특수 적분해 경계요소법에 의한 2차원 및 3차원 동적 탄소성 응력 해석

Inelastic Transient Dynamic Analysis of Two- and Three-dimensional Stress Problems by Particular Integral Boundary Element Method

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요 지

본 연구는 2차원 및 3차원 동적 탄소성 응력 해석을 위한 특수 적분해 경계요소법의 공식 개발을 제시한다. 정적 탄성에 대한 기본식이 일반해를 구하는데 이용되었으며, 전체형상함수 개념을 이용하여, 변위율과 traction rate의 특수 적분해를 구함으로써 지배 방정식의 가속도 부분을 근사화시켰다. 시간 적분을 위하여 Houbolt 시적분 방법을 이용하였으며, Newton-Raphson 알고리즘을 이용하여 수치 연산을 행하였다. 제시된 공식에 따른 예제 해석을 통하여 그 방법의 유효성과 정확성을 설명하였다.

핵심용어 : 특수 적분해 경계요소법, 동적 탄소성 해석, Newton-Raphson 알고리즘

Abstract

The particular integral formulation for two(2D) and three(3D) dimensional inelastic transient dynamic stress analysis is presented. The elastostatic equation is used for the complementary solution. Using the concept of global shape function, the particular integrals for displacement and traction rates are obtained to approximate acceleration of the inhomogeneous equation. The Houbolt time integration scheme is used for the time-marching process. The Newton-Raphson algorithm for plastic multiplier is used to solve the system equation. Numerical results of four example problems are given to demonstrate the validity and accuracy of the present formulation.

Keywords : *BEM, particular integral, inelastic transient dynamic, newton-raphson algorithm*

1. Introduction

The Boundary Element Method(BEM) has developed into a powerful numerical method for solving inelastic transient dynamic stress problems(Banerjee, 1994 Banerjee and Butterfield, 1981 Beskos, 1995, 2003). Because of acceleration and initial stress terms in the governing equation, the direct application of the BEM to the inelastic transient dynamic stress problems generates domain integrals. By treating both plastic stresses and inertial forces by internal cells, the domain BEM technique was applied for 2D and 3D inelastic transient dynamic problems(Carrer and Telles, 1992; Coda and Venturini, 2000; Hatzigeorgiou and Beskos, 2002).

Some attempts have been made to eliminate the

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acceleration volume integral. The time-domain BEM formulation, using the elastodynamic fundamental solution and time-stepping techniques, has been developed for 2D and 3D inelastic transient dynamic stress analyses(Banerjee et al., 1989; Ahmad and Banerjee. 1990; Israil and Banerjee, 1992). Kontoni and Beskos(1993) proposed the volume integral conversion method for 2D inelastic transient dynamicstress problem by transforming the inertial volume integrals into surface integrals.

This paper presents the particular integral formulation for 2D and 3D inelastic transient dynamic stress problems. The particular integral method is a classical technique, obtaining the total solution as the sum of a complementary solution for the homogeneous part of the differential equation and a particular solution for the total governing inhomogeneous differential equation(Yang et al., 2002; Park and Banerjee, 2002a, 2002b, 2006, 2007).

In order to approximate the acceleration term of the inhomogeneous equation, a global shape function is considered and then the particular integrals for displacement and traction rates are derived for 2D and 3D formulations. The solution of elastostatic equation is used as the complementary solution. The Houbolt time integration scheme is used for the time-marching process. A standard predictor-corrector method, together with the Newton-Raphson algorithm for plastic multiplier(Gao and Davies, 2002), is used to solve the system equation. The numerical results for example problems are compared with those by ABAQUS/STANDARD(ABAQUS Inc., 2004) to demonstrate the validity and accuracy of the present formulation.

2. Particular Integral Formulation

The governing differential equation for inelastic transient dynamic stress analysis of a homogeneous, isotropic body in the absence of body force can be written in an incremental form as

$$(\lambda+\mu)\Delta u_{j,ji}+\mu\,\Delta u_{i,jj}=\rho\,\Delta\ddot{u}_i+\Delta\sigma^o_{ij,j} \tag{1}$$

where u_i is the displacement, $\ddot{u}_i$ is the acceleration, σ^o_{ij} is the initial stress, ρ is the mass density, and are Lame's constants, Δ denotes incremental quantity, commas represent differentiation with respect to spatial coordinates, and $i, j=1,2(3)$ for two(three) dimensions. The incremental initial stress rate is defined as

$$\Delta\sigma^o_{ij}=\Delta\sigma^e_{ij}-\Delta\sigma^{ep}_{ij} \tag{2}$$

where $\Delta\sigma^e_{ij}=D^e_{ijkl}\,\Delta\varepsilon_{kl}$, $\Delta\sigma^{ep}_{ij}=D^{ep}_{ijkl}\,\Delta\varepsilon_{kl}$, ε_{kl} is the strain and D^e_{ijkl}, D^{ep}_{ijkl} are the elastic and elastoplastic constitutive tensors respectively. The solution of Eq. (1) can be represented as a sum of complementary function u^c_i satisfying the homogeneous equation

$$(\lambda+\mu)\Delta u^c_{j,ji}+\mu\,\Delta u^c_{i,jj}=0 \tag{3}$$

and particular integral u^p_i satisfying the inhomogeneous equation

$$(\lambda+\mu)\Delta u^p_{j,ji}+\mu\,\Delta u^p_{i,jj}=\rho\,\Delta\ddot{u}_i+\Delta\sigma^0_{ij,j} \tag{4}$$

where superscripts c and p indicate complementary and particular solutions respectively. Then the total solutions for displacement, traction and stress rates can be expressed as

$$\Delta u_i=\Delta u^c_i+\Delta u^p_i \tag{5a}$$

$$\Delta t_i=\Delta t^c_i+\Delta t^p_i \tag{5b}$$

$$\Delta\sigma_{ij}=\Delta\sigma^c_{ij}+\Delta\sigma^p_{ij} \tag{5c}$$

where t^c_i, σ^c_{ij} and t^p_i, σ^p_{ij} are the complementary functions and particular integralsfor traction and stress rates, respectively.

2.1 Complementary solutions

The boundary integral equation related to the complementary functions, u^c_i and t^c_i, can be written as(Banerjee, 1994)

$$C_{ij}(\xi)\,\Delta u^c_i(\xi)=\int_S\left[G_{ij}(\mathbf{x},\xi)\Delta t^c_i(\mathbf{x})-F_{ij}(\mathbf{x},\xi)\Delta u^c_i(\mathbf{x})\right]dS(\mathbf{x}) \tag{6}$$

where G_{ij}, F_{ij} are the fundamental solutions for elastostatic equation and $C_{ij}(\xi)=1$, 0 and 1/2 depending on the point ξ being in the interior, outside or on a smooth boundary point respectively. The complementary function for the interior stress rate can be written by using the stress-strain relationship as(Banerjee, 1994)

$$\Delta\sigma_{ij}^{c}(\xi)=\int_{S}\left[G_{kij}^{\sigma}(\mathbf{x},\xi)\Delta t_{k}^{c}(\mathbf{x})-F_{kij}^{\sigma}(\mathbf{x},\xi)\Delta u_{k}^{c}(\mathbf{x})\right]dS(\mathbf{x}) \quad (7)$$

where G_{kij}^{σ}, F_{kij}^{σ} are the kernel functions for stresses.

2.2 Particular integrals

Eq. (4) contains acceleration term as well as the initial stress rate term. In order to eliminate the domain integrals due to the acceleration term, the concept of global shape function can be used. By introducing the global shape function $C_{ik}(\mathbf{x},\xi_n)$, the acceleration $\ddot{u}_i(\mathbf{x})$ can be approximated as

$$\Delta\ddot{u}_i(\mathbf{x})=\sum_{n=1}^{\infty}C_{ik}(\mathbf{x},\xi_n)\Delta\ddot{\phi}_k(\xi_n) \quad (8)$$

where $\ddot{\phi}_k(\xi_n)$ is the fictitious function.

Substitution of Eq. (8) into Eq. (4) gives

$$(\lambda+\mu)\Delta u_{j,ji}^{p}(\mathbf{x})+\mu\,\Delta u_{i,jj}^{p}(\mathbf{x})$$
$$=\rho\sum_{n=1}^{\infty}C_{ik}(\mathbf{x},\xi_n)\Delta\ddot{\phi}_k(\xi_n)+\Delta\sigma_{ij,j}^{0}(\mathbf{x}) \quad (9)$$

Then the corresponding particular integrals for displacement, stress and traction rates can be obtained using the following relations

$$\Delta u_i^p(\mathbf{x})=\sum_{n=1}^{\infty}U_{ik}(\mathbf{x},\xi_n)\Delta\ddot{\phi}_k(\xi_n) \quad (10)$$

$$\Delta\sigma_{ij}^p(\mathbf{x})=\sum_{n=1}^{\infty}S_{ikj}(\mathbf{x},\xi_n)\Delta\ddot{\phi}_k(\xi_n) \quad (11)$$

$$\Delta t_i^p(\mathbf{x})=\sum_{n=1}^{\infty}T_{ik}(\mathbf{x},\xi_n)\Delta\ddot{\phi}_k(\xi_n) \quad (12)$$

The details of the derivation for particular integrals related to the acceleration term is given in Appendix.

3. Numerical Implementation

The boundary integral equation (6) and stress equation (7) can be written in matrix form as

$$[G]\{\Delta t^c\}-[F]\{\Delta u^c\}=0 \quad (13)$$

$$\{\Delta\sigma^c\}=[G^\sigma]\{\Delta t^c\}-[F^\sigma]\{\Delta u^c\} \quad (14)$$

Considering the total solutions of Eq. (5) the complementary functions in Eqs. (13) and (14) can be eliminated as

$$[G]\{\Delta t\}-[F]\{\Delta u\}=[G]\{\Delta t^p\}-[F]\{\Delta u^p\}+[M]\{\Delta\sigma^o\} \quad (15)$$

$$\{\Delta\sigma\}=[G^\sigma]\{\Delta t\}-[F^\sigma]\{\Delta u\}-[M^\sigma]\{\Delta\sigma^o\}$$
$$-([G^\sigma]\{\Delta t^p\}-[F^\sigma]\{\Delta u^p\})+\{\Delta\sigma^p\} \quad (16)$$

$[M]$ and $[M^\sigma]$ are the matrices related to the volume integral of the initial stress term, such as

$$[M]=\int_V B_{ikj}(\mathbf{x},\xi)\Delta\sigma_{ik}^{o}(\mathbf{x})dV(\mathbf{x}) \quad (17)$$

$$[M^\sigma]=\int_V B_{klij}^{\sigma}(\mathbf{x},\xi)\Delta\sigma_{kl}^{o}(\mathbf{x})dV(\mathbf{x})+J_{klij}^{\sigma}\Delta\sigma_{kl}^{o}(\xi) \quad (18)$$

where

$$B_{ikj}(\mathbf{x},\xi)=\frac{1}{2}\left(G_{ij,k}+G_{ik,j}\right) \quad (19)$$

The details of Eqs. (17)~(19) can be found in the reference(Banerjee, 1994).

If a finite number of ξ_n are chosen, the particular integrals for displacement, traction and stress rates can be written as

$$\{\Delta u^p\}=[U]\{\Delta\ddot{\phi}\} \quad (20a)$$

$$\{\Delta t^p\}=[T]\{\Delta\ddot{\phi}\} \quad (20b)$$

$$\{\Delta\sigma^p\}=[S]\{\Delta\ddot{\phi}\} \quad (20c)$$

Considering the fictitious nodal values as

$$\{\Delta\ddot{\phi}\}=[C]^{-1}\{\Delta\ddot{u}\} \quad (21)$$

one can obtain the following equations

$$[G]\{\Delta t\}-[F]\{\Delta u\}=[N]\{\Delta\ddot{u}\}+[M]\{\Delta\sigma^o\} \tag{22}$$

$$\{\Delta\sigma\}=\lfloor G^\sigma\rfloor\{\Delta t\}-\lfloor F^\sigma\rfloor\{\Delta u\}-\lfloor N^\sigma\rfloor\{\ddot{u}\}-\lfloor M^\sigma\rfloor\{\Delta\sigma^o\} \tag{23}$$

where

$$[N]=\left([G][T]-[F][U]\right)[C]^{-1} \tag{24}$$

$$[N^\sigma]=\left([G^\sigma][T]-[F^\sigma][U]-[S]\right)[C]^{-1} \tag{25}$$

Several time integration schemes have been proposed to deal with equations of the type of Eq. (22) in FEM. In this study, the Houbolt method is used such that

$$\Delta\ddot{u}_{t+\Delta t}=\frac{1}{\Delta t^2}\left(2\,\Delta u_{t+\Delta t}-5\,\Delta u_t+4\,\Delta u_{t-\Delta t}-\Delta u_{t-2\Delta t}\right) \tag{26}$$

Considering the equilibrium equation (22) at time t+Δt one can obtain

$$\left(\frac{2}{\Delta t^2}[N]+[F]\right)\{\Delta u\}_{t+\Delta t}=[G]\{\Delta t\}_{t+\Delta t}+$$
$$\frac{1}{\Delta t^2}[N]\left(5\{\Delta u\}_t-4\{\Delta u\}_{t-\Delta t}+\{\Delta u\}_{t-2\Delta t}\right)-[M]\{\Delta\sigma^o\}_{t+\Delta t} \tag{27}$$

To solve the system equation (27), boundary variables as well as displacement at interior points are simultaneously treated as the unknown variables. So Eq. (25) is rewritten as

$$[N]_{b+I}=\left(\begin{bmatrix}[G]_{bb}\\ [G]_{Ib}\end{bmatrix}[T]_b-\begin{bmatrix}[F]_{bb} & [0]\\ [F]_{Ib} & [I]\end{bmatrix}\begin{bmatrix}[U]_b\\ [U]_I\end{bmatrix}\right)[C]^{-1}_{b+I} \tag{28}$$

where $[I]$ is identity matrix, subscript Idenotes interior values and subscript bindicates boundary values.

By substituting the boundary conditions at time $t+\Delta t$ and taking all the unknowns to the left-hand side, the final system of equation can be rewritten as

$$[A]\{\Delta x\}_{t+\Delta t}=\{\Delta y\}_{t+\Delta t}-[M]\{\Delta\sigma^o\}_{t+\Delta t} \tag{29}$$

where x is unknown vector of displacement and traction including displacement at interior points, y is a known vector and A is the coefficient matrix, which is obtained by rearranging Eq. (27) so that the known $\{\Delta u\}_{t+\Delta t}$ and $\{\Delta t\}_{t+\Delta t}$ values form one vector $\{\Delta y\}_{t+\Delta t}$ and the unknown values another vector $\{\Delta x\}_{t+\Delta t}$.

In a similar way, the combined form of the stress at boundary nodes and interior points can be written as

$$\begin{Bmatrix}\{\Delta\sigma\}_b\\ \{\Delta\sigma\}_I\end{Bmatrix}=\begin{bmatrix}[G^\sigma]_{bb}\\ [G^\sigma]_{Ib}\end{bmatrix}\{\Delta t\}_b-\begin{bmatrix}[F^\sigma]_{bb}\\ [F^\sigma]_{Ib}\end{bmatrix}\{\Delta u\}_b$$
$$-\begin{bmatrix}[0] & [0]\\ [N^\sigma]_{Ib} & [N^\sigma]_{II}\end{bmatrix}\{\Delta\ddot{u}\}_{b+I}-\begin{bmatrix}[0] & [0]\\ [M^\sigma]_{Ib} & [M^\sigma]_{II}\end{bmatrix}\{\Delta\sigma^o\}_{b+I} \tag{30}$$

Traction recovery method(Banerjee, 1994; Gao and Davies, 2002) is commonly used to evaluate the boundary stress and so the explicit calculation of $[G^\sigma]_{bb}$ and $[F^\sigma]_{bb}$ is not needed. Then Eq. (30) can be rearranged as

$$\{\Delta\sigma\}=[A^\sigma]\{\Delta x\}+\{\Delta y^\sigma\}-[M^\sigma]\{\Delta\sigma^o\} \tag{31}$$

where x is the unknown variables obtained by solving Eq. (29), y^σ denotes the vector of known boundary values. Eqs. (29) and (31) are nonlinear system due to the unknown initial stress vector σ^o. In this study, the Newton-Raphson algorithm for the plastic multiplier(Gao and Davies, 2002) is employed for elastoplastic solution. The detailed exaplanation can be found in the reference(Gao and Davies, 2002).

4. Numerical Examples

In order to test the validity of the present particular integral formulation and the resulting computer program in both two and three dimensions, four examples of application are solved. The results of displacement history are compared with those obtained from ABAQUS/STANDARD(2004). While the present program can be used for the different material models, such as Tresca, von Mises, Mohr-Coulomb and Drucker-Prager models, only the von Mises model is considered.

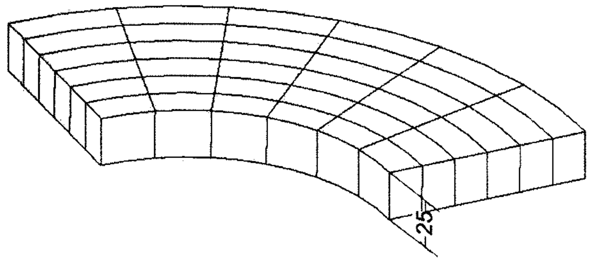

Figure 1 Modeling mesh(Example 1)

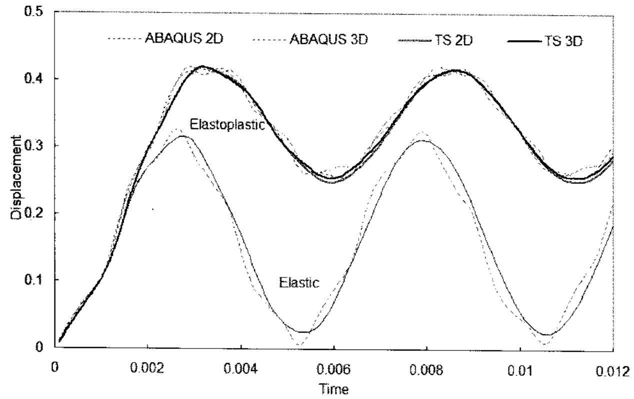

Figure 2 Radial displacement history at the inner surface(Example 1)

4.1 Example 1: A thick-walled cylinder subjected to internal pressure

The first example considers a thick-walled circular cylinder subjected to a suddenly applied uniform internal pressure(p=185). The inner radius a=100 and the outer radius b= 200. The body is modeled by using 96 quadratic boundary elements and 36 quadratic volume cells in 3D analysis(Fig. 1), while 32 quadratic boundary elements with 64 quadratic volume cells are used in 2D analysis. In 3D analysis, the thickness of cylinder is taken to be 25 unit. The material properties are: E=210000, ν =0.3, σ_Y =355 and ρ =7.85×10^{-6}. The time step Δt is 1×10^{-4}. Employing symmetry condition, only the positive octant of the cylinder is investigated, and symmetric constraints are imposed at the cutting face as the roller boundary condition. Numerical results for radial displacement history at the inner surface are shown in Fig. 2 for both 2D and 3D analyses. Generally, good agreement between the present study(TS) and ABAQUS can be seen for both 2D and 3D analyses.

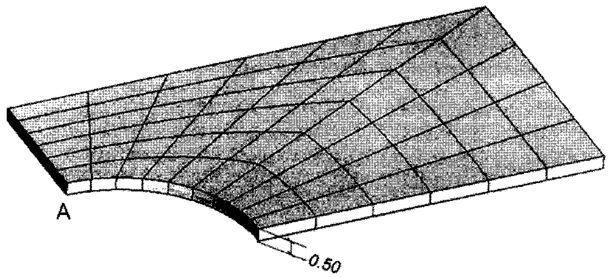

Figure 3 Modeling mesh(Example 2)

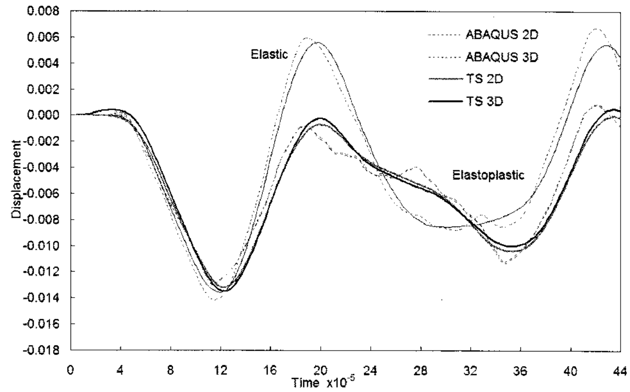

Figure 4 Displacement history at point A(Example 2)

4.2. Example 2: A rectangular strip with hole subjected to uniaxial tension

The second example considers the analysis of a rectangular strip with hole. Uniform tension(p=7.5 ×10^{5}) is suddenly applied along the free end. Due to the symmetry, only a quarter of the model is analyzed (Fig. 3), and symmetric constraints are imposed at the cutting face as the roller boundary condition. The modeling mesh contains 32 quadratic boundary elements with 60 quadratic volume cells(2D), and152 quadratic boundary elements with 60 quadratic volume cells(3D). The material properties are: E = 21×10^{8}, ν = 0.3, σ_Y = 235×10^{4}, E_t =5×10^{7} and ρ =7.85×10^{-3}. The time step Δt is 4×10^{-6}. Fig. 4 shows the numerical results of displacement history at point A. Good agreement of the results can be seen for 2D and 3D analyses, while apparently different results from ABAQUS can be noticed in elastoplastic range.

4.3 Example 3: A Cantilever beam subjected to shearload

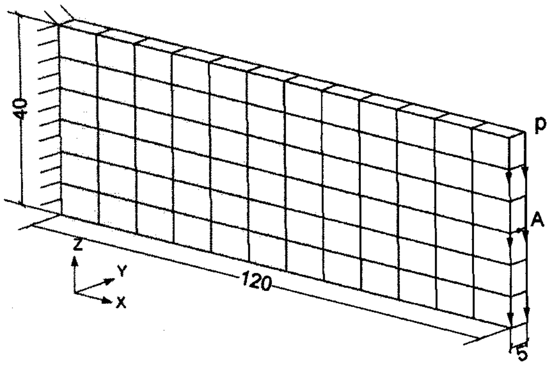

Figure 5 Modeling mesh(Example 3)

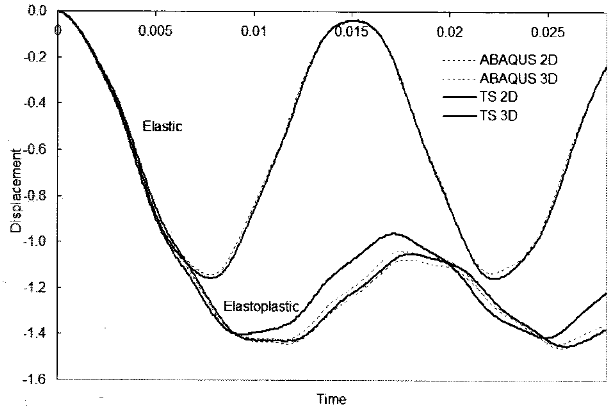

Figure 6 Vertical displacement history at point A(Example 3)

The third example considers the analysis of a cantilever beam subjected to suddenly applied surface traction(p=25) at the free end(Fig. 5). The modeling meshes consists of 286 quadratic boundary elements and 80 quadratic volume cells(3D), and 30 quadratic boundary elements and 50 quadratic volume cells(2D). The material properties used are: E=200000, ν =0.3, σ_Y =400 and ρ =7.85×10^{-6}. The time step Δt is taken to be 1×10^{-4}.

Fig. 6 shows the results of vertical displacement history at point A. Generally, good agreement between TS and ABAQUS can be seen for both 2D and 3D analyses.

4.4 Example 4: A hollow sphere subjected to internal pressure

The final example considers a thick-walled hollow sphere subjected to a suddenly applied uniform internal pressure(p=7.5). The inner radius a=1.0 and the outer radius b=2.0. The modeling mesh contains 192 quadratic boundary elements with 192 quadratic volume cells(Fig. 7). The material properties are: E=100, ν =0.3, σ_Y =10 and ρ =0.1. The time step Δt is taken to be 2.5×10^{-3}. Symmetric boundary conditions are employed and so only the positive octant of the hollow sphere is investigated. While somewhat different between TS and ABAQUS results can be seen in Fig. 8, the reason of the difference may be the use of different time integration schemes in the BEM and the FEM analyses.

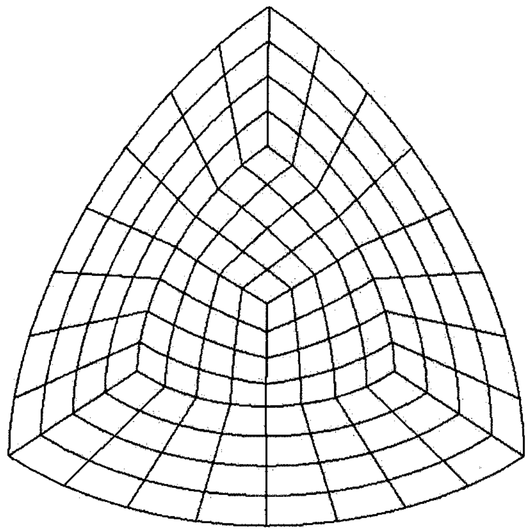

Figure 7 Modeling mesh(Example 4)

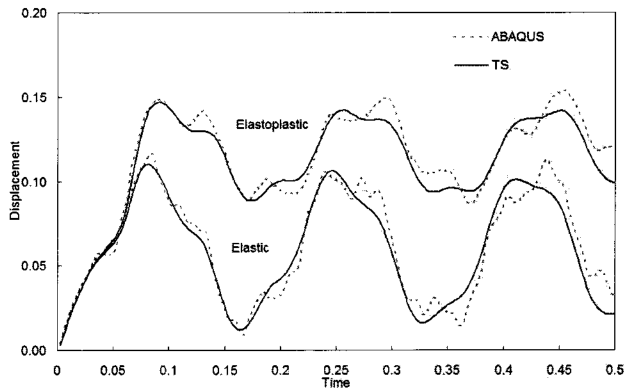

Figure 8 Radial displacement history at the inner surface(Example 4)

5. Conclusions

The application of BEM to 2D and 3D inelastic transient dynamic stress problems is described by using particular integrals. The elastostaticequation is used for the complementary solution and thus the computer program for the present formulations can beeasily implemented from any available program for elastostatic problems by including the Houbolt time integration scheme and the Newton-Raphson algorithm. While the volume integrals due to inelasticity effect are still necessary, the volume integrals for the inertial effect are eliminated.

The present formulation wasverified by comparing the results of four example problems with those by ABAQUS. Generally good agreement among the results was obtained. The present formulation needs to be extended to a multi-region form for a large scale practical application.

Appendix: Particular integrals for acceleration term

By using Galerkin vector F_i (Fung, 1965), the particular integral for displacement u_i^p related to the acceleration term can be expressed as

$$\Delta u_i^p(\mathbf{x}) = \frac{1-\nu}{\mu}\Delta F_{i,ll}(\mathbf{x}) - \frac{1}{2\mu}\Delta F_{l,li}(\mathbf{x}) \quad \text{(A1)}$$

where ν is the Poisson's ratio.

Substituting of Eq. (A1) into Eq. (4), without the consideration of the initial stress term, yields

$$(1-\nu)\Delta F_{i,jjll}(\mathbf{x}) = \rho\,\Delta\ddot{u}_i(\mathbf{x}) \quad \text{(A2)}$$

By introducing the global shape function $C_{ik}(\mathbf{x},\xi_n)$ in Eq. (8) and assuming

$$\Delta F_i(\mathbf{x}) = \sum_{n=1}^{\infty} E_{ik}(\mathbf{x},\xi_n)\,\Delta\ddot{\phi}_k(\xi_n) \quad \text{(A3)}$$

Eq. (A2) can be written as

$$(1-\nu)E_{ik,jjll} = \rho C_{ik} \quad \text{(A4)}$$

By using the following functions

$$C_{ik}(\mathbf{x},\xi_n) = \delta_{ik}\frac{\mu}{\rho}(A-r) \quad \text{(A5)}$$

$$E_{ik}(\mathbf{x},\xi_n) = \delta_{ik}\,\mu\,(E_1 A - E_2 r)\,r^4 \quad \text{(A6)}$$

and substituting Eqs. (A5) and (A6) into (A4), the relationships among coefficients can be derived as

$$E_1 = \frac{1}{8d(2+d)(1-\nu)}\ ;\ E_2 = \frac{1}{15(1+d)(3+d)(1-\nu)} \quad \text{(A7)}$$

where d is the dimensionality of the problem, that is, 2(3) for two (three) dimensions.

Then the particular integrals for displacement, stress and traction can be found as

$$\Delta u_i^p(\mathbf{x}) = \sum_{n=1}^{\infty} U_{ik}(\mathbf{x},\xi_n)\,\Delta\ddot{\phi}_k(\xi_n) \quad \text{(A8)}$$

$$\Delta\sigma_{ij}^p(\mathbf{x}) = \sum_{n=1}^{\infty} S_{ikj}(\mathbf{x},\xi_n)\,\Delta\ddot{\phi}_k(\xi_n) \quad \text{(A9)}$$

$$\Delta t_i^p(\mathbf{x}) = \sum_{n=1}^{\infty} T_{ik}(\mathbf{x},\xi_n)\,\Delta\ddot{\phi}_k(\xi_n) \quad \text{(A10)}$$

where

$$U_{ik}(\mathbf{x},\xi_n) = (U_1 A + U_2 r)\,\delta_{ik} r^2 + (U_3 A + U_4 r)\,y_i y_k \quad \text{(A11)}$$

$$S_{ikj}(\mathbf{x},\xi_n) = \delta_{ij} y_k\,(S_1 A + S_2 r) + (\delta_{ik} y_j + \delta_{jk} y_i)\,(S_3 A + S_4 r) + S_5\frac{y_i y_j y_k}{r} \quad \text{(A12)}$$

$$T_{ik}(\mathbf{x},\xi_n) = S_{ikj}(\mathbf{x},\xi_n)\,n_j(\mathbf{x}) \quad \text{(A13)}$$

$$U_1 = 2\{2(2+d)(1-\nu)-1\}E_1$$

$$U_2 = -\frac{5}{2}\{2(3+d)(1-\nu)-1\}E_2$$

$$U_3 = -4E_1 \quad U_4 = \frac{15}{2}E_2$$

$$S_1 = \lambda\{2U_1 + (1+d)U_3\} + 2\mu U_3\ ;$$

$$S_2 = \lambda\{3U_2 + (2+d)U_4\} + 2\mu U_4$$

$$S_3 = \mu(2U_1 + U_3)\ ;\ S_4 = \mu(3U_2 + U_4) \quad S_5 = 2\mu U_4$$

$n_j(\mathbf{x})$ = unit normal at x in the j-th direction.

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