PID Type Iterative Learning Control with Optimal Gains

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Abstract: Iterative learning control (ILC) is a simple and effective method for the control of systems that perform the same task repetitively. ILC algorithm uses the repetitiveness of the task to track the desired trajectory. In this paper, we propose a PID (proportional plus integral and derivative) type ILC update law for control discrete-time single input single-output (SISO) linear time-invariant (LTI) systems, performing repetitive tasks. In this approach, the input of controlled system in current cycle is modified by applying the PID strategy on the error achieved between the system output and the desired trajectory in a last previous iteration. The convergence of the presented scheme is analyzed and its convergence condition is obtained in terms of the PID coefficients. An optimal design method is proposed to determine the PID coefficients. It is also shown that under some given conditions, this optimal iterative learning controller can guarantee the monotonic convergence. An illustrative example is given to demonstrate the effectiveness of the proposed technique.

Keywords: Iterative learning control, monotonic convergence, optimal design, PID type ILC.

1. INTRODUCTION

Iterative learning control (ILC) is a technique to control the systems doing a defined task repetitively and periodically in a limited and constant time interval. Examples of such systems are robot manipulators that are required to repeat a given task with high precision, chemical batch processes or, more generally, the class of tracking systems. Motivated by human learning, the basic idea behind iterative learning control is to use information from the previous executions of the task in order to improve the performance from trial to trial in the sense that tracking error is sequentially reduced. Thus, the principle of ILCS is that, during the execution of control algorithm in the j th iteration, some data as errors are recorded. These are used by the learning algorithm in the execution $j+1$ for improving the control inputs and progressively reducing the output errors and increasing the performance of close loop system. Finally after a number of repeated trials, the system should obtain an appropriate control input, so that this input produces the desired output.

Since the iterative learning control concept was proposed [1] (widely credited to Arimoto), a very large number of approaches have been considered.

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Detailed literature reviews and recent developments on ILC research can be found in [2-4]. Early research efforts on ILC schemes were mainly on their analysis, without explicit design or synthesis procedures. However, the convergence conditions found in the literature are typically not sufficient for actual ILC applications. Therefore, in recent years increasing efforts have been made on the design issue of ILC. There are some efforts in [5,6] to use the parametric optimization approach to design the ILC algorithms. A novel model-based method was presented in [7], so that ILC based on a quadratic performance criterion was revisited and generalized for time-varying linear constrained systems with deterministic, stochastic disturbances and noises. The robust optimal design problem and convergence properties analysis of iterative learning control approaches are discusses in [8]. The LMI approach was studied for analysis and controller design for discrete linear repetitive systems in [9]. A new control framework for batch and repetitive processes was proposed in [10]. The presented framework provided a pertinent means to incorporate RFC (real-time feedback control) into ILC so that the performance of ILC was virtually separated from the effects of real-time disturbances. In [11] a model reference adaptive ILC strategy was presented for continuous time single-input single-output linear time-invariant systems with unknown parameters, performing repetitive tasks. A 2D (2-dimensional) systems theory based technique was offered in [12]. The problem of decentralized iterative learning control for a class of large scale interconnected dynamical systems was considered in [13]. An overview of the ILC technique, which can be used to improve tracking control performance in batch processes, was given by [14].

Despite the continual advances in control theory, PID (proportional plus integral and derivative) controller is still the most commonly used controller in the process control industry [15,16]. This is mainly due to its noticeable effectiveness, simple structure and its robustness. Hence, we are motivated to use the PID strategy in designing the iterative learning control schemes. For this purpose some efforts are done to use the PID type control approaches in iterative learning.

The monotonic convergence property of P-type iterative learning controller was studied [17]. In [18] a P-type ILC was designed for a class of discrete-time linear system, which guarantees the monotonic convergence. A P-type iterative learning control for systems that contain resonance was studied in [19]. In [20] a PID-type ILC algorithm was presented that has robustness property against initial state errors. The question "what's the use of the error integral in ILC updating law?" is answered in [21], through analysis and illustrations, both simulation and extreme cases studies, it is shown that the role of the tracking error integral term (I-component) in ILC updating scheme is helpful in achieving a monotonic convergence. In [22] a PD- type ILC was given for the trajectory tracking of a pneumatic X-Y table, experimental results show the under the disturbances, the PD- type ILC controller is superior to the P- type one and can effectively control the system to track the given trajectory. The PD- type ILC design of a class of affine nonlinear time-delayed systems with external disturbances was considered in [23], the simulation examples show the effectiveness of proposed scheme. Also another PID type ILC can be found in [24-30].

From above literature reviews, one can summarize the specific advantage role of each of PID modes in the ILC action as follows:

The P-component has a stabilizer role in the ILC system and causes monotonic convergence; the I-component rejects the effect of non-zero initial errors and increases the convergence rate, while D-term can reduce the effect of disturbance inputs.

For above merits of each of PID components in the ILC action, the PID controller is a popular scheme in the designing of ILCS. Hence, presenting any new PID type controller in ILC domain is a significant task.

The aim of this paper is presentation a new PID type iterative learning method, which causes the monotonic convergence. The paper is organized as follows. Section 2 defines the problem. In Section 3, the defined problem is solved and the convergence of its solution procedure is analyzed. Section 4 discusses the monotonic convergence and an optimal manner is presented for determination the controller parameters. An illustrative simulation test example is given in section 5. Finally Section 6 concludes the paper.

2. PROBLEM FORMULATION

Let an operation, or trial, of the system to be controlled be denoted by the subscript "j" and let time during a given trial be denoted by "i," where $i \in [0, M]$. Both i and j are integers. Hence, the underlying discrete-time, linear, time invariant plant can be described by:

$$\begin{cases} x_j(i+1) = Ax_j(i) + Bu_j(i) \\ y_j(i) = Cx_j(i) \\ x_j(0) = x_0 \\ i = 0,1,\dots,M \quad j = 0,1,\dots, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are input and output of the system respectively. A, B and C are real-valued coefficients with appropriate dimensions. Also x_0 is the system initial condition.

We define the problem of this paper as follows.

Consider (1) and make the following reasonable assumptions:

A1) The system initial condition x_0 is unknown.

A2) The scalar CB is nonzero.

A3) A desired output trajectory $y_d(i)$ is given.

Utilizing the PID strategy, determine the control input sequence of system (1), such that with increasing the number of repetition the error between $y_j(i)$ and $y_d(i)$ become small as possible so that the following tracking can be established:

$$\lim_{j\to\infty} (y_d(i) - y_j(i)) = 0 \quad \text{for } i = 1,2,\dots,M. \tag{2}$$

Comment 1: The assumption (A2) is a standard assumption in ILC design which guarantees the existence of the learning gains. This is not really a restriction, because it can be satisfied by choosing proper sampling period in discretizing the continuous -time systems.

3. PROBLEM SOLUTION METHOD

3.1. PID type iterative learning control

We consider the following updating law to determine the input of system (1):

$$\begin{aligned} &u_{j+1}(i) = u_j(i) + \Delta u_{j+1}(i) \\ &i = 0,1,\dots,M-1\ ,\ j = 0,1,\dots, \end{aligned} \tag{3}$$

where $\Delta u_{j+1}(i)$ is a modifier term.

Here, according to PID strategy $\Delta u_{j+1}(i)$ is chosen as follows:

$$\Delta u_{j+1}(i) = k_P e_j(i+1) + k_I \sum_{m=1}^{i+1} e_j(m) + k_D\left(e_j(i+1) - e_j(i)\right), \quad (4)$$

where

$$e_j(i) = y_d(i) - y_j(i) \text{ for } 1 \le i \le M \text{ and } e_j(0) \triangleq 0 \quad (5)$$

and k_P, k_I, and k_D are real constant gains (coefficients), we call them proportional, integration and derivative learning gains respectively.

According to the proposed control law, modifying term $\Delta u_{j+1}(i)$ is formed from summation of three expressions:

1) Proportional expression:

The value $k_P e_j(i+1)$ that is proportional value of the error in time $i+1$ from iteration j.

2) Summed (integral) expression:

The value $k_I \sum_{m=1}^{i+1} e_j(m)$ that is proportional of the summed errors from time 1 up to time $i+1$ in iteration j.

3) Differential expression:

The value $k_D\left(e_j(i+1) - e_j(i)\right)$ that is proportion to the difference of error in two sequential time i and $i+1$.

Now the problem is to determine the gains k_P, k_I and k_D so that the (2) can be satisfied.

3.2. Convergence analysis

From (1) the following relation is obtained easily:

$$Y(j) = GU(j) + G_0 x_0 \quad j = 0,1,..., \quad (6)$$

where $U(j)$ and $Y(j)$ are input and output vectors respectively in iteration j, and are as follows:

$$U(j) = \left[u_j(0) \;\; u_j(1) \;\; \cdots \;\; u_j(M-1)\right]^T, \quad Y(j) = \left[y_j(1) \;\; y_j(2) \;\; \cdots \;\; y_j(M)\right]^T, \quad (7)$$

where T denotes the Transpose.

Also, G and G_0 are following matrices:

$$G = \begin{bmatrix} g_1 & 0 & 0 & \dots & 0 \\ g_2 & g_1 & 0 & & 0 \\ g_3 & g_2 & g_1 & & 0 \\ \vdots & & & \ddots & \vdots \\ g_M & g_{M-1} & g_{M-2} & \cdots & g_1 \end{bmatrix}, \quad G_0 = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^M \end{bmatrix}, \quad (8)$$

where

$$g_k = CA^{k-1}B \quad \text{for } k = 1,2,...,M. \quad (9)$$

The elements of G are the standard Markov parameters of the system (1), and also these parameters can be extracted using the impulse response of system.

From (6) we get:

$$Y(j+1) = Y(j) + GV(j) \quad j = 0,1,..., \quad (10)$$

where

$$V(j) = U(j+1) - U(j). \quad (11)$$

From (10), the error dynamic of open-loop system is obtained as follows:

$$E(j+1) = E(j) - GV(j) \quad j = 0,1,..., \quad (12)$$

where $E(j)$ is the error vector in iteration j and it is defined below:

$$E(j) = Y_d - Y(j) = \left[e_j(1) \;\; e_j(2) \cdots e_j(M)\right]^T, \quad Y_d = \left[y_d(1) \;\; y_d(2) \cdots y_d(M)\right]^T. \quad (13)$$

By using relations (3) and (4) we have:

$$u_{j+1}(i) = u_j(i) + k_P e_j(i+1) + k_I \sum_{m=1}^{i+1} e_j(m) + k_D(e_j(i+1) - e_j(i)), \quad 0 \le i \le M-1. \quad (14)$$

Utilizing the definitions of $V(j)$ and $E(j)$, which are given respectively by (11) and (13), we can write the set of relations (14) as following compact form:

$$V(j) = \{(k_P + k_D)I + k_I F_1 - k_D F_2\} E(j) \quad \text{for } j = 0,1,..., \quad (15)$$

where $I \in \mathbb{R}^{M \times M}$ is identity matrix and F_1, $F_2 \in \mathbb{R}^{M \times M}$ are defined as follows:

$$F_1 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 1 & & 0 & 0 \\ \vdots & \vdots & & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & & 1 & 0 \\ 1 & 1 & 1 & \cdots & 1 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & & 0 & 0 \\ \vdots & \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}. \quad (16)$$

Substituting $V(j)$ from (15) into (12) yields:

$$E(j+1) = G_c E(j) \quad j = 0,1,..., \quad (17)$$

where G_c is a matrix given by:

$$G_c = I - (k_P + k_D)G - k_I GF_1 + k_D GF_2. \quad (18)$$

The relation (17) is the error dynamic of closed-loop system and it determines the variation of the error vector $E(j)$. Therefore to achieve (2) is depended to G_c and we achieve to this goal by appropriate selection of the gains k_P, k_I and k_D.

For analyzing the convergence of the obtained ILCS (iterative learning control system), a definition and a Theorem are presented.

Definition 1: The proposed ILCS is said to be convergent, if for any $E(0)$, that is for any initial input $u_0(i)$, it generates an input sequence $u_j(i)$ for system (1), such that (2) holds, meaning:

$$\lim_{j\to\infty} E(j) = 0. \quad (19)$$

Theorem: The presented ILCS is convergent if and only if the coefficients k_P, k_I and k_D are chosen in the following interval:

$$|1 - g_1(k_P + k_I + k_D)| < 1. \quad (20)$$

Proof: We define the operator Ψ from space $\mathbb{R}^M$ to space $\mathbb{R}^{M\times M}$ as follows:

$$\forall a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{bmatrix} : \Psi(a) = \begin{bmatrix} a_1 & 0 & 0 & \dots & 0 \\ a_2 & a_1 & 0 & & 0 \\ a_3 & a_2 & a_1 & & 0 \\ \vdots & & & \ddots & \vdots \\ a_M & a_{M-1} & a_{M-2} & \dots & a_1 \end{bmatrix}. \quad (21)$$

Thus the operator Ψ produces a low triangular Toeplitz matrix from vector a. We define this operator because that matrices I, G, F_1 and F_2, which form G_c according to (18), are kind of low triangular Toeplitz and we have:

$$I = \Psi(\alpha),\ F_1 = \Psi(\beta),\ F_2 = \Psi(\gamma),\ G = \Psi(g) \quad (22)$$

where $\alpha, \beta, \gamma, g \in \mathbb{R}^M$ are as follows:

$$\alpha = [1\ \ 0\ \ 0\ \ 0\ \ \cdots\ \ 0]^T,\ \beta = [1\ \ 1\ \ 1\ \dots\ 1]^T,$$
$$\gamma = [0\ \ 1\ \ 0\ \ 0\ \ \cdots\ \ 0]^T,\ g = [g_1\ \ g_2\ \ g_3 \dots g_M]^T. \quad (23)$$

It is easy to show that the operator Ψ has two following properties:

1) This operator is linear, that is for any two arbitrary vectors $a, b \in \mathbb{R}^M$ and two arbitrary scalars $c_1, c_2 \in \mathbb{R}$ we have:

$$\Psi(c_1 a + c_2 b) = c_1 \Psi(a) + c_2 \Psi(b). \quad (24)$$

2) If the multiplication $\odot$ between two vectors $a = [a_1\ a_2 \cdots a_M]^T$ and $b = [b_1\ b_2 \cdots b_M]^T$ is defined as follows:

$$a \odot b = \begin{bmatrix} a_1 b_1 \\ a_2 b_1 + a_1 b_2 \\ a_3 b_1 + a_2 b_2 + a_1 b_3 \\ \vdots \\ \sum_{m=1}^{l} a_m b_{l+1-m} \\ \vdots \\ \sum_{m=1}^{M} a_m b_{M+1-m} \end{bmatrix}, \quad (25)$$

then we will have:

$$\Psi(a \odot b) = \Psi(a)\Psi(b). \quad (26)$$

Therefore according to these properties and (22), from (18) matrix G_c can be written as the following form

$$\begin{aligned} G_c &= \Psi(\alpha) - (k_P + k_D)\Psi(g) \\ &\quad - k_I \Psi(g)\Psi(\beta) + k_D \Psi(g)\Psi(\gamma) \\ &= \Psi(\alpha) - (k_P + k_D)\Psi(g) \\ &\quad - k_I \Psi(g \odot \beta) + k_D \Psi(g \odot \gamma) \\ &= \Psi\big(\alpha - (k_P + k_D)g - k_I(g \odot \beta) + k_D(g \odot \gamma)\big) \end{aligned}$$

or

$$G_c = \Psi(g_c), \quad (27)$$

where

$$g_c = \alpha - (k_P + k_D)g - k_I(g \odot \beta) + k_D(g \odot \gamma). \quad (28)$$

Using above relation, vector g_c is calculated as follows:

$$g_c = \begin{bmatrix} g_{c1} & g_{c2} & g_{c3} & \cdots & g_{cM} \end{bmatrix}^T$$
$$= \begin{bmatrix} 1 - g_1 k_P - g_1 k_I - g_1 k_D \\ -g_2 k_P - (\sum_{i=1}^{2} g_i) k_I - (g_2 - g_1) k_D \\ -g_3 k_P - (\sum_{i=1}^{3} g_i) k_I - (g_3 - g_2) k_D \\ \vdots \\ -g_M k_P - (\sum_{i=1}^{M} g_i) k_I - (g_M - g_{M-1}) k_D \end{bmatrix}. \quad (29)$$

Considering the definition of Ψ, from (27) we conclude that G_c is a low triangular Toeplitz matrix which is formed by g_c, Thus:

$$G_c = \begin{bmatrix} g_{c1} & 0 & 0 & \cdots & 0 & 0 \\ g_{c2} & g_{c1} & 0 & & 0 & 0 \\ g_{c3} & g_{c2} & g_{c1} & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ g_{c(M-1)} & g_{c(M-2)} & g_{c(M-3)} & & g_{c1} & 0 \\ g_{cM} & g_{c(M-1)} & g_{c(M-2)} & \cdots & g_{c2} & g_{c1} \end{bmatrix}. \tag{30}$$

Considering the low triangular form of G_c, leads to the following characteristic polynomial for it:

$$\Delta_{G_c}(\lambda) = \det(\lambda I - G_c) = (\lambda - g_{c1})^M .$$

The dynamical equation (17) results that the necessary and sufficient condition for converging the presented iterative learning control system is that all eigenvalues of G_c must lie into the unit circle. That is:

$$|g_{c1}| < 1. \tag{31}$$

The above condition is equivalent to (20). □

Comment 2: According to assumption (A2) in the definition of our problem in section 2, since the scalar $g_1 \triangleq CB$ is nonzero we can find numerous real numbers for k_P, k_I and k_D which they satisfy inequality (20).

4. MONOTONIC CONVERGENCE ANALYSIS AND OPTIMUM SELCTION OF THE CONTROLLER PARAMETERS

4.1. Monotonic convergence

The monotonic convergence that means the better and better operation from trial to trial is defined as follows:

Definition 2: The proposed ILCS is said to be monotonic convergent, if for any $E(0)$, the following condition holds:

$$\begin{aligned} &\|E(j+1)\|_\lambda < \|E(j)\|_\lambda \quad \text{if } E(j) \neq 0, \\ &\|E(j+1)\|_\lambda = \|E(j)\|_\lambda \quad \text{if } E(j) = 0, \\ &\text{for } \lambda = 1,2,\infty \quad \text{and} \quad j = 0,1,2,...., \end{aligned} \tag{32}$$

where $\| \ \|_\lambda$ denotes the λ-norm.

In the previous section we obtained the necessary and sufficient condition for the converging of presented ILCS. But this condition doesn't guarantee the convergence be monotonic. For monotonic convergence the following Lemma is presented:

Lemma: The presented ILCS has monotonic convergence if:

$$\|g_c\|_1 < 1. \tag{33}$$

Proof: By taking of norm of two sides of (17), we get:

$$\|E(j+1)\|_\lambda \le \|G_c\|_\lambda \|E(j)\|_\lambda$$
$$\text{for } \lambda = 1,2,\infty \quad \text{and} \quad j = 0,1,\dots .$$

Thus for monotonic convergence, it is sufficient we have:

$$\|G_c\|_\lambda < 1 \quad \text{for } \lambda = 1,2,\infty. \tag{34}$$

But analyzing to confirm the above condition is not easy. Hence, we use the especial form of G_c and extract a sufficient condition for holding (34). Since G_c is a low triangular Toeplitz matrix which is produced by g_c, it is easy to show:

$$\|G_c\|_1 = \|G_c\|_\infty = \|g_c\|_1 . \tag{35}$$

On the other hand we have:

$$\|G_c\|_2^2 = \lambda_{\max}\left(G_c^T G_c\right), \tag{36}$$

where $\lambda_{\max}(\ )$ indicates the maximum eigenvalue.

In the reference [31] has been shown that for every symmetrical matrix Γ we have:

$$\lambda_{\max}(\Gamma) \le \|\Gamma\|_1 . \tag{37}$$

Therefore (36) and (37) result:

$$\begin{aligned} \|G_c\|_2^2 &= \lambda_{\max}\left(G_c^T G_c\right) \le \left\|G_c^T G_c\right\|_1 \le \left\|G_c^T\right\|_1 \|G_c\|_1 \\ &= \left(\|G_c\|_1\right)^2 \end{aligned}$$

or

$$\|G_c\|_2 \le \|G_c\|_1 . \tag{38}$$

Considering (35) and (38), we conclude that $\|g_c\|_1 < 1$ is a sufficient condition for (34), and thus for monotonic convergence of the presented ILCS from view point of three norms $\lambda = 1,2,\infty$. □

Comment 3: According to the above Lemma it is better if we can select suitable values for gains k_P, k_I and k_D so that the value of $\|g_c\|_1$ gets as possible as small. Using nonlinear numerical programming methods (such as optimization toolbox of MATLAB), one can determine k_P, k_I and k_D so that $\|g_c\|_1$ be minimize. But for two following reasons instead of minimizing $\|g_c\|_1$, here we minimize a upper bound of it:

1) Minimizing of $\|g_c\|_1$ doesn't give a closed form and explicit formula for parameters k_P, k_I and k_D, whereas, here we are fond to achieve a closed form formula for these parameters.

2) A critical and important problem in numerical optimization methods is the selection of initial values for variables. Hence, we can use the obtained values for k_P, k_I and k_D from minimizing of upper bound of $\|g_c\|_1$ as initial values in numerical minimizing of $\|g_c\|_1$.

4.2. Minimization of a $\|g_c\|_1$ upper bound

Since g_c has M components, it is easy to see that:

$$\|g_c\|_1 < \sqrt{M}\,\|g_c\|_2. \quad (39)$$

Therefore we determine k_P, k_I and k_D, so that the following index function be minimum:

$$\rho = \|g_c\|_2^2 = g_c^T g_c. \quad (40)$$

Using (29), we can write g_c as follows:

$$g_c = \alpha - HK, \quad (41)$$

where α is introduced by (23) and $K \in \mathbb{R}^3$, $H \in \mathbb{R}^{M\times 3}$ are defined below:

$$K = \begin{bmatrix} k_P \\ k_I \\ k_D \end{bmatrix}, H = \begin{bmatrix} g_1 & g_1 & g_1 \\ g_2 & g_1+g_2 & g_2-g_1 \\ g_3 & g_1+g_2+g_3 & g_3-g_2 \\ \vdots & \vdots & \vdots \\ g_M & g_1+g_2+\cdots g_M & g_M-g_{M-1} \end{bmatrix}. \quad (42)$$

Thus index ρ gets into form:

$$\rho = 1 - 2\alpha^T HK + K^T H^T HK. \quad (43)$$

Gradient (derivation) of ρ respect to K is as follows:

$$\frac{\nabla\rho}{\nabla K} = -2H^T\alpha + 2H^T HK. \quad (44)$$

Solving equation $\frac{\nabla\rho}{\nabla K} = 0$ yields the optimum value for K:

$$K^* = (H^T H)^{-1} H^T \alpha = g_1 (H^T H)^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \quad (45)$$

Optimal value for K exists if matrix $H^T H$ be invertible. This matrix is invertible if and only if H has full column rank. Since $H^T H$ is a symmetrical 3×3 matrix, therefore its inverse (if exists) will be symmetrical, which is shown as follows:

$$(H^T H)^{-1} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_2 & h_4 & h_5 \\ h_3 & h_5 & h_6 \end{bmatrix}. \quad (46)$$

Using (45) and (46) we obtain:

$$\begin{cases} k_P^* = g_1(h_1 + h_2 + h_3) \\ k_I^* = g_1(h_2 + h_4 + h_5) \\ k_D^* = g_1(h_3 + h_5 + h_6). \end{cases} \quad (47)$$

From (44) we get:

$$\frac{\nabla^2\rho}{\nabla K^2} = 2H^T H. \quad (48)$$

If K^* exists that is H has full column rank, then the symmetrical matrix $H^T H$ will be positive definite. So K^* causes index ρ gets global minimum.

Substituting K^* from (45) into (43) yields the global minimum of ρ as follows:

$$\begin{aligned} \rho^* &= 1 - \alpha^T H (H^T H)^{-1} H^T \alpha \\ &= 1 - g_1^2 (h_1 + 2h_2 + 2h_3 + h_4 + 2h_5 + h_6). \end{aligned} \quad (49)$$

If $\rho^* < M^{-1}$, then from (39) and (40) we have $\|g_c\|_1 < 1$, that is the following condition is a sufficient condition for monotonic convergence:

$$\rho^* < M^{-1}. \quad (50)$$

Satisfying the above condition depends on parameters of the system (1), that is $\{g_1, g_2, \ldots, g_M\}$ and M, or more precise to the Markov parameters of the system and duration of iteration. For any given system, we can calculate the value of ρ^* using (49). If the calculated value is less than M^{-1}, we can ensure that the proposed optimal approach to control the system causes monotonic convergence. Otherwise we cannot judge about monotonic convergence. For example suppose a repetitive system with following coefficient matrices:

$$A = \begin{bmatrix} 0.5 & 0 \\ -2 & 0.8 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 2 & -0.4 \end{bmatrix}$$

and duration of iteration $M = 100$.

For this system is calculated:

$$\rho^* = 0.004427.$$

It shows that $\rho^* < M^{-1} = 0.01$, hence controlling this system using the presented method certainly leads to monotonic convergence.

Comment 4: from (47) we have:

$$k_P^* + k_I^* + k_D^* = g_1(h_1 + 2h_2 + 2h_3 + h_4 + 2h_5 + h_6),$$

therefore

$$1 - g_1(k_P^* + k_I^* + k_D^*)$$
$$= 1 - g_1^2(h_1 + 2h_2 + 2h_3 + h_4 + 2h_5 + h_6).$$

The above term is equal with ρ^*, which is given by (49). Since according to (40) index ρ is a nonnegative function, we conclude its global minimum that is ρ^* is also nonnegative. On the other hand H has full column rank (the existence condition of K^*) and hence the symmetrical matrix $(H^T H)^{-1}$ is positive definite. Also α that is defined by (23) is a nonzero vector. Thus, $\alpha^T H(H^T H)^{-1} H^T \alpha$ is a positive number and consequently from (49) it is resulted that $\rho^* < 1$. Thus we have:

$$0 \le 1 - g_1(k_P^* + k_I^* + k_D^*) < 1.$$

Hence, the computed optimal values for controller parameters satisfy the constraint (20), which is needed for convergence of presented ILCS.

5. SIMULATION RESULTS

In this section, an example is given to demonstrate the effectiveness of the proposed method. As Fig. 1, we consider a DC motor which its armature is driven by a constant current source but its field winding current is variable. So that the motor rotational angle control is done by varying the voltage of the source connected to the field winding. The motor rotates a mechanical load. In this situation the state space equations of the motor are as follows [32]:

$$\dot{x}(t) = Ax(t) + Bv_f(t),$$
$$y(t) = Cx(t), \quad t \ge 0,$$

where

$$x(t) = \begin{bmatrix} i_f(t) & \omega(t) & \theta(t) \end{bmatrix}^T, \quad y(t) = \theta(t),$$

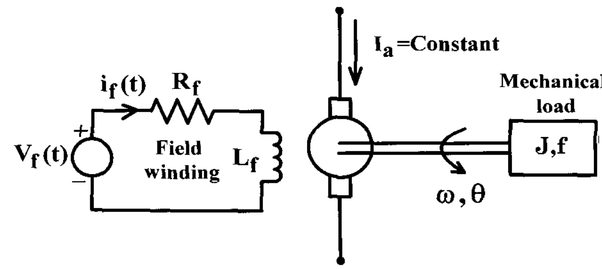


Fig. 1. DC motor with constant armature current.

$$A = \begin{bmatrix} -\frac{R_f}{L_f} & 0 & 0 \\ \frac{k_m}{J} & \frac{-f}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L_f} \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix},$$

and R_f, L_f are the field winding resistance and inductance respectively, k_m is the motor torque ratio, J and f are the mechanical load inertia momentum and friction ratio respectively. Also $v_f(t)$, $i_f(t)$ are respectively the field winding source voltage and current, $\omega(t)$ and $\theta(t)$ are the motor shaft rotational speed and angle respectively.

We purpose to determine the input voltage of motor $(v_f(t))$, so that the motor output $y(t)$ periodically follows the desired given signal $y_d(t)$ in time interval $[0, t_f]$, such that by increasing the iterations number, error between $y(t)$ and $y_d(t)$ vanishes. To determine the input voltage of motor, we use the proposed method in this paper. For this reason the state equations of the motor should be written as discrete-time form. We discretize the motor state equations by choosing the sampling period $T = 0.01$ sec and the following amounts for parameters:

$$R_f = 20\Omega, \quad L_f = 1\text{H}, \quad k_m = 100\,\tfrac{\text{Nm}}{\text{A}},$$
$$f = 0.5\,\tfrac{\text{Nms}}{\text{rad}}, \quad \text{J} = 2\,\tfrac{\text{Nms}^2}{\text{rad}}, \quad t_f = 12\,\text{sec}.$$

By considering variable j as the iteration number the obtained discrete state equations are as follows:

$$\begin{cases} x_j(i+1) = A_D x_j(i) + B_D V_{fj}(i) \\ y_j(i) = C_D x_j(i) \\ i = 0,1,\ldots,1200, \; j = 0,1,\ldots, \end{cases}$$

where the coefficient matrices are as below:

$$A_D = \begin{bmatrix} 0.8187 & 0 & 0 \\ 0.4526 & 0.9975 & 0 \\ 0.0023 & 0.0100 & 1 \end{bmatrix},$$

$$B_D = \begin{bmatrix} 0 \\ 0.0197 \\ 0.0211 \end{bmatrix}, \ C_D = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

The desired output trajectory, which is shown in Fig. 2, is chosen as follows:

$$y_d(t) = 10\left(1 + \sin(\frac{2\pi}{t_f}t - \frac{\pi}{2})\right) \quad 0 < t \le t_f,$$

$$t_f = 12 \text{ sec}$$

or

$$y_d(i) = 10\left(1 + \sin(\frac{2\pi}{M}i - \frac{\pi}{2})\right)$$

$$1 \le i \le M, \ M = \frac{t_f}{T} = 1200.$$

Motor input voltage at first iteration (say $j = 0$), that the controller hasn't any previous experience, is selected equal to 10. Simulation is done in two following cases.

Case 1: Non optimum selection of gains k_P, k_I and k_D.

In this situation the parameters k_P, k_I and k_D is selected so that just the convergence condition (20) is satisfied. According to the motor state equations (in discrete form), g_1 is obtained as:

$$g_1 = C_D B_D = 0.0211.$$

Then using (20) convergence condition for the obtained iterative learning control system will be:

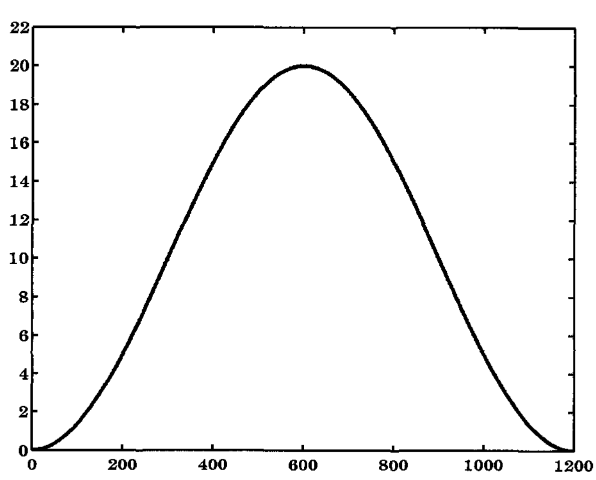


Fig. 2. The desired output trajectory $y_d(i)$.

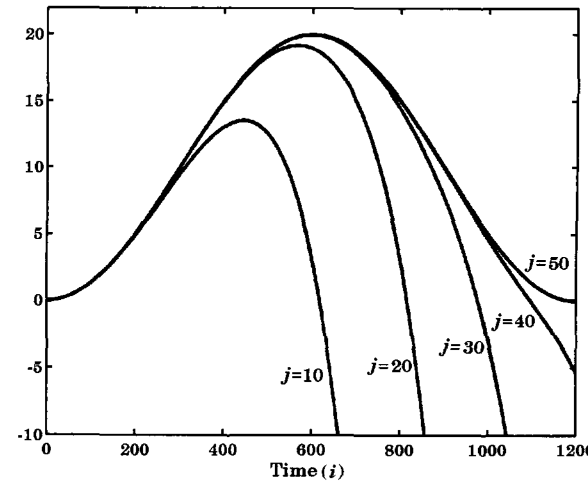


Fig. 3. The motor output (rotational angle) at the iterations $j = 10, 20, 30, 40, 50$ in case 1.

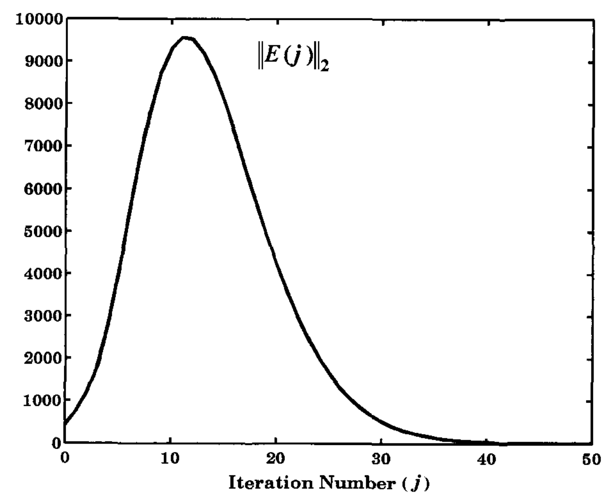


Fig. 4. The norm 2 of error vector with respect to the iteration number j in case 1.

$$0 < k_P + k_I + k_D < 94.7867.$$

We select

$$k_P = -0.2 \ , \ k_I = 0.0005 \ , \ k_D = 20.$$

This selection satisfies the convergence condition.

Simulation results are shown in Figs. 3 and 4. These results indicate that by increasing the iteration number, the output of the motor converges to the given desired trajectory. But Fig. 4 shows obviously that the convergence at this situation is not monotonic.

Case 2: Optimum selection of gains k_P, k_I and k_D.

In this case we select the coefficients k_P, k_I and k_D according to the (47) as follows:

$$k_P^* = -0.0988 \ , \ k_I^* = 1.2548 \times 10^{-4} \ , \ k_D^* = 47.1736.$$

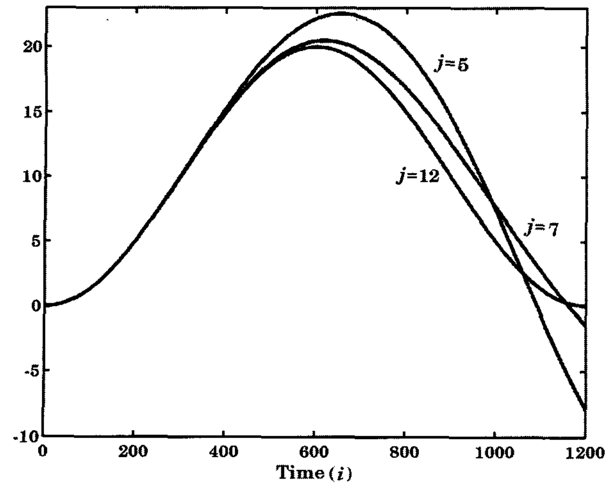


Fig. 5. The motor output (rotational angle) at the iterations $j = 5,7,12$ in case 2.

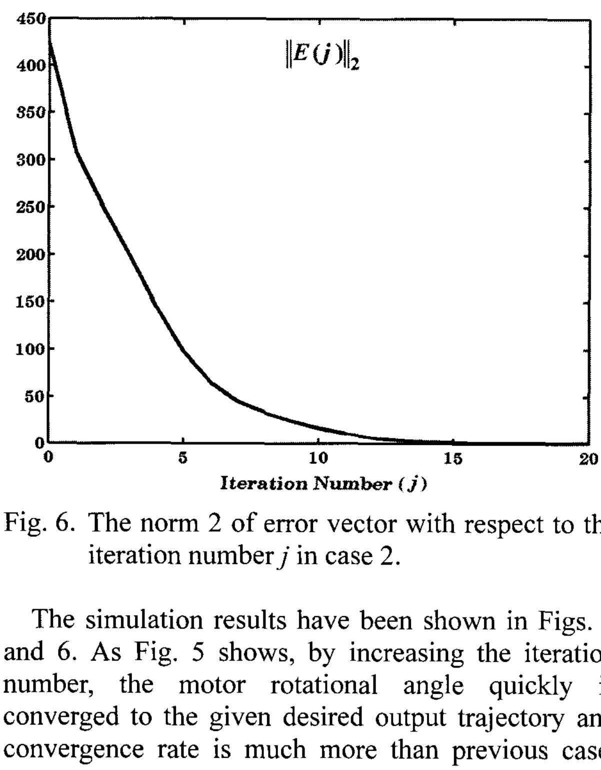


Fig. 6. The norm 2 of error vector with respect to the iteration number j in case 2.

The simulation results have been shown in Figs. 5 and 6. As Fig. 5 shows, by increasing the iteration number, the motor rotational angle quickly is converged to the given desired output trajectory and convergence rate is much more than previous case. Fig. 6 indicates that the convergence in this situation is monotonic. Therefore optimum selection of k_P, k_I and k_D using (47), not only increases the convergence rate but also causes the monotonic convergence.

6. CONCLUSION

In this paper, we proposed a straightforward extension of standard PID scheme to linear repetitive discrete time systems in order to improve the transient tracking performance through iterative learning. The convergence of the presented method was analyzed and its convergence condition achieved in terms of PID gains. Then we focus on the monotonic convergence of the presented learning process and an optimal design method for PID-type ILC scheme is proposed. A sufficient condition is obtained in terms of system Markov parameters for guarantee the using of proposed optimal PID approach to control the system causes the monotonic convergence. An illustrative simulation test example confirmed the effectiveness of the presented approach. However, the extending the proposed method to control the multi input and multi output (MIMO) repetitive systems merit further research.

REFERENCES

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123-140, 1984.

[2] Z. Bien and J.-X. Xu, *Iterative Learning Control - Analysis, Design, Integration and Applications*, Kluwer Academic Publishers, 1998.

[3] Y. Q. Chen and C. Wen, *Iterative Learning Control: Convergence, Robustness and Applications*, vol. LNCIS-248 of Lecture Notes series on Control and Information Science, Springer-Verlag, London, 1999.

[4] J.-X. Xu and Y. Tan, *Linear and Nonlinear Iterative Learning Control*, Springer-Verlag, New York, 2003.

[5] D. H. Owens and J. Hatonen, "Iterative learning control - An optimization paradigm," *Annual Reviews in Control*, vol. 29, no. 1, pp. 57-70, 2005.

[6] J. Hatonen, D. H. Owens, and K. Feng, "Basis functions and parameter optimization in high-order iterative learning control," *Automatica*, vol. 42, no. 2, pp. 287-294, 2006.

[7] J. H. Lee, K. S. Lee, and W. C. Kim, "Model-based iterative learning control with a quadratic criterion for time-varying linear systems," *Automatica*, vol. 36, no. 5, pp. 641-657, 2000.

[8] J.-X. Xu and Y. Tan, "Robust optimal design and convergence properties analysis of iterative learning control approaches," *Automatica*, vol. 38, no. 11, pp. 1867-1880, 2002.

[9] K. Galkowski, E. Rogers, S. Xu, J. Lam, and D. H. Owens, "LMIs-a fundamental tool in analysis and controller design for discrete linear repetitive processes," *IEEE Trans. on Circ. and Sys. -I: Fundamental Theory and Applications*, vol. 49, no. 6, pp. 768-778, 2002.

[10] I. Chin, S. J. Qin, K. S. Lee, and M. Cho, "A two-stage iterative learning control technique combined with real-time feedback for independent disturbance rejection," *Automatica*, vol. 40, no. 11, pp. 1913-1922, 2004.

[11] A. Tayebi, "Model reference adaptive iterative learning control for linear systems," *International Journal of Adaptive Control and*

Signal Processing, vol. 20, no. 9, pp. 475-489, 2006.

[12] T. Al-Towaim, A. D. Barton, P. L. Lewin, E. Rogers, and D. H. Owens, "Iterative learning control - 2D control systems from theory to application," *International Journal of Control*, vol. 77, no. 9, pp. 877-893, 2004.

[13] H. Wu, "Decentralized iterative learning control for a class of large scale interconnected dynamical systems," *Journal of Mathematical Analysis and Applications*, vol. 327, no. 1, pp. 233-245, 2007.

[14] J. H. Lee and K. S. Lee, "Iterative learning control applied to batch processes: an overview," *Control Engineering Practice*, vol. 15, no. 10, pp. 1306-1318, 2007.

[15] K. J. Astrom, H. Panagopoulos, and T. Hagglund, "Design of PI controllers based on non-convex optimization," *Automatica*, vol. 34, no. 5, pp. 585-601, 1998.

[16] R. Toscano, "Robust synthesis of a PID controller by uncertain multimodel approach," *Information Sciences*, vol. 177, no. 6, pp. 1441-1451, 2007.

[17] K. L. Moore, "An observation about monotonic convergence in discrete-time, P-type iterative learning control," *Proc. of the IEEE International Symposium on Intelligent Control*, pp. 45-49, MX, USA, 2001.

[18] K. L. Moore, Y. Q. Chen, and V. Bahl, "Feedback controller design to ensure monotonic convergence in discrete-time, P-type iterative learning control," *Proc. of the 4th Asian Control Conference*, pp. 440-445, Singapore, September 2002.

[19] J. D. Ratcliffe, J. J. Hatonen, P. L. Lewin, E. Rogers, T. J. Harte, and D. H. Owens, "P-type iterative learning control for systems that contain resonance," *International Journal of Adaptive Control and Signal Processing*, vol. 19, no. 10, pp. 769-796, 2005.

[20] K.-H. Park, Z. Bien, and D.-H. Hwang, "A study on the robustness of a PID-type iterative learning controller against initial state error," *International Journal of Systems Science*, vol. 30, no. 1, pp 49-59, 1999.

[21] Y. Q. Chen and K. L. Moore, "PI-type iterative learning control revisited," *Proc. of the American Control Conference*, pp. 1262-1267, Anchorage, Alaska, May 2002.

[22] C. K. Chen and J. Hwang, "PD-type iterative learning control for trajectory tracking of a pneumatic X-Y table with disturbances," *JSME International Journal, Series C*, vol. 49, no. 2, pp. 520-526, 2006.

[23] Z. Baolin, T. Gongyou, and Z. Shi, "PD-type iterative learning control for nonlinear time delay system with external disturbance," *Journal of Systems Engineering and Electronics*, vol. 17, no. 3, pp. 600-605, 2006.

[24] D. Wang, "On D-type and P-type ILC designs and anticipatory approach," *International Journal of Control*, vol. 73, no. 10, pp. 890-901, 2000.

[25] Y. Q. Chen and K. L. Moore, "An optimal design of PD-type iterative learning control with monotonic convergence," *Proc. of the 17th IEEE International Symposium on Intelligent Control*, pp. 548-553, Vancouver, British Columbia, Canada, October 2002.

[26] S. S. Saab, "Stochastic P-type/D-type iterative learning control algorithms," *International Journal of Control*, vol. 76, no. 2, pp. 139-148, 2003.

[27] Y. C. Huang, M. Chan, Y. P. Hsin, and C. C. Ko, "Use of PID and iterative learning controls on improving intra-oral hydraulic loading system of dental implants," *JSME International Journal, Series C*, vol. 46, no. 4, pp 1449-1455, 2003.

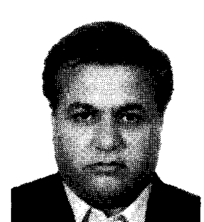
[28] J. Xu, L. Sun, and T. Cai, "An optimal design of open and close loop PID type ILC algorithm," *Proc. of the SPIE - The International Society for Optical Engineering, Fifth International Symposium on Instrumentation and Control Technology*, pp. 751-755, 2003.

[29] G. Ji and Q. Luo, "An open-closed-loop PID-type iterative learning control algorithm for uncertain time-delay systems," *Proc. of the International Conference on Machine Learning and Cybernetics*, pp. 1154-1159, 2005.

[30] C. J. Chien, "An adaptive PID-type iterative learning controller for nonlinear systems with non-repeatable control tasks," *Transactions of the Chinese Institute of Engineers, Series A*, vol. 29, no. 2, pp. 279-287, 2006.

[31] D. S. Bernstein, *Matrix Mathematics: Theory, Facts, and Formulas with Application to Linear Systems Theory*, Princeton University Press, 2005.

[32] R. C. Dorf and R. H. Bishop, *Modern Control Systems*, 10th edition, Prentice Hall, 2004.



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