)y_{,j}\right]d\Omega=-\int_\Omega \overline{N_A}Sd\Omega \qquad (4.2)$$

Brezis et al (1978) showed the existence and uniqueness of the pressure $p$ field to the penalized problem, in the limit as $\epsilon$ tends to zero. In the first iteration of the solution procedure, the penalty function $H(p_\epsilon)$ is assumed to be unity throughout the entire domain. In regions of negative pressure the step function is applied, and the problem re-solved. In subsequent iterations, the stiffness matrix and forces vectors are pressure dependent. The iterative solution procedure terminates when the pressure field satisfies Eq. (4.2).

### 4.1 Slope Stability Analysis with De-coupled Seepage

In the following, de-coupled slope stability problems are solved with pore pressure fields resulting from seepage analysis by the strength reduction methods discussed in Swan and Seo (1999). The basic equilibrium field equations solved are

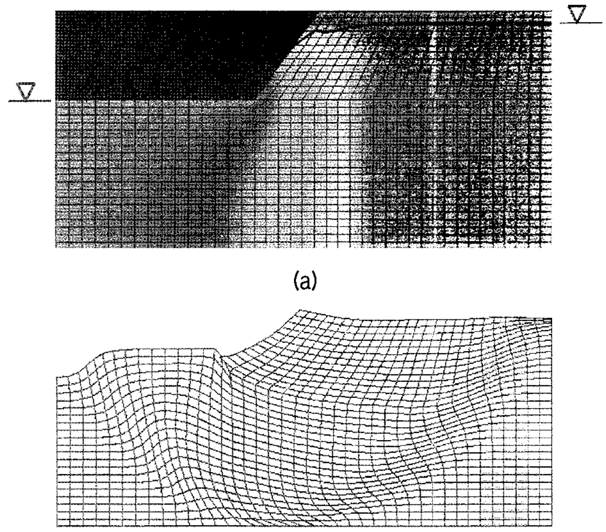
$$\nabla\cdot(\sigma'-p1)+\rho d=0 \quad \text{in} \quad \Omega \qquad (4.3)$$

where the pressure field $p(x)$ is solved from de-coupled seepage analysis and imposed on the slope domain; $\sigma'$ denotes the effective stress in the soil; and $\rho$ denotes the mass density of the soil (dry density above the phreatic surface and saturated density below the phreatic surface).

In the examples that follow the intention is to compare the stability characteristics of slopes both with and without seepage. The example problems include earthen slopes having both purely cohesive soils and purely frictional soils. The soil strength parameters used are the same as those listed in Table 1.

Table 1. Clay and sand material parameters used in slope analysis

| Material Parameter | Clay Values | Sandy Values |
|---|---|---|
| $\rho$ | $1800\,kg/m^3$ | $1800\,kg/m^3$ |
| $\mu$ | $2.00\,MPa$ | $12.0\,MPa$ |
| $K$ | $3.33\,MPa$ | $20.0\,MPa$ |
| $\alpha$ | $228\,kPa$ | $0.03\,kPa$ |
| $\lambda$ | $0.00\,kPa$ | $153\,kPa$ |
| $\beta$ | $3.48\times10^{-6}\,Pa^{-1}$ | $3.48\times10^{-6}\,Pa^{-1}$ |



(a)

(b)

Fig. 2. Undeformed configuration with free surface and deformed failure mechanisms for 30 meter clay slope with a response angle of 49°

### *Non-Frictional, Purely Cohesive Soils*

In this analysis of a purely cohesive soil slope, the free surface of the headwater is taken to be $5m$ below the ground surface and tailwater level corresponds to the toe level of the slope. The calculated free surface and pore pressure field are shown in Figure 2a. Slope stability analysis is then preformed and the deformed shape of the slope at the limit state is shown in Figure 2b. An analysis of the same slope was considered in Swan and Seo (1999) without seepage effects. While the mechanisms of failure are virtually the same, stability analysis without seepage yielded $(FS)_{sr} = 3.03$ while in this analysis $(FS)_{sr} = 2.02$. This represents a reduction of thirty three percent in the factor of safety.

### *Analysis with Purely Frictional Soils*

In these examples for various heights of frictional sandy slopes, the free surface is also calculated from a certain level of the slope as shown in Figure 3. The free surface head water was also taken as the same with non-frictional, purely cohesive soil. The deformed shapes and compared factor of safeties are shown in Figure 4. In general, the results indicate that the presence of flowing water in the slopes modeled can reduce the stability factors by between 18% and 22%, with the larger reductions corresponding to higher slopes.

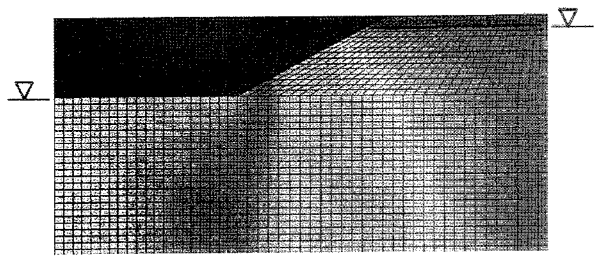


Fig. 3. Undeformed configuration of a 20° slope with free surface and piezometric head distribution

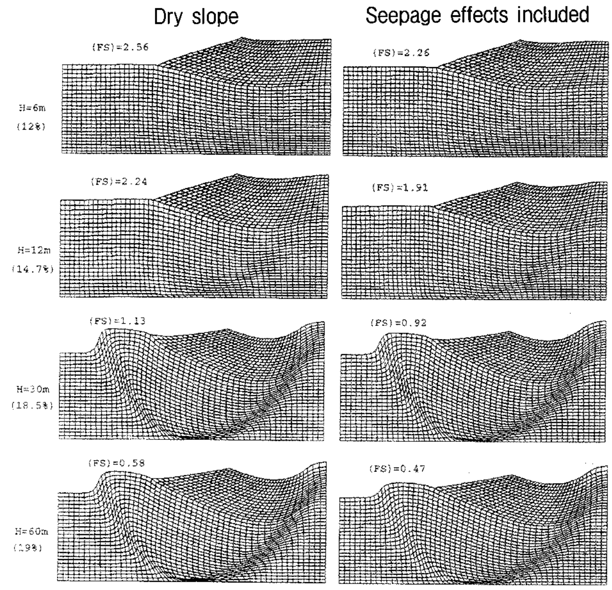



Fig. 4. Limit state mechanisms and stability factors computed for a 20° purely sand slope of varying heights and saturated conditions with seepage.

## 5. Summary and Closure

The strength reduction method was applied to the earthern slopes, in which active, unconfined steady state seepage is occurring. As an approximation, the problem is de-coupled from the fully coupled problem of slope stability analysis. In the first step of analysis, the unconfined seepage was performed for the pressure field in the slope. Then the fluid pore pressure field is imposed on the slope stability problem. In the example of the boundary problem some of published problems were computed and compared. The seepage induced slop analysis was then performed to compare the results of dry slope which was done in Swan and Seo (1999). The results show that the presence of water can reduce the factors of safety.

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